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Theoretical Estimates of Heavy Quarkonium and B_c Mesons Mass Spectra in a Non-Relativistic Model

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Abstract: *In this paper, The radial Schrödinger equation solution is used to investigate the mass spectra of heavy quarkonia and B_c mesons for different states. For this an alternative non-relativistic potential model is applied. The Nikiforov-Uvarov method is used to solve Schrodinger equation with general non relativistic potential model. Its effective method to get the energy eigenvalues and eigen functions for heavy quarkonia. Based on the findings, our findings are consistent with those of other experimental and theoretical studies. The results are acceptable when compared to those of other experimental studies.*

Key words: *Mesons Mass Spectrum, Nikiforov-Uvarov Method, Schrödinger equation, Cornell Potential.*

1. Introduction:

Solving the Schrödinger equation with spherically symmetrical particles is one of the most difficult problems in quantum physics. The Cornell potential is one of these possibilities because it accurately describes the properties of quarkonium. Despite the fact that this potential has been used for a long time, the precise solution to the Schrödinger equation with this potential is unknown. To find analytical solutions to this equation, use a direct numerical solution of the boundary conditions given on the wave functions as well as various approximation methods. The Schrodinger equation [SE] solution plays an important role in high energy physics. Because the properties of heavy quarkonium, which consists of heavy quarks and antiquarks, are well described by the Schrödinger equation, the solution of this equation with a spherically symmetric potential is of great importance in describing quarkonia spectra [1]. The Cornell potential, a hold type potential, is widely used in modelling the interaction potentials of meson systems [2].

The Nikiforov - Uvarov method is used to obtain asymptotic expressions for the eigen functions and the eigenvalues of the Schrödinger equation for the chosen potential. This method can be used to calculate approximate analytical formulas for energy levels, which can be useful when qualitatively analyzing the spectrum of a model system. The mass spectrum of quarkonia and B_c mesons is calculated using solution's expressions.

2. Theoretical Nikiforov - Uvarov (NU) formalism:

The second order differential order form of SE is obtained by using Nikiforov - Uvarov method [3]

$$\Psi'' = \frac{\tilde{\tau}\,(r)}{\sigma(r)}\Psi'' + \frac{\tilde{\sigma}\,(r)}{\sigma^2(r)}\Psi = 0 \qquad (1)$$

Where polynomial σ(r) and $\tilde{\tau}(r)$ are of maximum second degree and τ̃ (r) is of first degree polynomial. To find the solution of equation (1) by separation of variables using the transformation

$$\Psi(r) = \Phi(r)y(r) \qquad (2)$$

The hyper geometric type of the form of eqn.(1) can be obtained due to transformation

$$\sigma(r)y'' + \tau(r)y' + \lambda y = 0 \qquad (3)$$

The value of function $\varphi(r)$ is written as in [4]

$$\frac{\varphi'(r)}{\varphi(r)} = \frac{\pi(r)}{\sigma(r)} \qquad (4)$$

Where

$$\tau(r) = \tilde{\tau}(r) + 2\pi(r) = 0 \qquad (5)$$

The following expressions are used to calculate the eigenvalue and eigen function of $\pi(r)$ and constant parameter λ.

$$\pi(x) = \frac{\sigma' - \tilde{\tau}}{2} \pm \sqrt{\left(\frac{\sigma' - \tilde{\tau}}{2}\right) - \tilde{\sigma} + K\sigma} \qquad (6)$$

and

$$\lambda_n = K + \pi'(r) \qquad (7)$$

In equation (6), the value of K is possible. if the expression in square root sets the discriminant to zero[3]. The new eigenvalue can take the form.

$$\lambda_n = -n\tau'(r) - \frac{n(n-1)}{2}\sigma''(r) = 0, \quad n = 0,1 \ldots \qquad (8)$$

3. The Schrodinger equation for the Potential:

For the two particle systems, we solve the SE for the wave function Via Potential $\boldsymbol{U}(\boldsymbol{r})$ in N=3 dimensional space [4]

$$\Psi''(r) = 2\mu_R\,[\varepsilon - \upsilon(r)]\Psi(r) = 0 \qquad (9)$$

Where μ_R is reduced of two meson systems and $\upsilon(r)$ is the interacting potential for mesons. The distance between particle is r . As variable in eqn.(1) is separable [5] for given potential. Setting the wave function$R_{nl}(x) = \chi_{nl}(r)$ as

$$\frac{1}{x^2}\frac{d}{dx}\left(x^2\ \frac{dR_{nl}(x)}{dx}\right)+\left[2\mu(\varepsilon-v(r)-\frac{l(l+1)}{r^2}\right]R_{nk}(x)=0 \qquad (10)$$

Where $\frac{d^2}{dr^2}\chi_{nk}(r)$=$\chi''_{nk}(r)$ and $V(r)$ is chosen potential of the for

$$V(r)=\alpha r-\frac{b}{r'}+\gamma r^2-S \qquad (11)$$

This potential has the following characteristics: the first term αr is a Confinement part for long distances, and the second $-\frac{b}{r'}$ is an Asymptotic part for short distances. Include the quadratic potential [6-7] term γr^2 in the potential, which plays an important role and S is arbitrary constant. To improve quarkonium properties, we extend the potential to include the quadratic potential [6-7] term αr in the potential, which has a significant impact. We obtain by substituting it in eqn.(10).

$$\frac{d^2\chi_{nk}(k)}{dx^2}=\frac{2\gamma}{x^2}+\ \frac{d\chi_{nk}(x)}{drx}+\frac{2\mu_R}{x^4}\left[\left(\varepsilon-\frac{\alpha}{x}+br-\frac{\gamma}{2\mu_R}r^2\right)\right]\chi_{nk}(x)=0 \qquad (12)$$

Where $\gamma=l(l+1)$ assume the term $\frac{\alpha}{x}$ for the approximation scheme and r_0 for the meson's characteristic radius. The scheme is based on the expansion $\frac{\alpha}{x}$ in power series around the characteristic radius r_0, i.e. around $\delta=\frac{1}{r_0}$ in r space limit to second order. If the dependent term $\frac{\alpha}{x}$ does not exist, the approximation is similar to the Pekeris-type approximation, which aids in deforming the centrifugal term. The improved potential can be solved using the NU method.

We expand $\frac{\alpha}{x}$ into a powers series around y = 0 by arranging it as $y=x-\delta\ where\ \ \delta=\frac{1}{r_0}$

$$\frac{\alpha}{x}=\frac{\alpha}{\delta\left(1+\frac{y}{\delta}\right)}\approx\frac{\alpha}{\delta}\left(1-\frac{y}{\delta}+\frac{y^2}{\delta^2}\right)$$

$$=\frac{\alpha}{\delta^3}(3\delta^2\ -3\delta x+x^2) \qquad (13)$$

Notice: Within the approximation, there is an additional model parameter called. If $\alpha=0$, it removes δ from the calculation, and the confining potential vanishes with it. Only the confining field problem remains after the approximation. We get equation by substituting it in (8).

$$\frac{d^2\chi_{nl}(x)}{dx^2}=\frac{2\gamma}{x^2}+\ \frac{d\chi_{nl}(x)}{dx}+\frac{2\mu_R}{x^4}\ (-a+cx-dx^2)\chi_{nl}(x)=0 \qquad (14)$$

Where, $a = -\mu_R\left(\varepsilon - \frac{3\alpha}{\delta}\right), c = \mu_R\left(b + \frac{3\alpha}{\delta^2}\right)$ $and\ d = \left(\frac{x}{2\mu_R} + \frac{\alpha}{\delta^3}\right)$ match eqn.(12) with eqn.(1), we have $\tilde{\tau} = 2x, \sigma = x^2$ $and\ \tilde{\sigma} = 2(-a + cr - dr^2)$, As a result, we can use the NU method in eqn.(14) to

$$U_{nk} = \frac{3\alpha}{\delta} - \frac{2\mu_R\left(b+\frac{3\alpha}{\delta^2}\right)}{\left[(2n+1\pm\sqrt{1+4l(l+1)+\frac{8\mu_R\alpha}{\delta^3}}\right]^2} \quad (15)$$

The wave function $\chi_{nk}(x)$ of equation (10) has the following form

$$\chi_{nk}(x) = Z_{nk}\ r^{\frac{c}{\sqrt{2a}}}\, e^{\frac{\sqrt{2a}}{x}}\ \frac{d^n}{dx^n}\left(\gamma^{2n} r^{\frac{c}{\sqrt{2a}}} e^{\frac{2\sqrt{2a}}{x}}\right) \quad (16)$$

The normalisation constant Z_n is used in this calculation. Its value is based on

$$\int |R_{nk}(r)|^2 dr = 1 \quad (17)$$

Using $\chi_{nk}(r) = rR_{nk}(x))$ and setting $x = \frac{1}{r}$ in (12) for n = 0,1,2,..... becomes

$$R_{nk} = Z_{nk}\ r^{\frac{c}{\sqrt{2a}}-1}\, e^{-\sqrt{2a}x} - (r^2\frac{d}{dr})^n \left(r^{-2n\frac{c}{\sqrt{2a}}}\, e^{-2\sqrt{2a}x}\right) \quad (18)$$

We compute the mass spectrum of meson systems with quark and antiquark flavours.
The relationship [4][8] is used.

$$M = 2m_{q\bar{q}} + U_{nk} \quad (19)$$

$$M = 2m_{q\bar{q}} + \frac{3\alpha}{\delta} - \frac{2\mu_R\left(b+\frac{3\alpha}{\delta^2}\right)}{\left[(2n+1\pm\sqrt{1+4l(l+1)+\frac{8\mu_R\alpha}{\delta^3}}\right]^2} \quad (20)$$

Where $m_q, m_{\bar{q}}$ are the masses of bottominium and charmonium mesons, respectively. For the states 1S to 5S, we calculate the masses of $b\bar{b}$, $c\bar{c}$ and B_c mesons and compare our calculated values with PDG and two other theoretical predictions as shown in tables. The masses of bottominium and charmonium mesons are represented by $m_q, m_{\bar{q}}$, respectively. The masses of $b\bar{b}$, $c\bar{c}$ and B_c mesons are calculated for the different states and compare our results to experimental values and two other theoretical researchers tabulated.

The Bottomonium system masses in GeV
($\delta = 0.360$GeV, a $= 0.2$GeV^2 b $= 1.560$, m_b= 4.713GeV)

n	Type	Present	Ref [9]	Ref [10]	Ref [11]	Expt.[12]
1	1S	9.461	9.428	9.398	9.390	9.398
2	2S	10.025	9.979	10.023	10.015	10.023
3	3S	10.356	10.359	10.355	10.383	10.355
4	4S	10.577	10.683	10.586	10.597	10.579
1	1P	9.798	9.806	9.859	9.864	9.859
2	2P	10.175	10.205	10.223	10.220	10.232
1	1D	9.997	10.075	10.161	10.153	10.163
2	2D	10.299	10.423	10.449	10.436	-

The Charmonium system masses in GeV
(a $= 0.2$GeV^2, $\delta = 0.240$GeV, m_c=1.220 GeV, b $= 1.215$)

n	Type	Present	Ref [13]	Ref [14]	Ref [15]	Expt.[16]
1	1S	3.098	3.096	3.096	3.078	3.097
2	2S	3.688	3.686	3.686	4.187	3.686
3	3S	4.042	3.984	4.040	5.297	4.039
4	4S	4.240	4.150	4.269	6.407	-
5	5S	4.427	-	-	-	-
1	1P	3.526	3.433	3.255	3.415	3.511
2	2P	3.770	3.910	3.779	4.143	3.927

The B_c meson systems masses in GeV (m_c=1.210 GeV
m_b=4.823GeV, $\delta = 0.230$GeV,a $= 0.3$GeV^2,b $= 1.245$)

n	Type	Present	Ref [9]	Ref [10]	Expt.[12]
1	1S	6.279	6.272	6.272	6.275
2	2S	6.867	6.864	6.842	6.275
3	3S	7.183	7.306	7.226	-
4	4S	7.370	7.684	7.585	-
1	1P	6.487	6.868	6.699	-
2	2P	6.974	7.146	7.094	-
3	3P	7.244	7.536	7.474	-
1	1D	6.773	6.990	7.029	-
2	2D	7.127	7.399	7.405	-
3	3D	7.338	7.761	7.750	-

4. Conclusions and Discussion:

In this work, the energy eigenvalues and wave functions in 3-dimensional space are investigated analytically by solving the Schrödinger equation (SE) using the NU method. The Cornell potential has been increased to include the quadratic potential, which accurately describes heavy mesons. The mass spectra of heavy mesons were calculated and compared to experimental data [12][16] as well as relevant others theoretical works

[9][10][11][13][14][15] that used different models, as shown in tables. The model has been noted to be an improvement. At N = 3, the masses of heavy mesons are computed. We found that the existing results are compatible with the experimental data. The results errors for the charmonium are 0.0302, 0.0361 for the bottomonium, and 0.005 for the Bc meson, which is better than other published works of different potential models.

5.References:

[1] W. Flokowski, Singapore world scientific,P.416(2010)

[2] E. Eichten, K. Gottfried, T. Kinoshita, J. Kogut, K. D. Lane, and T. -M. Yan. Spectrum of Charmed Quark-antiquark Bound States, Phys. Rev. Lett.,34,369-372(1975)

[3] A.F. Nikiforov and V.B. Uvarov, Special Functions of Mathematical Physics, Birkhauser, Basel (1998).

[4] S.M. Kuchin and N.V. Maksimenko, Univ. J. Phys. Appl. 7,295(2013)

[5] L. D. Landau and E. M. Lifshits, Quantum Mechanics (Nonrelativistic Theory) [in Russian], Fizmatgiz, Moscow, (1989).

[6] E. Eichten, T. Kinoshita, K. Gottfried, et al., Spectrum of Charmed Quark-Antiquark Bound States, Phys. Rev. Lett., 34, 369–372 (1975).

[7] R.N. Faustov, V.O. Galkin, A.V. Tatarintsev, A. S. Vshivtsev, Spectral problem of the radial Schrödinger equation with confining power potentials, Teor. Matem. Fiz.,. 113, 1530 – 1542,(1997)

[8] R. Kumar, F. Chand. Commun. Theor. Phys. 59(2013) 528

[9] Q ¯ Q (Q ∈ {b, c}) spectroscopy using Cornell potential, N. R. Soni,∗ B. R. Joshi,† R. P. Shah,‡ H. R. Chauhan,§ and J. N. Pandya, European Physical Journal C 78(7), (2017).

[10] D. Ebert, R. N. Faustov, and V. O. Galkin, Spectra of heavy mesons in the Bethe-Salpeter approach, Eur. Phys. J. C 71, 1825 (2011).

[11] C. S. Fischer, S. Kubrak, and R. Williams, Spectra of heavy mesons in the Bethe-Salpeter approach, Eur. Phys. J. A 51, 1409.5076, 10 (2015).

[12] J. Beringer et al. [Particle Data Group], Phys. Rev. D 86, 1(2012); Byungsik Hong, Overview of quarkonium production in heavy-ion collisions at LHC, published by EDP Sciences(2015).

[13] Ramesh Kumar and Fakir Chand, Series solutions to the N-dimensional radial Schrödinger equation for the quark–antiquark interaction potential, Phys. Scr., 85, 055008 (4pp), (2012).

[14] Ahmed Al-Jamel1 & Hatem Widyan, Heavy Quarkonium Mass Spectra in A Coulomb Field Plus Quadratic Potential Using Nikiforov-Uvarov Method, App. Phys. Res., 4, 94 – 99, (2012).

[15] M. Abu-Shandy, N-dimensional Schrodinger equation at finite temperatures using Nikiforov- Uvarov Method .J. Egypt. Math. Soci.23, 4(2016)

[16] K.A. Olive et al.(particle Data Group) Chin. Phys. C 38, 090001(2014)