Generation of Pythagorean triangle with Area/ Perimeter as a Wagstaff prime numbers

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Abstract - *In this paper, we express the ratio Area/Perimeter as a Wagstaff Prime number for Pythagorean triangle. Some of the interesting patterns and sides of a triangle are shown.*

Key Words: **Pythagorean Triangle, Wagstaff prime number**.

1.INTRODUCTION

Mathematical topics include arithmetic, number theory, formulas and associated structures, shapes and the spaces in which they are contained (geometry), quantities and their changes, formulas and related structures. The Greek word "Mathema," which means knowledge, study, and learning, is where the word "mathematics" originates. The area of pure mathematics known as number theory is extensively used to study integer and integer value functions. In the field of number theory, the concept of the Pythagorean triangle is a fascinating one. A wide variety of interesting problems can be found in [7, 12, 15]. In [4, 5], the authors related the Pythogorean triangle with polygonal numbers. Special types of numbers like Dhuruva numbrs, nasty numbers and Jarasandha numbers are available in [1, 2, 11, 13]. Pythogorean triangle with nasty number as a leg is discussed in [6]. Special pairs of Pythogorean triangle and rectangles with Jarasandha numbers are given in [8–10]. Common sided Pythagorean triples are discussed in [14] .In [3], connection between pythagorean triangle and dodecic number is explained. Also [16] deals with the relationship between the Pythagoren triangle and woodall primes in which the number of Pythagoren triangle generated by expressing Area/ Perimeter as woodall prime numbers.

In this paper, we express the ratio Area/ Perimeter of a Pythogorean triangle as a Wagstaff prime number and we observe some entralling results.

2. BASIC DEFINITIONS

Definition 1: Let (r, s, t) be a Pythagorean triple if it satisfies the Pythagorean equation $r^2 + s^2 = t^2$ where $r, s, t \in N$. A Pythagorean triangle contains the sides as a Pythagorean triple and itis denoted by $T(r, s, t) : r^2 + s^2 = t^2$ where r, s are legs and t is hypotenuse.

Definition 2: Most suitable solution of the Pythagoren equation is $r = m_1^2 - m_2^2$, $s = 2m_1m_2$ and $t = m_1^2 + m_2^2$, where $m_1, m_2 \in N$ such that $m_1 > m_2$. If m_1, m_2 are of opposite parity and $gcd(m_1, m_2) = 1$, then the solution is said to be primitive.

3. WAGSTAFF PRIME NUMBER

In Number Theory, Wagstaff Prime number is a prime number of the form (2p+1)/3, where p is an odd prime.

The prime pages attribute the naming of the Wagstaff primes, which are named after the mathematician Samuel S. Wagstaff Jr., to Fran¸cois Morain, who did so during a lecture at the Eurocrypt 1990 conference. The New Mersenne conjecture mentions Wagstaff primes, and they are used in cryptography.

Ryan Propper revealed the discovery of the Wagstaff probable prime 2021 which has slightly more than 4.5 million decimal digits.

4. METHOD OF ANALYSIS

The symbols A_1 and P_1 denotes the Area and Perimeter of a Pythagoren Triangle, respectively.

Assume

$$A_1/P_1 = \text{Wagstaff Prime number.}$$

This relationship results in the following equation

$$m_2(m_1-m_2)/2 = \text{Wagstaff Prime number.}$$

Case 1 :
When

$m_2(m_1-m_2)/2 = 3$ (one digit Wagstaff Prime number)

m_2	m_1-m_2	m_1	$r = m_1^2 - m_2^2$	$s = 2m_1m_2$	$t = m_1^2 + m_2^2$	A_1	P_1	A_1/P_1
1	6	7	48	14	50	336	112	3
2	3	5	21	20	29	210	70	3
3	2	5	16	30	34	240	80	3
6	1	7	13	84	85	546	182	3

Table 1

Case 2 :
When

$m_2(m_1-m_2)/2 = 11$ (two digit Wagstaff Prime number)

m_2	m_1-m_2	m_1	$r = m_1^2 - m_2^2$	$s = 2m_1m_2$	$t = m_1^2 + m_2^2$	A_1	P_1	A_1/P_1
1	22	23	528	46	530	12144	1104	11
2	11	13	165	52	173	4290	390	11
11	2	13	48	286	290	6864	624	11
22	1	23	45	1012	1013	22770	2070	11

Table 2

Case 3 :
When

$m_2(m_1-m_2)/2 = 43$ (two digit Wagstaff Prime number)

m_2	m_1-m_2	m_1	$r = m_1^2 - m_2^2$	$s = 2m_1m_2$	$t = m_1^2 + m_2^2$	A_1	P_1	A_1/P_1
1	86	87	7568	174	7570	658416	15312	43
2	43	45	2021	180	2029	181890	4230	43
43	2	45	176	3870	3874	340560	7920	43
86	1	87	173	14964	14965	1294386	30102	43

Table 3

Case 4 :
When

$m_2(m_1-m_2)/2 = 683$ (three digit Wagstaff Prime number)

m_2	m_1-m_2	m_1	$r = m_1^2 - m_2^2$	$s = 2m_1m_2$	$t = m_1^2 + m_2^2$	A_1	P_1	A_1/P_1
1	1366	1367	1868688	2734	1868690	2554496496	3740112	683
2	683	685	469221	2740	469229	642832770	941190	683
683	2	685	2736	935710	935714	1280051280	1874160	683
1366	1	1367	2733	3734644	3734645	5103391026	7472022	683

Table 4

Case 5 :
When

$m_2(m_1-m_2)/2 = 2731$ (four digit Wagstaff Prime number)

m_2	m_1-m_2	m_1	$r = m_1^2 - m_2^2$	$s= 2m_1m_2$	$t = m_1^2 + m_2^2$	A_1	P_1	A_1/P_1
1	5462	5463	29844368	10926	29844370	163039782384	59699664	2731
2	2731	2733	7469285	10932	7469293	40827111810	14949510	2731
2731	2	2733	10928	14927646	14927650	81564657744	29866224	2731
5462	1	5463	10925	59677812	59677813	325990048050	119366550	2731

Table 5

Case 6 :
When

$m_2(m_1-m_2)/2 = 43691$ (five digit Wagstaff Prime number)

m_2	m_1-m_2	m_1	$r = m_1^2 - m_2^2$	$s= 2m_1m_2$	$t = m_1^2 + m_2^2$	A_1	P_1	A_1/P_1
1	87382	87383	7635788688	174766	7635788690	667238122923504	15271752144	43691
2	43691	43693	1909078245	174772	1909078253	166826711517570	3818331270	43691
43691	2	43693	174768	3817981726	3817981730	333630515144784	7636138224	43691
87382	1	87383	174765	15271402612	15271402613	1334453338743090	30542979990	43691

Table 6

Case 7 :
When

$m_2(m_1-m_2)/2 = 174763$ (six digit Wagstaff Prime number)

m_2	m_1-m_2	m_1	$r = m_1^2 - m_2^2$	$s= 2m_1m_2$	$t = m_1^2 + m_2^2$	A_1	P_1	A_1/P_1
1	349526	349527	122169123728	699054	122169123730	42701407309276700	244338946512	174763
2	174763	174765	30542805221	699060	30542805229	10675626708896100	61086309510	174763
174763	2	174765	699056	61084911390	61084911394	21350886908323900	122170521840	174763
349526	1	349527	699053	244337548404	244337548405	85402448112230700	488675795862	174763

Table 7

5. OBSERVATIONS

1. There are 4 Pythogorean triangle for all the above 7 cases, out of which 2 triangles areprimitive and the remaining 2 triangles are non- primitive.

2. ¼ (r+s-t) is a wagstaff prime number.

3. Out of 4 triangles in all the cases, $t - r$ is a even prime for one non-primitive triangle and cubic number for primitive triangle.

4. For the Wagstaff prime number 3, $2(r + s - t)$ = Nasty number.

5. In all the cases, the sides s, t are consecutive for one of the primitive triangles.

6. s+t , 2(t-r) are perfect squares.

6. CONCLUSION

In this work, generation of Pythagoren Triangles with Area/Perimeter as a Wagstaff prime number and entralling observations are shown. Further, One may find the PythagorenTriangles for any other number pattern.

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