Design and Analysis of Induction Motor thermal Model for Numerical Protection

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Abstract - *As it employs high speed DSP algorithms in microprocessor based relays, present day numerical technology is able to perform much more protection functions in a more reliable and effective manner as compared to its earlier counterparts. The paper discusses one such advancement that became possible in the field of motor protection. IEEE Standard C37.96-2000, Guide for AC Motor Protection [11], recommends the use of overcurrent relays for overload and locked rotor protection. But the simple dynamics of an over current relay cannot provide adequate thermal protection for a motor. In the paper the author derives, the thermal model from the thermal limit curves of the motor, which can be used in the relay algorithm to calculate the temperature rise on real time. The relay can be programmed to initiate a trip command, when the heating reaches the threshold. Correlation between the temperature and current has been established, for a 280KW, 6600V motor.*

Key Words: **Numerical Protection, Thremal model, Thermal limit curves, adiabatic process, locked rotor.**

1.INTRODUCTION

The simple dynamics of an over current relay cannot provide adequate thermal protection for a motor. The thermal limit curves are the characteristics of thermal models that enable microprocessor relays to continuously calculate and monitor motor temperature in real time. Employing thermal limit curves of thermal protection instead of over current relays is one way of optimizing motor protection, because of the varying thermal effects during starting and running of an induction motor.

II.THERMAL MODEL FUNDAMENTALS

The first order thermal model of an induction motor derived considering motor as a heated homogeneous body following non-adiabatic principle of thermal dynamics is given below. Let the winding temperature rise, θ above the ambient be written as,

$$\theta = \theta_W - \theta_A \text{................ (1)}$$

where , θ is defined as the winding temperature rise

θ_W above ambient temperature θ_A.

The rate of increase of the temperature is given by the equation expressing the thermal equilibrium condition as,

$$\text{Power Supplied – losses} = C_s m \frac{d\,\theta_W}{dt} = C_s m \frac{d\,\theta}{dt} \quad \text{.......(2)}$$

Cs – Specific heat of the winding

m – Mass of the winding

$$\text{Power supplied} = I^2 r \text{...................(3)}$$

And,

$$\text{Losses} = \frac{\theta_W - \theta_A}{R} = \frac{\theta}{R} \text{..................... (4)}$$

where R is the thermal resistance in °C/Watt. Now the equation no.2may be rewritten as,

$$I^2 r - \frac{\theta}{R} = C_s m \frac{d\theta}{dt} \text{...........................(5)}$$

(Or)

$$I^2 r.R = C_s m.R \frac{d\theta}{dt} + \theta \;\ldots\ldots\ldots\ldots..(6)$$

It can be arranged and written for current as:

$$I^2 = C_s m.R.\left(\frac{1}{r.R}.\frac{d\,\theta}{dt}\right) + \frac{\theta}{r.R} \text{...................(7)}$$

The mass m multiplied by the specific heat Cs is known as C, the thermal capacity of the system with units of joules/°C. It represents the amount of energy in joules required to raise the system temperature by one degree centigrade. The product of the thermal resistance R and the thermal capacitance C has units of seconds and represents the thermal time constant.

$$\tau = R.m.Cs = R.C \text{................ (8)}$$

The solution in the time domain for the temperature, as a function of time and current is:

$$\theta = I^2 r.R\left[1 - e^{\frac{-t}{\tau}}\right] \text{.................. (9)}$$

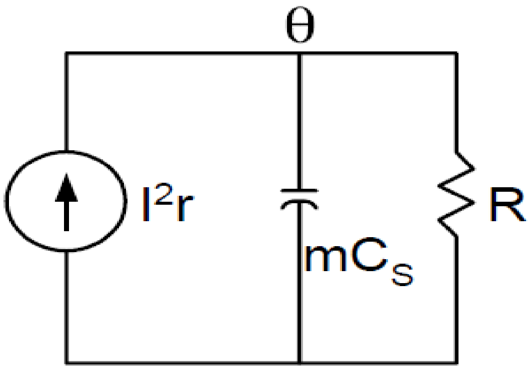
As θ is the winding temperature above the ambient,

$$\theta_W(t) = I^2 r.R\left[1 - e^{-\frac{t}{\tau}}\right] + \theta_A \text{............... (10)}$$

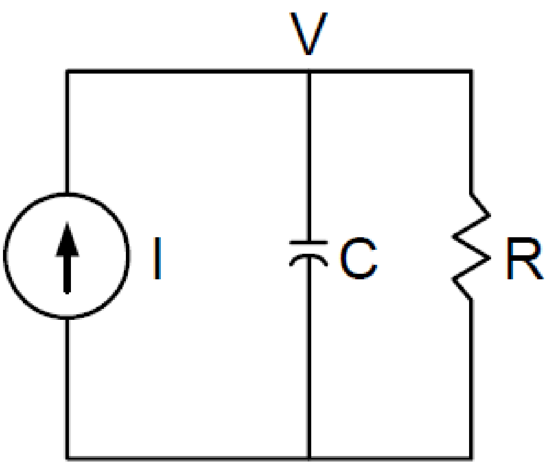
The final steady state temperature of the winding for a constant current I may be written as:

$$\theta_W(0 \to \infty) = I^2 r.R + \theta_A \text{..................... (11)}$$

Equation no.6 is a first order differential equation and is analogous to a RC parallel circuit supplied by a current source. The power supplied to the motor in the thermal process is equivalent to the current source supplying the RC circuit. The temperature in the thermal process is equivalent to the voltage across the capacitor in the RC circuit. The equivalence between the two systems is shown in Figure1.



(a) Load Current



(b) RC Equivalent

Fig. 1 Equivalence between the Thermal model and a Parallel RC Circuit

Table.1-Electrical Equivalence of thermal model

Thermal model	Parallel RC Circuit
$I^2 r.R = C_s m\frac{d\theta}{dt} + \frac{\theta}{R}$	$I = C\frac{dV}{dt} + \frac{V}{R}$
$\theta(t) = I^2 r.R\left[1 - e^{-\frac{t}{\tau}}\right]$	$V = I.R\left[1 - e^{-\frac{t}{\tau}}\right]$

III. EQUIVALENT TIME- CURRENT CURVE

If the winding temperature must not go beyond a maximum temperature θ_{max} then the equation with the time as a variable is:

$$\theta_{max} = I^2 r.R\left[1 - e^{-\frac{t}{\tau}}\right] + \theta_A \text{................ (12)}$$

Solving for t gives the time-current equation:

$$t = \tau \ln\left[\frac{I^2 r.R}{I^2 r.R - (\theta_{max} - \theta_A)}\right] \text{..................... (13)}$$

Let current Imax is the maximum current that can be supplied to the motor without the winding reaching maximum temperature as time goes to infinity. This maximum current would have to satisfy Equation 11 as in:

$$\theta_{max} = I_{max}^2 r.R + \theta_A \text{............. (14)}$$

(or)

$$I_{max}^2 r.R = \theta_{max} - \theta_A \text{.................... (15)}$$

Substituting θ_{max}-.θ_A in Equation no.13 gives the equation:

$$t = \tau \ln\left[\frac{I^2 r.R}{I^2 r.R - I_{max}^2 r.R}\right] = \tau \ln\left[\frac{I^2}{I^2 - I_{max}^2}\right] \text{..... (16)}$$

In Equation no.16, shows the total time to reach the hot-spot temperature as a function of the current. Equation no.16 is also remarkable because it replaces all the temperature constants with the maximum current I_{max}.

It should be noted that Equation no.16 has no solution unless:

I > Imax......... (17)

Any current less than I_{max} will raise the motor temperature to a steady temperature given by equation no.14. This temperature will be represented by a capacitor voltage in the equivalent circuit of Figure 1.

The steady temperature of the winding above the ambient for any operating current, starting from ambient may be written as:

$$\theta_{op} = I_{op}^2 r.R + \theta_A \text{ .... (18)}$$

The time response equation for the temperature rise is:

$$\theta_{op} = I^2 r.R\left[1 - e^{-\frac{t_{op}}{\tau}}\right] + \theta_A \text{ .... (19)}$$

Time for the temperature rise is:

$$t_{op} = \tau\, ln\left[\frac{I^2 r.R}{I^2 r.R - (\theta_{op} - \theta_A)}\right] \text{........ (20)}$$

As per equation no.18, t_{op} is:

$$t_{op} = \tau\, ln\left[\frac{I^2 r.R}{I^2 r.R - I_{op}^2 r.R}\right] = \tau\, ln\left[\frac{I^2}{I^2 - I_{op}^2}\right] \text{.......(21)}$$

Finally, the time for a current of magnitude I to heat the motor to its limiting value from an operating current or temperature (motor hot start) is provided by:

$$t = \tau\, ln\left[\frac{I^2}{I^2 - I_{max}^2}\right] - \tau\, ln\left[\frac{I^2}{I^2 - I_{op}^2}\right] = \tau\, ln\left[\frac{I^2 - I_{op}^2}{I^2 - I_{max}^2}\right] \text{.... (22)}$$

Finally when normalizing the current with respect to the operating current, the trip time of the relay will be:

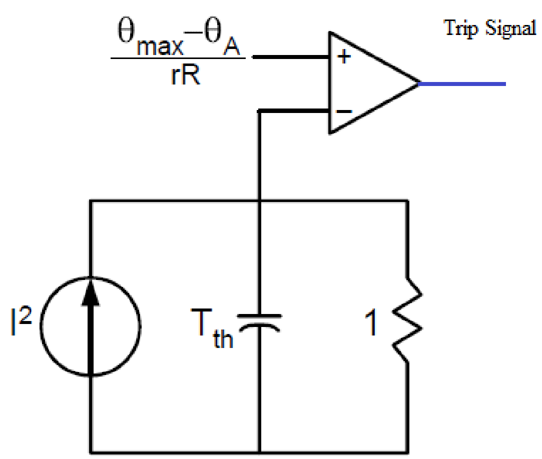
$$t = \tau\, ln\left[\frac{I_{pu}^2 - 1}{I_{pu}^2 - SF^2}\right] \text{................. (23)}$$

Where:

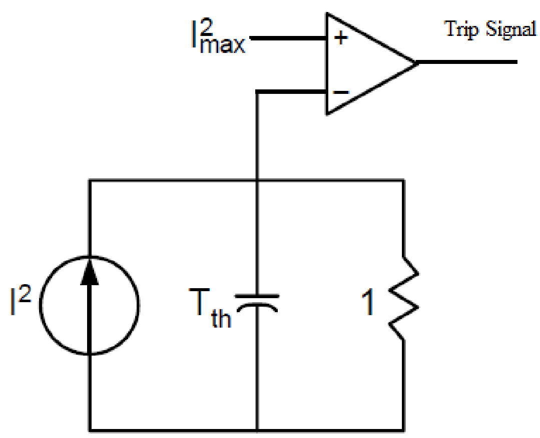
$$SF = \frac{I_{max}}{I_{op}} \text{ .... (24)}$$

Service factor ,SF is the percentage of overloading the motor can handle for short periods when operating normally within the correct voltage tolerances.

Figure 2 shows the thermal model with temperature and current as the tripping threshold.



(a) Temperature as tripping threshold



(b) Current as Tripping Threshold

Fig.2 Thermal model with Temperature and Current as Tripping Threshold

Let the pair of points (I_0, T_0) and (I_1, T_1) represent the co-ordinates on the time-current curve, corresponding to the maximum constant temperature, then:

$$I_1^2 r.R\left[1 - e^{-\frac{t_1}{\tau}}\right] = I_0^2 r.R\left[1 - e^{-\frac{t_0}{\tau}}\right] \text{.......... (25)}$$

It may also be written as:

$$\frac{\left[1 - e^{-\frac{t_1}{\tau}}\right]}{\left[1 - e^{-\frac{t_0}{\tau}}\right]} = \frac{I_0}{I_1} \text{ .... (26)}$$

III. CURVE DESIGN

By definition, the thermal limit curves give the time for current exceeding the service factor to raise the initial load temperature to an overload temperature that requires the motor to be disconnected. Each curve is a plot of specific limiting temperature. The relay thermal protection is modeled as per these motor curves. Fig.3 shows thermal curves for a 280 KW, 6600V cage rotor induction motor. The manufacturer has specified the initial temperature for a set of 'hot' and 'cold' overload and locked rotor curves.

As per IEC255, (Equation no.21), the general equation for trip time for hot and cold conditions in terms of per unit values are:

$$t_{H-curve} = \tau \ln \left[\frac{(I)^2 - (I_H)^2}{(I)^2 - (I_{SF})^2} \right] \text{............ (27)}$$

$$t_{C-curve} = \tau \ln \left[\frac{(I)^2 - (I_C)^2}{(I)^2 - (I_{SF})^2} \right] \ldots\ldots\ldots\ldots\ldots\ldots (28)$$

Where,

I = Motor current in per unit of full load.

I_{SF} = Current at the service factor

I_H = Current that raised the temperature to a predetermined final value with a hot initial condition.

I_C = Current that raised the temperature to a predetermined final value starting from ambient.

The fit is obtained by setting the threshold which equals the square of the service factor (SF^2). The equation for SF^2 is:

$$I^2 \left[1 - e^{-\frac{t_{H-curve}}{\tau}} \right] + I_H^2 . e^{\frac{-t_{H-curve}}{\tau}} = SF^2 \text{ ..... (29)}$$

(Or)

$$I^2 \left[1 - e^{-\frac{t_{C-curve}}{\tau}} \right] + I_C^2 . e^{\frac{-t_{C-curve}}{\tau}} = SF^2 \text{..... (30)}$$

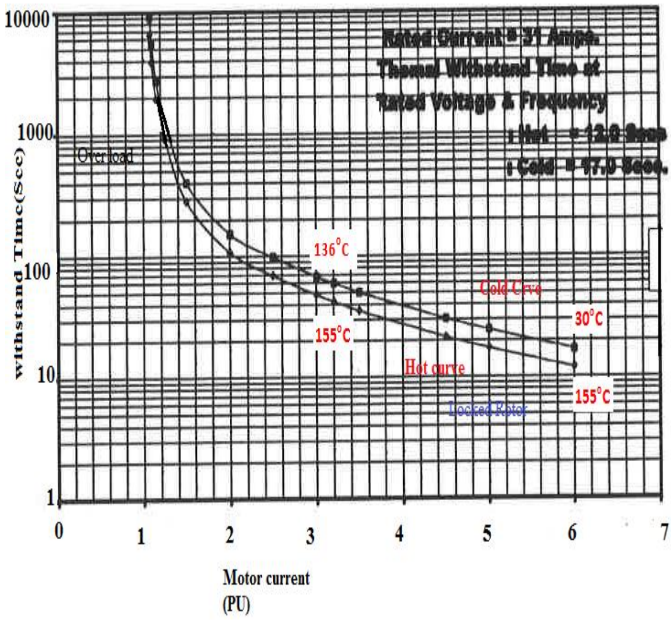


Fig.3 Thermal Limit Curves

If the curves obey a first order thermal process, we will be able to choose τ, I_H, and I_C so that equations fit the curves. Also, I_H and I_C must be in the ratio of the initial temperatures above ambient. A unique solution obtained with a basic assumption is:

$$\frac{I_H^2}{I_C^2} = \frac{Tem_{(H-initial)} - Tem_{(C-initial)}}{Tem_{(C-final)} - Tem_{(C-initial)}} \text{.............. (31)}$$

For the given curves, equation (31) will become:

$$\frac{I_H^2}{I_C^2} = \frac{155-30}{136-30} = \frac{125}{106} = 1.1792 \text{ .... (32)}$$

i.e, $I_H^2 = 1.1792\, I_C^2$.................... (33)

IV. CURVE FITTING PROCEDURE

1. Choose a current and read the corresponding time points from the hot (155°C) and the cold (100°C) overload curves in Figure 3. Enter the current and time values in Equations 29 and 30. For example, at 3per-unit current, the hot and cold times are $t_{H\text{-}CURVE}$ = 50 seconds and $t_{C\text{-}CURVE}$ = 65 seconds, respectively.

2. Choose τ and I_H so that equations 29 and 30 are satisfied. It is seen that Equations are satisfied for, τ = 1000 and I_H = 0.9 Pu.

3. Similarly the other points on the graph can also be checked.

For eg.:

$$3^2\left[1-e^{\frac{-50}{1000}}\right]+0.9^2.e^{\frac{-50}{1000}} = 1.1^2 = 1.21 \ldots\ldots\ldots (34)$$

(Or)

$$3^2\left[1-e^{\frac{-65}{1000}}\right]+\frac{0.9^2}{1.179}.e^{\frac{-65}{1000}} = 1.1^2 = 1.21 \ldots\ldots (35)$$

Table :1 thermal limit check points

I(pu)	$t_{H\text{-}curve}$	$t_{C\text{-}curve}$	T_{th}	I_H^2(pu)	I_C^2(pu)	I_{SF}(pu)
3.0	50	65	1000	0.81	0.687	1.10
2.0	100	170	1000	0.81	0.687	1.10
4.0	28	40	1000	0.81	0.687	1.10

When the unique values for τ, I_H and I_C are used, equations 27 and 28 may be used to calculate any curve point with precision.

$$t_{H-curve} = 1000\ ln\left[\frac{(I)^2-(0.81)^2}{(I)^2-(1.1)^2}\right] \ldots\ldots\ldots\ldots\ldots (36)$$

$$t_{C-curve} = 1000\ ln\left[\frac{(I)^2-(0.687)^2}{(I)^2-(1.1)^2}\right] \ldots\ldots\ldots\ldots (37)$$

The above equations are the solutions of a first order differential equation mentioned in equation no.6.

IV. CORRELATION BETWEEN CURRENT AND TEMPERATURS

According to equations 36 and 37 the squares of the initial currents for hot and cold conditions are 0.81pu and 0.687 pu respectively, which can be taken as the per unit temperatures in thermal model. While relating temperature and currents between hot and cold conditions, the equation obtained is,

$$\frac{\theta}{I^2} = \frac{(155-30)-(136-30)}{0.81-0.687} = 154.47 \ldots\ldots\ldots (38)$$

Where θ is the motor temperature above the ambient and I is the per unit current. The absolute steady state temperature Θ can be written as,

$$\theta = \frac{(155-30)-(136-30)}{0.81-0.687} I^2 + 30 = 154.47\ I^2 + 30 .. (39)$$

Tabulated values of Current and temperature is given in table 2.

Table. 2 Correlation of Current and Temperature

Motor current (pu)	Temperature
1.1	217^0C
0.9	155^0C
0.7	105^0C
0	30^0C

V. CONCLUSION

This paper, introduces the basic principles and mathematical equations found in thermal protection modeling of induction motor. The modeling has been done with respect to the 'hot' and 'cold' thermal limit curves available in the data sheet of the motor. Checkpoints have been tabulated according to a 280KW, 6600V squirrel cage induction motor. The correlation between temperature and current has been established, which proves that all the temperature dependent variables can be replaced by current, making the analysis much easier for an electrical engineer.

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