Investigation of Stress Concentrations in Laser-Cut Polymethylmethacrylate (PMMA) Material under Static and Centripetal Loading

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Abstract – *Stress concentrations play an important role in determining the strength, durability and functionality of structural components. A number of studies have looked at how resultant stress concentrations of laser cutting different polymeric materials can have a detrimental effect on finished parts in terms of their functionality and durability. Recently student projects at the University experienced issues with uncertainty in stress concentrations from laser cutting of polymeric materials. This study looks at conventional tensile testing to evaluate polymers such as PMMA and additive manufacturing materials subjected to tensile testing, varying the presence of fillet radii on section changes, and compare the findings to finite element analysis (FEA) models. In addition, parts will be used in dynamic testing to compare the effects of dynamic loading on the same specimens to that of standard tensile testing. The initial testing will avoid the effects of impulse on specimens but allow comparisons to be drawn between standard testing techniques, dynamic testing using a motor hub and FEA results, which will help guide future student projects.*

Key Words: Laser cutting, stress concentration, tensile strength, PMMA.

1. INTRODUCTION

"Strength of Materials" is a core subject taught on a variety of engineering degree courses. Students are introduced to the concepts of stress concentrations and fracture mechanics. Texts studied in this area are based on empirical testing and results based on photo-elastic stress for situations such as the analysis of gear teeth. Laser cutting is a popular technology used in a range of fabrication/manufacturing environments. Polymethylmethacrylate (PMMA) is a popular choice in laser cutting due to the excellent finish that can be achieved from the laser cutting process. There have been a number of recent articles on optimizing laser cutting parameters, including kerf taper, maintaining kerf width when laser cutting PMMA by Elsheikh, Deng and Showaib (2020) and Khoshaim et al (2021). However, the links between these areas of study and student practical-based project work were lacking.

A recent manufacturing project required students to laser cut polymeric blades. A range of different polymer sheet materials were available. This formed the basis of the research. Given the extensive use of laser cutting at Solent University, UK, the authors decided to investigate this further by conducting a study into the levels of stress concentration in PMMA laser cut test specimens and in addition to this a flat fan blade shape that could be subjected to centripetal load testing.

2. RESEARCH METHDOLOGY

2.1 Tensile Testing Considerations

In order to gauge the levels of stress concentration factors in static testing, samples of ASTM D638 tensile specimens (Figure 1) were laser-cut and tested to establish the tensile strength for this particular grade of PMMA sheet. By keeping these consistent and setting to the normal values required, a comparison can be made. The laser cut specimens were produced on the Hobarts Super Speed 660 laser cutter, from Hobart Lasers Ltd, a UK-based company; typical of the type used at universities and schools. By modifying the tensile test piece design, as shown in Figure 2, a series of stepped flat tension bars with varying shoulder fillet radii were created. Figure 3 shows the main parameters used in testing that help determine the stress concentration factors. The results could then be plotted on a stress concentration chart, as shown in Figure 4.

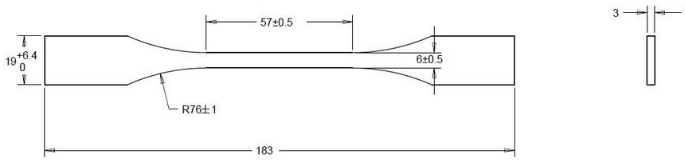

Figure 1. ASTM D638 Tensile Test Specimen

Specimens were weighed to determine density before tensile testing. The relatively low densities correlated with the lower-than-average tensile strengths for the ASTM D638 (around 50 MPa). These specimens all fracture close to the center of the specimen. The loading rate was set to 6 mm per minute. Stepped shoulder specimen lengths kept to ASTM length values.

The following laser-cutter settings were applied: power was set to 100%, cutting speed to 2% of full range and 1000 pulses per minute. For tensile testing, the loading rate was set to 6 mm per minute. Care was taken to ensure alignment of the specimen and no pre-stressing had taken place on clamping the specimen into the jaws, and that slippage was not occurring during the testing cycles. Load-extension graphs were produced for each specimen. Fractured specimens were examined; all ASTM D638 specimens fractured around the center of the specimen. Densities were checked and these correlated well to the average strength for this grade of cast polymer sheet.

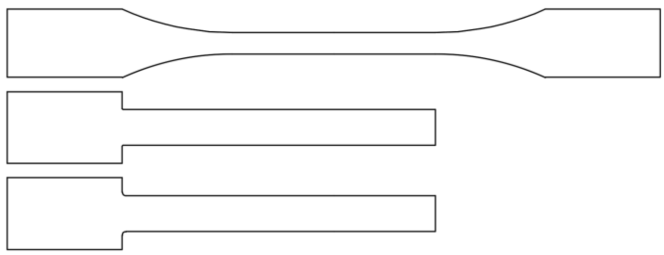

Figure 2. Specimens (from top to bottom: ASTMD638, 0.25 mm and 2 mm shoulder fillet radii on stepped flat tension bars with H/d ratio of 2).

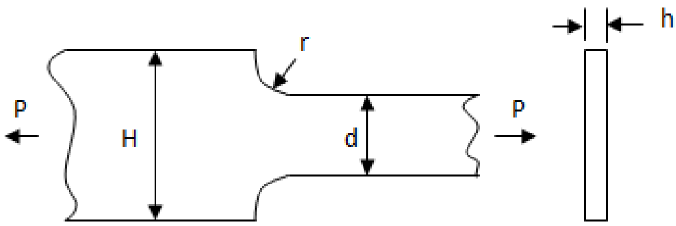

Figure 3. Diagram of a stepped flat tension bar showing associated parameters.

20 specimens per batch were tested and results computed in a spreadsheet. Due to the physical size of the tensile testing equipment, the range of H/d ratios was limited to 2 and 1.1, although a good range of r/d ratios were possible. Finite Element Analysis (FEA) was conducted using SolidWorks. This was used to predict fracture of the specimen under the loading conditions specified in the tensile testing.

2.2 Centripetal Testing Considerations

In addition to the tensile testing, centripetal testing was also conducted. This was selected based on first-year undergraduate student project workshop activities, with the aim to evaluate the level of stress concentration in the material. Although PMMA is a poor choice for an application such as a fan blade, the application suited the testing regime for investigating brittle fracture under centripetal loading conditions. Blades (Figure 4) were cut and using the dimensions of the blade spigots, holes were drilled into Delrin hubs which allowed for a slight interference fit. A hot glue gun was then used to apply a very small amount of glue and the blades were inserted with a small amount of adhesive. Acetal (Tufnel) bosses were used. These were mounted on to aluminium bosses.

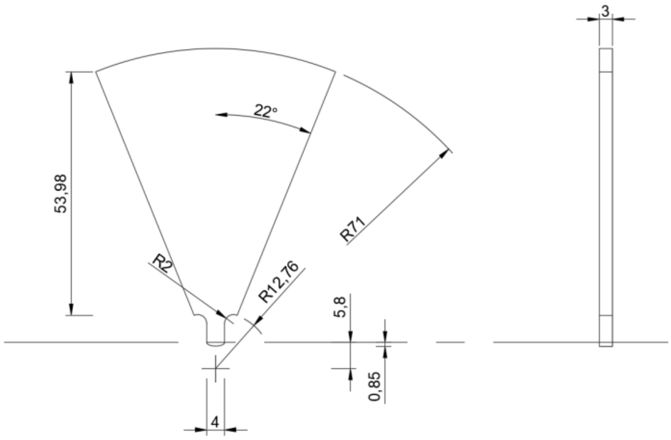

Figure 4. Centripetal Blade Specimen

The fan blade assembly consisted of two blades, 180° apart. Care was taken to ensure that the blades were flat without any attack angle. A variable power supply was connected to a motor and the voltage slowly increased in small increments. The power was slowly increased (to avoid impulse effects). A rev counter recorded the fracture speed of either 1 or 2 of the 2-blade system. The set up of the motor/fan blade assembly can be seen in Figure 5. The rotational speed of the motor was displayed by using a rev counter connected to the display module. The speeds at which one or both blades fractured due to excessive centripetal force were recorded. Regardless of the number of fractured blades, a new blade assembly then replaced the previous one for the next experiment. All blades were discarded before the next test. Four assemblies were tested for a range of spigot fillet radii. The results were then compared to FEA analysis, taking into account the angular acceleration and operating under centripetal loading.

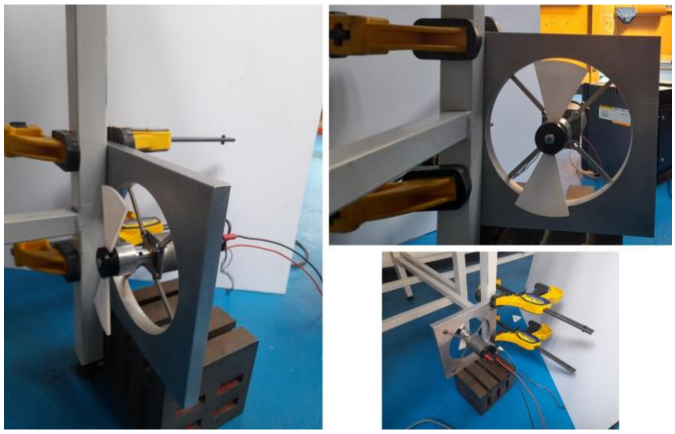

Figure 5. The set-up of the fan blade assembly for centripetal force testing.

3. ANALYSIS OF RESULTS

3.1 Tensile Test Theory

All ASTM D638 specimens fractured in the center and all stepped flat tension bars failed in a brittle manner at the shoulders. In order to generate the stress concentration factors the following equation is used:

$$K_t = \frac{\sigma_{\max}}{\sigma_{nom}}$$

The stress concentration factors were determined using a spreadsheet (Table 1).

Table 1. Tensile Results Example

Specimen Identifier	Width (mm)	Thickness (mm)	Max Load (N)	Stress (MPa)	K-Factor
1_i_01	9.78	2.96	580	20.03537	2.465208
1_i_02	9.86	2.97	580	19.8059	2.49377
1_i_03	9.76	2.97	580	20.00883	2.468478
1_i_04	9.72	2.98	530	18.29757	2.699341
1_i_05	9.65	3.02	660	22.64695	2.180928
1_i_06	9.66	3.02	690	23.65184	2.088267
1_i_07	9.87	2.97	780	26.60854	1.856223
1_i_08	9.78	2.96	420	14.50837	3.404335

3.3 Tensile Test Results

The brittle nature of PMMA and the way in which it fractures presents challenges for statistical analysis. Another approach was to look at the logarithmic plots that were used and the deviation of results from the line-of-best-fit. With the exception of the r/d ratio of 0.075, the other results showed good linearity on the logarithmic plot. The laser kerf was measured as 0.15 mm during this study and previous work on this particular laser cutter when cutting PMMA sheet 3 – 4 mm thick. The H/d ratio of 1.1 could only be applied to two specimen types, due to the constraints of the physical size of the machine jaws. This does not give enough data to plot a curve like the one shown in Figure 6 but revealed that for the r/d ratio of 0.025, the H/d ratio of 1.1 is exactly half of the H/d ratio of 2. For the r/d ratio of 0.05 the H/d ratio of 1.1 is 60% of that of the H/d ratio of 2. Given the lower densities of around 1170 kg/m3 (and corresponding tensile strengths from the standard tensile specimens of 50 MPa) results are only included for the grade of PMMA tested.

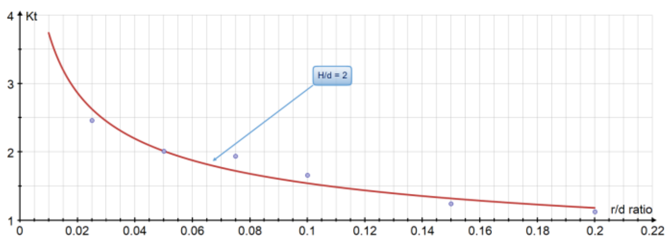

Figure 7. Stress concentration factor (Kt) chart for H/d ratio of 2 on the PMMA specimens.

The generated curve shows a much shallower profile to the existing stress concentration charts available in the literature as shown in figure 8.

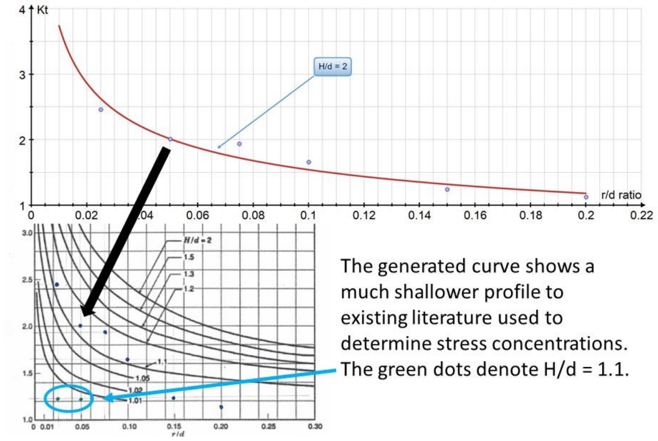

Figure 8. Stress concentration factor for H/d = 2 and H/d =1.1

In terms of structural strength and elongation, the specimen showed same result of load versus overall extension as of predicted from literature (Figure 9).

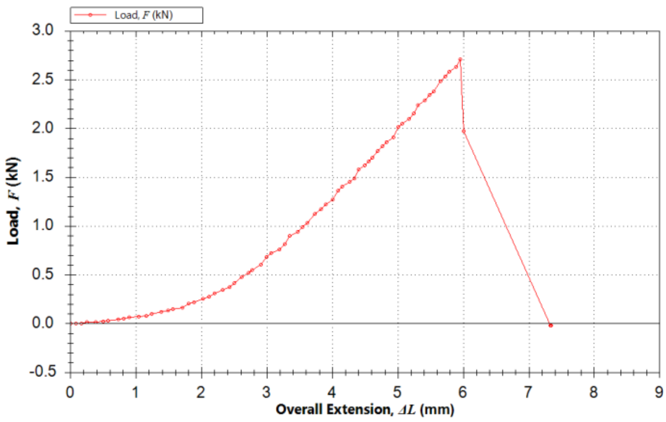

Figure 9. Load vs Extension Graph of Stepped Shoulder Specimen

One area that could be explored is the notch sensitivity, as literature is quite limited when considering PMMA. This would require a lot more testing with variable orientations of the PMMA and different grades. Previous research by Zheng, Wang and Yan (2003) has studied PMMA extensively for notch sensitivity. Other authors, such as Santos et al (2016) have modelled combined discontinuities in plates, but these are based on metallic plates with very high loading. In this paper tensile testing is conducted and compared to the FEA evaluation for both tensile stepped flat bars and rotating blades. When considering the fracture of the tension stepped bars, a comparison can be made to well established literature, such as Peterson's Stress Concentration Factors by Pilkey (2020). The findings of this research show that stress at fracture for PMMA tests results in lower stress concentration factors than those quoted for some of the H/d ratios, but this is based on a material that was not used in the early empirical testing. This reinforces the need for FEA validation.

3.4 Centripetal Test Results

On some occasions both fan blades fractured at the same rotational speed and other runs just one blade fractured (the hub was then discarded and replaced with a new hub and blade assembly). Due to the geometry of the blade, there are no charts to compare these to, in the same way the stepped tension bars can be compared to a limited extent. The fan blade testing was much more limited, as the set-up time was quite long. After collecting results from the first runs of blades, the fracture occurred at speeds between 3562 rev per min and 4806 rev per min. It was clear that the variance was likely to be high (due to the variables e.g., differences in interference fit, the levels in angular acceleration at the point of fracture). Unlike the tensile testing, establishing a clear relationship between the fillet radii and the stress concentration factor was not possible with the existing set up. Part of this was also down to the fact of the shape of the blade: the overall stress concentration due to the change in section was very large in comparison to the variation in fillet radii. It is not surprising that fracture speeds varied considerably so the focus of this part of the research is on the comparison to FEA simulation.

3.5. FEA Simulation based Results Evaluation for Tensile Testing.

Finite Element Analysis (FEA) conducted on the tensile specimens shows a good comparison across all r/d ratios up as shown in table 2.

Table 2. Yield Load from FEA

Specimen Identifier	h (mm)	H (mm)	d (mm)	r (mm)	H/d	r/d	Yield Load from FEA (N)
1_i	2.85	20	10	0.25	2	0.025	428
1_ii	2.85	20	10	0.5	2	0.05	545
1_iii	2.85	20	10	0.75	2	0.075	689
1_iv	2.85	20	10	1	2	0.1	651
1_v	2.85	20	10	1.5	2	0.15	727
1_vi	2.85	20	10	2	2	0.2	810
2_i	2.85	13.2	12	0.3	1.1	0.025	706
2_ii	2.85	22	20	1	1.1	0.05	1091
2_iii	2.85	24.2	22	1.65	1.1	0.075	1202
2_iv	2.85	24.75	22.5	2.25	1.1	0.1	1177

There was around 100 N difference when looking at averages, however due to the variance of the brittle polymer this a lot lower and in all but one of the ratios, the actual testing reached higher levels than the FEA simulation for both H/d ratio = 2 (Figure 10) and H/d ratio= 1.1 (Figure 11)

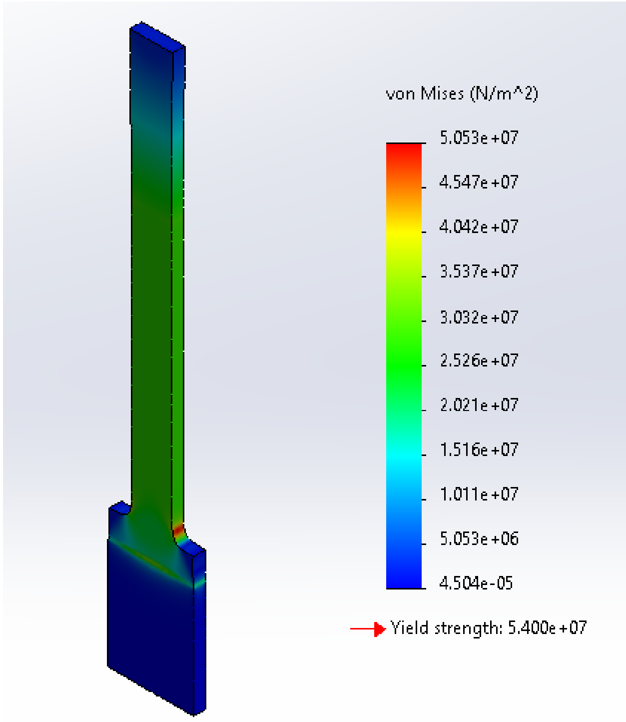

Figure 10. FEA stress model of a H/d ratio 2 in a 2mm fillet radius stepped flat tension bar.

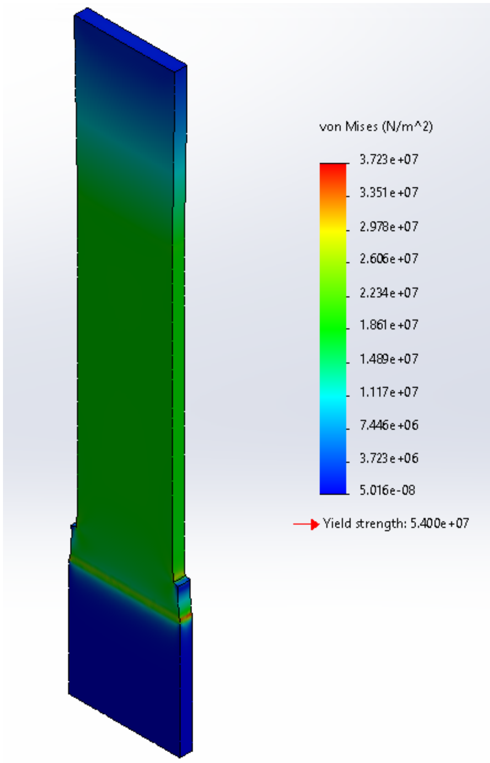

Figure 11. FEA stress model of a H/d ratio 1.1 in a 2.25mm fillet radius stepped flat tension bar.

Due to the brittle nature of the PMMA, Mohr's failure criterion was applied as a basis for the FEA. Whilst there are papers on stress concentration analysis, some are purely based on FEA whilst others include substantial testing. Some studies have looked at reducing the stress concentration in ASTM D638 specimens, such as composites with a Nylon 11 matrix containing Nd-Fe-B particulates by Garrell et al (2003) although these are based on von mises stresses due to the more ductile behavior of the nylon matrix. In this study the FEA has shown good reliability in the material meeting the simulated fracture load; increasing the sample size would be beneficial although given the limited range of sizes the specimens there is uncertainty for other sizes.

3.6. FEA Simulation based Results Evaluation for Centripetal Testing.

Simulation results for the fan blades using a 0.5 mm fillet radius based on the effects of centripetal force and the relatively small angular acceleration (due to the increasing voltage supplied to the motor) predicted fracture at 6250 rev per min. This at 62% of the actual fracture point. However, since few blades were evaluated and due to the large variance expected, more work is needed in this area to evaluate the reliable minimum speed at which fracture occurs at. The FEA does show a factor of safety of 1 at the critical point at where fracture is predicted to occur (at the base of the spigot). The FEA stress simulation (Figure 12) shows intense distortion around the spigot base and given the large stress concentration effect any material defects will of course lower the fracture speed. Variations in size and stress at the joints will also contribute to this. Care was taken not to cause any premature damage to the blades; however, the smallest defect could have significantly weakened the blade.

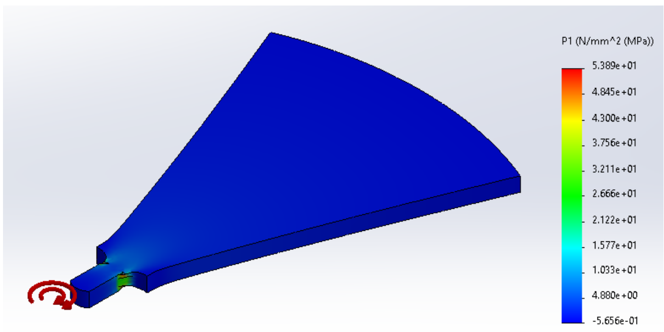

Figure 12. FEA stress model of fan blade subjected to centripetal loading at 6250 rpm.

3.6 Experimental Issues and Observations.

Although results show consistency, however experimental issues were encountered, and following observations have been made:

- One grade of PMMA was tested, using a laser cutter.
- All ASTM D638 specimens failed close to the center of the specimens.
- Slippage occurred on a few specimens, which were discarded.
- Due to the grip size, a limited range of H/d values could be cut for testing. Not enough specimens could be cut to generate a range of curves, but one curve was obtained.
- The blade specimens only used one geometry, due to time constraints.
- The rev counter was attached directly to the motor spindle.
- The speed was adjusted by limiting the current and increasing the voltage. The slow angular acceleration was essential to avoid the impulse effects.
- A slight press fit (found by previous project work) prevented slippage of the blades.
- All fractures were caused by the blade shattering from the effects of centripetal force.

4. CONCLUSIONS

The research study shows that the field of dealing with stress concentrations in design requires targeted FEA modelling, but in order to fully evaluate the nature of the concentrations empirical testing is also required. Dealing with brittle materials presents its own challenges. This study is limited to analyzing a range of specimen sizes, one type of geometry of blade. More time would have allowed different geometries of blade to be evaluated, a lower stress concentration design (such as a rectangular profile) would have made an interesting comparison. Higher grades (or different grades) of PMMA would be beneficial in this type of study. The testing arose as a point of interest from teaching strength of materials to undergraduate students, as well manufacturing principles involving laser cutting. In design work some undergraduate students overlook the potential problems of stress concentrations, and so this study helps demonstrate the importance of evaluating designs and following good design practice. Furthermore, the FEA modelling shows that the results do compare relatively well where controls are applied, and the material properties do not show a huge variance. Despite being a limited study, the results will prove useful for teaching undergraduates about the influence of stress concentrations in design work as well promote laboratory testing.

Overall, from this study, we can conclude that:

- Different grades of PMMA and sizes should be tested.

- The study should be increased to review other polymers and composites, including additive manufacturing technologies.
- Given the limitations, the focus on this study was to compare FEA modelling to experimental results.
- The fillet radii on the blades did not show any correlation to the level of stress concentration. Much larger samples are needed to fully evaluate this.
- However, the stress concentration due to change in section size is so great that the radius of fillet for the blades tested is not significant.
- Different blade geometries are required to evaluate the point at which fillet radii variations become a major factor in the strength of the component.
- This limited study has shown that FEA can provide reliable predictions for the strength of components containing stress concentrations.
- This study is useful for teaching undergraduate topics as it uses variations in specimen geometries to generate stress factor curves at the point of fracture.
- The centripetal testing is much more varied but shows how FEA can predict the critical fracture point and also reveals the scale of stress concentration factors that cannot be found easily in existing literature.

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BIOGRAPHIES

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“**Rob Benham** is a senior lecturer at Department of Science and Engineering, Solent University, UK. and currently a course leader for the HNC Engineering and Foundation Year Engineering. He has been teaching engineering courses at Solent University for over 15 years. Prior to the introduction of individual course leaders, he was programme leader on the old engineering programme. In 2003 Rob completed his PGCE (post-compulsory education) at Oxford Brookes University. He then taught in further education for one additional year. He continued some FE teaching and supply teaching when working part-time. In addition to this, in more recent times, Rob has delivered many taster days at Solent. He has strong research interests in manufacturing and materials. As a result, he brings a wide range of experience of educational settings with a diverse scope of learners.”