Numerical solution for two dimensional Non Newtonian boundary layer flow over a flat plate with suction/injection through porous media by using Collocation method

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Abstract - *In this paper, we are given a graphical presentation of the Falkner-Skan equation for the study of two-dimensional permeable steady boundary-layer viscous over a flat plate in the presence of Non-Newtonian power-law fluid and it is presented by a power-law model. Similarity transformation techniques are used to convert the boundary layer equations into a third order nonlinear differential equation. An equation containing three flow parameters like m is power-law relation parameter, omega is the porous parameter and beta is the Stream-wise pressure gradient. We converted the third order nonlinear differential equation into third order linear differential equation by using Quasi - linearization techniques. Results are obtained for the velocity profile, viscosity profile, and skin friction for the value of physical parameters is discussed in brief.*

Key Words: Boundary-layer equation, Falkner-Skan equation, Quasilinearization Techniques, Similarity transforms, Collocation Method, Non-Newtonian fluid, power-law fluid.

1.INTRODUCTION

Applications of Non-Newtonian and Newtonian fluids are very useful in an industrial and technologically. Air or water which is Newtonian fluids serves as a benchmark for the fluid flow behavior. However, the behavior of Non-Newtonian fluids is more important in the industry rather than Newtonian fluids. Non-Newtonian fluids like oil-water emulsions, foams, gas-liquid dispersions. Acrivosetal (1960) shows the thickness of boundary –layer for shear-thinning fluids is large compared to the shear-thickening fluids. Wu and Thomson (1996) that for moderate values of the Reynolds number, the boundary-layer equation for shear-thinning fluids provides an accurate solution. Andersson and Irgens (1998) show that the boundary layer equation predicts the finite-width of the boundary-layer for shear-thickening fluids. Results are obtained from Andersson and Irgens (1998) to support Filipuss et all (2001) gave rigorously mathematical analysis predicts the same finite-width of the boundary-layer. Denier and Dabrowiski (2004) have shown that these are double solution for the boundary-layer equations when a self-similar form is assumed.

Griffiths (2017) results show that the effects of shear-thinning areto stabilize the boundary-layer flow.

In this paper, we consider third order nonlinear ordinary differential equations with three boundary conditions. This third order nonlinear ordinary differential equation converted into a linear ordinary differential equation by using Quasilinearization techniques. Apply the Collocation Methodmethod (D F Griffiths-1978) to solve the third order linear differential equation. In this method, we have to consider the linear combination of the trail function with a constant coefficient. Then apply this method to find the constant coefficient. These constants put in the assumed solution and from the solution we identify the behavior of the boundary-layer flow of the Ostwald-de Waele fluid.

2. FORMULATION OF THE PROBLEM

Consider

$$\nabla.\vec{q} = 0 \tag{1}$$

$$\rho\left(\frac{\partial}{\partial t} + \vec{q}.\nabla\right)\vec{q} = -\nabla p + \nabla.\tau - kp \tag{2}$$

Which is the two-dimensional laminar boundary-layer flow of a viscous and incompressible fluid over flate through porous media with a Non-Newtonian power-law fluid. The above equation is express in the absence of body forces.

In equation (2) the parameter is defined as ρ is the fluid density, p is the pressure, k is the permeability of the porous medium and τ is the deviatoric stress tensor and is given by

$$\tau = \mu(\dot{q}) \tag{3}$$

Where $\dot{q}$ is the second invariant of the strain-rate tensor and the shear rate $\dot{q}$ is given by $\dot{q} = \frac{1}{2}(\dot{q} : \dot{q})^{1/2}$ (4)

with

$$\dot{q} = (\nabla\vec{u} + \nabla\vec{u}^T) \tag{5}$$

The constitutive viscosity relation μ for the Ostwald-de Waele power-law model is given by

$$\mu = K(\dot{q})^m \tag{6}$$

Where k is the material constant and m is the degree of shear thickening or shear thinning. If m=1 then it is called Newtonian viscosity relationship. If $m \succ 1$ then fluid is called pseudo-plastic or shear thickening fluids and $m \prec 1$ then the fluid is called dilatants or shear-thinning fluids. Bird et al (1987) can be referred to the through the account of the rheological data on them.

Consider $$\dot{q} = \left[(u_y + v_x)^2 + (u_x)^2 + (u_y)^2\right]^{\frac{1}{2}} \tag{7}$$

In (7) $\dot{q} = (u, v)$ is called velocity vector and u, v is called the velocity components in x and y direction respectively.

Let us consider the problem of two-dimensional incompressible and steady-state laminar boundary-layer flow over a wedge which moves with velocity U0w (x) in a non-Newtonian power-law fluid (2003). The positive x-axis is measured along the surface of the wedge with the apex as origin, and the positive y-axis is measured normal to the x-axis in the outward direction towards the fluid. Under these approximations the governing equations for the steady two-dimensional laminar viscous flow of a non-Newtonian fluid. It is considered We have

$$\left|\frac{\partial u}{\partial y}\right| \succ\succ \left|\frac{\partial u}{\partial x}\right| and \left|\frac{\partial p}{\partial y}\right| \prec\prec \left|\frac{\partial p}{\partial x}\right| \tag{8}$$

Thus equation (1) and (2) written into the form of

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + \frac{K}{\rho}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^m - kp$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{dp}{dy} + \frac{K}{\rho}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right)^m - kp$$

where $\dot{q} = \frac{\partial u}{\partial y}$

Consider $$U_0(x) = U_\infty x^{*n} \tag{9}$$

Here U_∞ denotes a positive constant and n is a constant related to the pressure gradient.

$$x = \frac{x^*}{L}, y = \frac{y^*}{\delta}, u = \frac{u^*}{U}, v = \frac{v^*}{U}, p = \frac{p^*}{p_\infty}$$

L, δ, U, P_∞ are certain reference values

The Ostwald-de Waele power-law fluid is

$$R_e = \frac{\rho \delta^n U^{2-n}}{Kv} \tag{10}$$

In the above equation, kinematic viscosity is given by v

And for a large Re the flow divides into near-field and far-field regions. In the boundary-layer region of the thickness of δ. The thickness of the boundary layer is given by δ then $\delta \prec\prec L$.

The basic approximation is $\left|\frac{\partial u}{\partial y}\right| \succ\succ \left|\frac{\partial u}{\partial x}\right| \left|\frac{\partial p}{\partial y}\right| \prec\prec \left|\frac{\partial p}{\partial x}\right|$

It means that p in the boundary layer is a function of x only.

Equations (5) may be written into the form of

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + \frac{K}{\rho}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^m - \frac{v}{k}(u - U)$$

$$0 = \frac{\partial p}{\partial y} \tag{11}$$

k is said to be consistency coefficient and m is non-dimensional and the dimension of k depends on the value of m.

Equations (8) can be reduced into similarity form, we assume that the boundary conditions are as given by below.

$$\begin{aligned} &y = 0 : u = 0, v = V_w(x) \\ &y \to \infty : u \to U_{0\infty} \end{aligned} \tag{12}$$

$U_0 w(x)$ is stretching surface velocity. This stretching surface obeys the rule of power-law relation $U_0 w(x) = U_{0\infty} x^m$. Velocity approaches to infinity mean velocity approaches the mainstream flow far-away from the wedge surface.

We know that the pressure is uniform throughout the flow field from the Bernoulli's equation, with $u = U_{0\infty}$ outside the boundary layer, we have

$$-\frac{1}{\rho}\frac{dp}{dx} = U_0 \frac{dU_0}{dx}. \tag{13}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U_0\frac{dU_0}{dx} + \frac{K}{\rho}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^m - \frac{v}{k}(u - U_0(x)) \tag{14}$$

Stream function $\psi(x,y)$ is defined as

$$u=\frac{\partial\psi}{\partial y}, v=-\frac{\partial\psi}{\partial x}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}=U_0(x)\frac{dU_0(x)}{dx}+\frac{K}{\rho}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^m-\frac{v}{k}(u-U_0(x)) \quad (15)$$

With boundary condition is given by

$$y=0:\frac{\partial\psi}{\partial x}=0, \frac{\partial\psi}{\partial y}=V_w(x)^n, y\to\infty:\frac{\partial\psi}{\partial y}=U_{0\infty}(x)^n \quad (16)$$

The similar solution of an equation (15) can be obtained by a similarity transformation

$$\psi(x,y)=\sqrt{\frac{2vKU_\infty x^{1+m^*}}{\rho(n+1)}}f(\eta), \eta=\sqrt{\frac{(n+1)\rho U_\infty x^{-1+m^*}}{2Kv}}y$$

$$m^*=\frac{(3n-1)(m-1)}{(m+1)} \quad (17)$$

Putting (17) in to (15) we get the following ordinary differential equation

$$\mu_0 f''' + \frac{2}{m+1}ff'' + \frac{\beta}{1+(m-1)\beta}(1-f'^2)-\Omega(f'-1)=0 \quad (18)$$

With new boundary conditions

$$f(0)=\alpha, f'(0)=0, f'(\infty)=1 \quad (19)$$

Here $\mu_0(\eta)=m\left|f''(\eta)\right|^{m-1}$ and prime denotes differentiation with respect to η. $\alpha=\sqrt{\frac{2x}{(m+1)vU(x)}}V_w(x)$ is the suction or injection parameter. α is positive then it is called suction and

If α is negative then it is called injection and if α is zero then it is called impermeable of the plate. β is called an adverse pressure gradient and $\Omega=\frac{2\left(\frac{U_\infty}{v}\right)^{(m-2)}}{k(m+1)R_e^{(m-1)}}$ is the permeability.

For $\beta=0=\Omega$ the above problem is called Blasius flow. We solve the equation (15) and (16) by numerical method and first, the equation converted into Cauchy Linearization techniques and then apply the Collocation method to solve the problem numerically.

3. NUMERICAL SOLUTION

Consider the equation (18) and (19) is again rewrittens as

$$\mu_0 f''' + \frac{2}{m+1}ff'' + \frac{\beta}{1+(m-1)\beta}(1-f'^2)-\Omega(f'-1)=0$$

With new boundary conditions

$$f(0)=\alpha, f'(0)=0, f'(\infty)=1$$

The above problem is can be converted into the linear ordinary differential equation by using Cauchy Linearization techniques and the obtain equation is

$$f'''+\left[\begin{array}{l}\left(\frac{2}{m^2+m}\right)ff''^{(1-m)}+(1-m)\left(\frac{2}{m^2+m}\right)\left(f''\right)^{-m}ff''\\+(1-m)\left(\frac{\beta}{m^2\beta-m\beta+m}\right)\left(f''\right)^{-m}(1-f'^2)+(1-m)\left(\frac{\Omega}{m}\right)\left(f''\right)^{-m}(1-f')\end{array}\right]f''+$$

$$\left[\left(\frac{-2\beta}{m^2\beta-m\beta+m}\right)f'f''^{(1-m)}-\left(\frac{\Omega}{m}\right)\left(f''\right)^{(1-m)}\right]f'+\left[\left(\frac{2}{m^2+m}\right)f''(f'')^{(1-m)}\right]f$$

$$-2\left(\frac{2}{m^2+m}\right)ff''^{(2-m)}+\left[2\left(\frac{\beta}{m^2\beta-m\beta+m}\right)f'+\frac{\Omega}{m}\right]f'f''^{(1-m)}.$$

With the boundary condition

$$f(0)=\alpha, f'(0)=0, f'(\infty)=1$$

Where $\beta=\frac{m(1+n)}{r}, M=\frac{\sigma\mu^2H_0^2(1+n)}{c\sigma r}$ (20)

We take $f(\eta)$ as before to satisfy the boundary conditions (four-term solution) and condition given in equation no.17 and take $\infty=5$ to restricted the interval [0, 5]

Consider

$$f(\eta)=\alpha+(0.013n^3)+c_2(\eta^2-0.13n^3)+c_4(n^4-6.67n^3)+c_5(n^5-41.666n^3) \quad (21)$$

$$R(\eta)$$
$$=[(0.079)+c_2(-0.7999)+c_4(24\eta-40)+c_5(60\eta^2-250)]+$$
$$\left[\left(\frac{2}{m^2+m}\right)\left(\alpha+\frac{\eta^2}{10}\right)\left(\frac{1}{5}\right)^{(1-m)}+(1-m)\left(\frac{2}{m^2+m}\right)\left(\frac{1}{5}\right)^{-m}\left(\alpha+\frac{\eta^2}{10}\right)\left(\frac{1}{5}\right)\right.$$
$$\left.+\left(\frac{\beta}{m^2\beta-m\beta+m}\right)(1-m)\left(\frac{1}{5}\right)^{-m}\left(1-\frac{\eta^2}{25}\right)+\left(\frac{\Omega}{m}\right)(1-m)\left(\frac{1}{5}\right)^{-m}(1-\frac{\eta}{5})\right]$$

$$\left((0.079\eta)+c_2(2-0.799\eta)+c_4(12\eta^2-40\eta)+c_5(20\eta^3-250\eta)\right)$$
$$+\left[\left(\frac{-2\beta}{m^2\beta-m\beta+m}\right)\left(\frac{\eta}{5}\right)\left(\frac{1}{5}\right)^{(1-m)}-\left(\frac{\Omega}{m}\right)\left(\frac{1}{5}\right)^{(1-m)}\right]$$
$$\left((0.039\eta^2)+c_2(2\eta-0.3999\eta^2)+c_4(4\eta^3-20\eta^2)+c_5(5\eta^4-125\eta^2)\right)$$
$$+\left[\left(\frac{2}{m^2+m}\right)\left(\frac{1}{5}\right)\left(\frac{1}{5}\right)^{(1-m)}\right]$$
$$\left(\alpha+(0.013\eta^3)+c_2(\eta^2-0.13333x^3)+c_4(\eta^4-6.66666\eta^3)+c_5(\eta^5-41.66666\eta^3)\right)$$
$$-2\left(\frac{2}{m^2+m}\right)\left(\alpha+\frac{\eta^2}{10}\right)\left(\frac{1}{5}\right)^{(2-m)}+\left[2\left(\frac{\beta}{m^2\beta-m\beta+m}\right)\left(\frac{\eta}{5}\right)+\frac{\Omega}{m}\right]\left(\frac{\eta}{5}\right)\left(\frac{1}{5}\right)^{(1-m)}$$
$$+m\left(\frac{1}{5}\right)^{(1-m)}\left[\left(\frac{2}{m^2+m}\right)\left(\alpha+\frac{\eta^2}{10}\right)\left(\frac{1}{5}\right)+\left(\frac{\beta}{m^2\beta-m\beta+m}\right)\left(1-\frac{\eta^2}{25}\right)+\frac{\Omega}{m}\left(1-\frac{\eta}{5}\right)\right]$$

(22)

We selected the values of η = 2.5 , 3.5 and 4 and put in an equation (22) in this way that we obtained the numerical solution nearest to exact solution with very minimize error

We obtain the equation with unknown constants $c_{2,}c_{4,}c_5$ and $\alpha,\beta,\Omega.$.

The numerical solution of a (18) for different parameters $\alpha,\beta,\Omega.$ have been obtained. Results for velocity profile and numerical solutions are reported.

4. RESULTS AND DISCUSSION

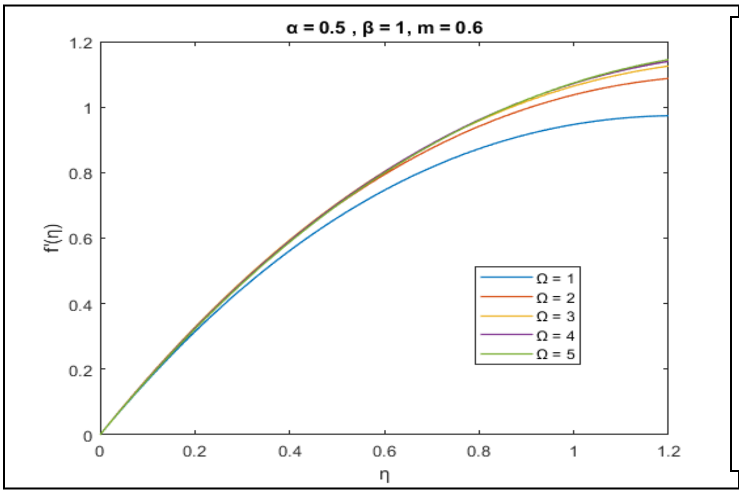

Figure1 (a): $f'(\eta)$ with η for various values of Ω , $\alpha=0.5,\beta=1,m=0.6$

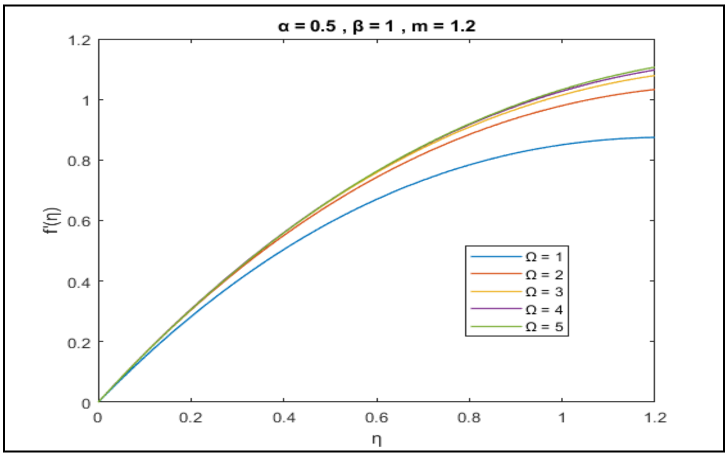

Figure1 (b): Variation of Velocity profiles $f'(\eta)$ with η for various values of Ω , $\alpha=0.5,\beta=1,m=1.2$

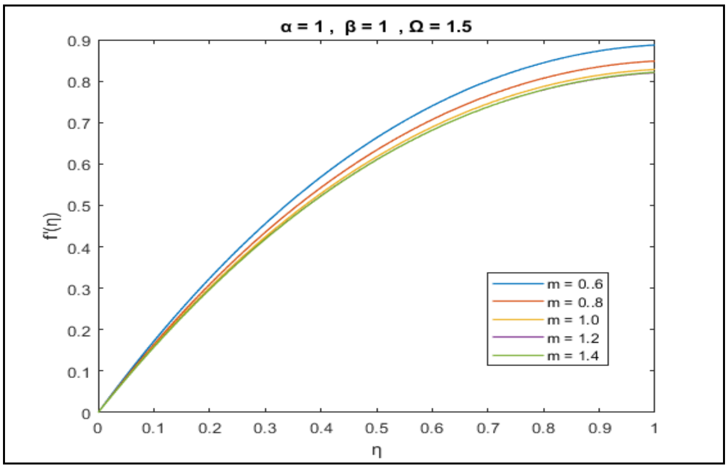

Figure2 (a): Variation of Velocity profiles $f'(\eta)$ with η for various values of m , $\alpha=1,\beta=1,\Omega=1.5$

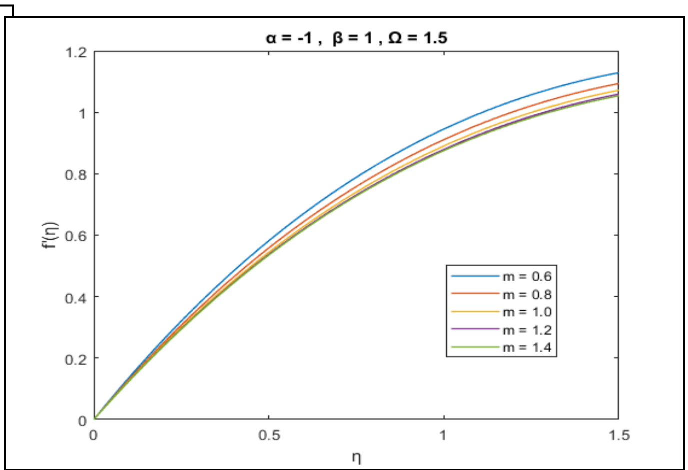

Figure2 (b): Variation of Velocity profiles $f'(\eta)$ with η for various values of m , $\alpha=-1,\beta=1,\Omega=1.5$

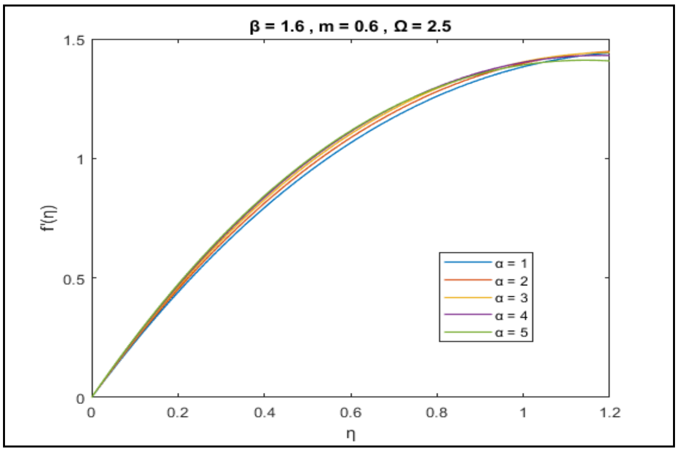

Figure3 (a): Variation of Velocity profiles $f'(\eta)$ with η for various values of α, $m = 0.6, \beta = 1.6, \Omega = 1.5$

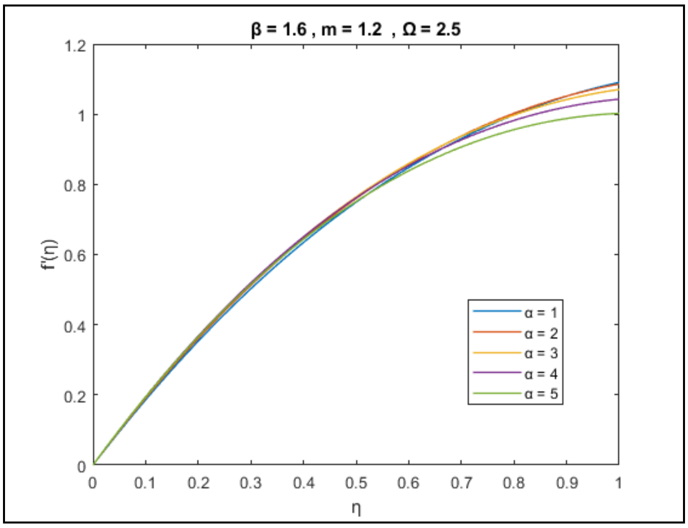

Figure3 (b): Variation of Velocity profiles $f'(\eta)$ with η for various values of α, $m = 1.2, \beta = 1.6, \Omega = 2.5$

5. CONCLUSION

From Figures 1 and 3 we can say that the variation of velocity profile as a function of η for different values of permeability parameter. There have been simulated using Collocation numerical method that is described in section 3.it is clear to notice that the thickness of the boundary layer thickness is an increase for increase permeability. It is also very clear that from the boundary layer shear-thickening when $m \succ 1$ this means for dilatants fluids and when $m \prec 1$ the boundary layer shear-thickening this means that for pseudo-plastic fluids for fixed values of α, β, m.

From figure-2 we can say that the variation of velocity profile as a function of η.For fixed α, β, Ω the boundary layer decrease as an increase m .

For fixed α, β, Ω, the effect of the Non-Newtonian parameter m on velocity profiles is depicted. When α are positive say1this means that α is said to be suction then decrease in m as an increase in velocity profile exponentially and increase monotonically as a decrease in m for injection α .

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