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UNSTEADY MHD FORCED CONVECTION FLOW AND MASS TRANSFER ALONG A VERTICAL STRETCHING SHEET WITH HEAT SOURCE / SINK AND VARIABLE FLUID PROPERTIES

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Abstract- *In this paper, unsteady magnetohydrodynamic forced convection flow of a viscous incompressible, electrically conducting fluid and mass transfer along a vertical porous stretching sheet is investigated, in the presence of heat source /sink with variable viscosity and thermal conductivity. The governing coupled non-linear partial differential equations are reduced to ordinary differential equations using similarity transformation and solved numerically using the Runge-Kutta fourth order method along with shooting technique. The effects of various flow parameters on the velocity, temperature and concentration distributions are analyzed and presented graphically. Skin-friction coefficient, Nusselt number and Sherwood number are derived at the sheet, discussed numerically and their numerical values for various values of physical parameters are presented through tables.*

Key Words : *MHD, variable viscosity, variable thermal conductivity, stretching sheet, heat source / sink.*

1. Introduction

The magnetohydrodynamics heat and mass transfer flow in the boundary layer induced by a moving surface in a fluid finds important applications in chemical engineering and meteorology. MHD thermal boundary layer flow with variable fluid properties has received a great deal of attention due to its important roles and wide applications in geophysics and thermal insulation engineering. Erikson et al. (8) studied heat and mass transfer on a moving continuous plate with suction and injection. Gehart and Pera (9) observed nature of vertical natural convection flows resulting from the combined buoyancy effects of thermal and mass diffusion. Chakrabarti and Gupta (4) investigated hydromagnetic flow and heat transfer over stretching sheet. Apelblat (1) presented mass transfer with a chemical reaction of first order with effects of axial diffusion. Forced convection over a flat plate submersed in a porous medium with variable viscosity is investigated by Ling and Dybbs (16). Chen and Char (5) discussed heat transfer of a continuous stretching surface with suction or blowing. Lai and Kulacki (17) analyzed effects of variable viscosity on convective heat transfer along a vertical surface in a saturated porous medium. Das et al. (7) studied effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. Hossain and Takhar (12) obtained radiation effect on mixed convection along a vertical plate with uniform surface temperature. Heat and mass transfer in the boundary layers on an exponentially stretching continuous surface was considered by Magyari and Keller (18). Hossain et al. (11) discussed the effect of radiation on free convection flow of fluid with variable viscosity from a porous vertical plate. Heat and mass transfer on a laminar flow along a semi-infinite horizontal plate with temperature dependent viscosity and chemical reaction was investigated by Ghay and Seddek (10). Seddeek and Salama (26) analyzed the effects of temperature dependent viscosity and thermal conductivity on unsteady MHD convective heat transfer past a semi infinite vertical porous moving plate with variable suction. Mukhopadhyay and Layek (22) found effects of thermal radiation and variable fluid viscosity on free convection flow and heat transfer past a porous stretching surface. Mukhopadhyay (21) presented unsteady boundary layer flow and heat transfer past a porous stretching sheet in presence of variable viscosity and thermal diffusivity. Olajuwon (25) analyzed convection heat and mass transfer in an electrical conducting power law flow over a heated vertical porous plate. Rahman and Salahuddin (24) studied hydromagnetic heat and mass transfer flow over an inclined heated surface with variable viscosity and electric conductivity. Dual solutions in boundary layer flow stagnation-point flow and mass transfer with chemical reaction past a stretching/ shrinking sheet was studied by Bhattacharyya (3). Hunsain et al. (13) analyzed heat and mass transfer in unsteady boundary layer flow through porous media with variable viscosity and thermal diffusivity. Makinde (19) discussed effects of variable viscosity on boundary layer over a permeable flat plate with radiation and a convective surface boundary condition. Nadeem et al. (23) observed MHD three dimensional casson fluid flow past a porous linear stretching sheet. Conjugated forced convection heat transfer from a heated flat plate of finite thickness and temperature dependent thermal conductivity was analyzed by Mohammed and Nourazar (20). Chen (6) studied mixed convection unsteady stagnation-point flow towards a stretching sheet with slip effects.

The objective of the paper is to investigate effect of variable viscosity and thermal conductivity on unsteady MHD forced convection and mass transfer flow of a viscous incompressible, electrically conducting fluid along a porous stretching vertical sheet in the presence of heat source/sink .

2. Formulation of the Problem

The x-axis is oriented about the vertical plate in the upward direction and y-axis is normal to the plate. Unsteady two dimensional incompressible viscous fluid flows on a heated vertical porous stretching plate in the region $y > 0$ is considered. The sheet is stretching in its own plane with velocity

$$U_w(x,t) = \frac{ax}{(1-\alpha t)}. \quad \text{.... (1)}$$

$a(>0)$ is the stretching parameter and $\alpha(>0)$ is the unsteadiness parameter and both have dimensions of time^{-1}.

The temperature $T_w(x,t)$ of the sheet is different from that of the ambient medium and $C_w(x,t)$ is concentration distribution near the sheet and both vary with time t and that distance x along the sheet. It is assumed that the external electric field is zero and Hall effects are negligible. It is also assumed that the induced magnetic field is negligibly small. The level of concentration of foreign mass is assumed to be low, so that the Soret and Dufour effects are negligible. The fluid velocity and thermal conductivity are assumed to vary linearly with temperature. The system influenced by an external transverse magnetic field of strength B defined as

$$B(t) = B_0(1-\alpha t)^{-1/2}. \quad \text{...(2)}$$

The volumetric rate of heat generation/absorption is given as

$$Q(t) = Q_0(1-\alpha t)^{-1}. \quad \text{... (3)}$$

Under above assumptions, the governing equations of continuity, momentum, energy and concentration are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad \text{...(4)}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\left(\frac{\partial \mu^*}{\partial y}\frac{\partial u}{\partial y} + \mu^*\frac{\partial^2 u}{\partial y^2}\right) + g\beta(T-T_\infty)$$
$$+g\beta^*(C-C_\infty) - \frac{\sigma B_0^2}{\rho}u, \quad \text{...(5)}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\rho C_P}\left(\frac{\partial \kappa^*}{\partial y}\frac{\partial T}{\partial y} + \kappa^*\frac{\partial^2 T}{\partial y^2}\right)$$
$$+\frac{Q}{\rho C_p}(T-T_\infty), \quad \text{... (6)}$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2}, \quad \text{...(7)}$$

where u and v are the velocity components along the $x-$ and $y-$directions respectively, ρ is the density of the fluid, g is the gravitational acceleration, β is the thermal expansion coefficient, β^* is the concentration expansion coefficient, σ is the electrical conductivity, T is fluid temperature inside the thermal boundary layer, C is the species concentration in boundary layer, T_∞ is the temperature far away from the sheet , C_∞ is the species concentration far away from the sheet. C_p is the specific heat at constant pressure, κ^* is the variable thermal conductivity, D is the mass diffusion coefficient.

Variation of the viscosity and thermal conductivity with temperature are assumed to be of the form given below

$$\mu^* = \mu_0\left\{b + b_1\left(T_w - T\right)\right\}, \quad \text{... (8)}$$

$$\kappa^* = \kappa_0\left\{1 + d\left(\frac{T - T_w}{T_w - T_\infty}\right)\right\}, \quad \text{... (9)}$$

where μ_0 is the constant value of coefficient of viscosity far away from the plate, b, b_1 are constants κ_0 is the conductivity of the fluid at temperature T_w, d is the parameter that depends on nature of the fluid.

The corresponding boundary conditions are given by

$$y = 0:\ u = U_w(x,t),\quad v = v_w(t),$$
$$T = T_w(x,t),\quad C = C_w(x,t)$$
$$y \to \infty:\ u \to 0,\quad T \to T_\infty,\quad C \to C_\infty. \quad \text{...(10)}$$

Here, $v_w\left(t\right) = -V_0\left(\frac{\upsilon^* a}{1-\alpha t}\right)^{1/2}$ is the suction velocity, $T_w\left(x,t\right) = T_\infty + \frac{a}{2\upsilon^* x^2}\left(1-\alpha t\right)^{-3/2}$ is the temperature of the sheet, $C_w\left(x,t\right) = C_\infty + \frac{a}{2\upsilon^* x^2}\left(1-\alpha t\right)^{-3/2}$ is the concentration distribution near the sheet, V_0 is the Cross-flow velocity of the fluid and $\upsilon^* = \mu_0 / \rho$ is the kinematic viscosity. It is implicitly assumed that the mathematical problem is defined only for $x \geq 0$.

3. Method of Solution

Introducing the similarity variable η, dimensionless functions f, θ and ϕ, and physical parameters as given below

$$\eta = \left(\frac{a}{\upsilon^*}\right)^{1/2}\left(1-\alpha t\right)^{-1/2} y,\quad f(\eta) = \Psi(x,y,t)\left[\frac{\upsilon^* a x^2}{\left(1-\alpha t\right)}\right]^{-1/2},$$

$$\theta(\eta) = \frac{(T - T_\infty)}{\left(T_w - T_\infty\right)},\quad T_w - T_\infty = \frac{a}{2\upsilon^* x^2}\left(1-\alpha t\right)^{-3/2}$$

$$\phi(\eta) = \frac{(C - C_\infty)}{\left(C_w - C_\infty\right)},\quad C_w - C_\infty = \frac{a}{2\upsilon^* x^2}\left(1-\alpha t\right)^{-3/2}$$

$$A = \alpha / a,\quad \lambda_T = b_1\left(T_w - T_\infty\right),\quad M = \frac{\sigma B_0^2 (1-\alpha t)}{\rho a},$$

$$Gr = \frac{g\beta x\left(T_w - T_\infty\right)}{U_w^2}, Gm = \frac{g\beta^* x\left(C_w - C_\infty\right)}{U_w^2},$$

$$S = \frac{Q\left(1-\alpha t\right)}{a\rho C_p},\quad \Pr = \frac{\upsilon^* \rho C_p}{\kappa_0},\quad Sc = \frac{\upsilon^*}{D}; \quad \text{...(11)}$$

where $\Psi(x,y,t)$ is the physical stream function. Stream function assures mass conservation automatically. The velocity components are obtained as

$$u = \frac{\partial \Psi}{\partial y} = ax\left(1-\alpha t\right)^{-1} f'(\eta), \quad \text{... (12)}$$

$$v = -\frac{\partial \Psi}{\partial x} = -\left(\upsilon^* a\right)^{1/2}\left(1-\alpha t\right)^{-1/2} f\left(\eta\right). \quad \text{... (13)}$$

Substituting (11) into (5) to (7), we obtain

$$A\{f'+(\eta/2)\,f''\}+(f')^2-ff''=$$
$$(b+\lambda_T-\lambda_T\theta)\,f'''-\lambda_T f''\theta'-Mf'$$
$$+Gr\theta+Gm\phi, \qquad ...(14)$$

$$\theta''+d\theta\theta''+d\left(\theta'\right)^2=$$
$$\Pr\left[A\left\{(3/2)\theta+(\eta/2)\theta'\right\}-\right.$$
$$\left.2f'\theta-f\theta'-S\theta\right] \qquad ...(15)$$

$$\phi''=Sc\left[A\left\{(3/2)\phi+(\eta/2)\phi'\right\}-\right.$$
$$\left.2f'\phi-f\phi'\right] \qquad ...(16)$$

where prime indicates differentiation with respect to η, A is the dimensionless measure of the unsteadiness, λ_T is the temperature-dependent viscosity parameter, M is the magnetic parameter, Gr is the Grashof number, Gm is the modified Grashof number, S is the heat generation/absorption parameter, $\Pr$ is the Prandtl number and Sc is the Schmidt number.

The corresponding boundary conditions are reduced to

$$\eta=0:\quad f(\eta)=V_0,\; f'(\eta)=1,\; \theta(\eta)=1,\; \phi(\eta)=1;$$
$$\eta\to\infty:\quad f'(\eta)\to 0,\; \theta(\eta)\to 0,\; \phi(\eta)\to 0.$$

...(17)

In order to obtain numerical solution of the equations (14) to (16) under the boundary condition (17) the problem is transformed into a system of first order equations as given by

$$f=f_1,\quad f'=f_2,\quad f''=f_3,\quad f'''=f_3',$$
$$\theta=f_4,\quad \theta'=f_5,\quad \theta''=f_5',$$
$$\phi=f_6,\quad \phi'=f_7,\quad \phi''=f_7', \qquad ...(18)$$

$$f_3'=\left[\lambda_T f_3 f_5+Mf_2-Grf_4-Gmf_6-f_1f_3+\right.$$
$$\left.f_2^2+A\left\{f_2+(\eta/2)\,f_3\right\}\right]/(b+\lambda_T-\lambda_T f_4),$$

... (19)

$$f_5'=\left[A\Pr\left\{(3/2)\,f_4+(\eta/2)\,f_5\right\}-df_5^2-\right.$$
$$\left.\Pr\left(Sf_4+2f_2f_4+f_1f_5\right)\right]/\left(1+df_4\right) \qquad ...(20)$$

$$f_7'=Sc\left[A\left\{(3/2)\,f_6+(\eta/2)\,f_7\right\}-2f_2f_6-f_1f_7\right].$$

...(21)

The corresponding boundary conditions are reduced to

$$\eta=0:\quad f_1=V_0,\; f_2=1,\; f_4=1,\; f_6=1;$$
$$\eta\to\infty:\; f_2\to 0,\; f_4\to 0,\; f_6\to 0.$$

...(22)

To solve eq. (19), (20) and (21) with boundary conditions (22), as an initial value problem we need the values of $f_3(0),\; f_5(0)\; and\; f_7(0)$ $\left[i.e.\; f''(0),\; \theta'(0)\; and\; \phi'(0)\right]$, but no such values are given. The initial guess values for $f''(0),\; \theta'(0)\; and\; \phi'(0)$ are chosen and using the fourth order Runge-Kutta method, the values are obtained. We compare the calculated values of $f'(\eta),\; \theta(\eta)\; and\; \phi(\eta)$ at a finite value of $\eta\to\infty$ with the given boundary conditions $f'(\infty)=0,\; \theta(\infty)=0,\; \phi(\infty)=0$ and adjust the values $f''(0),\; \theta'(0)\, and\; \phi'(0)$ to give a better approximation for the solution. The step-size is taken as $\Delta\eta=0.01$. The process is repeated until we obtain results correct up to the desired accuracy level of 10^{-5} as the criterion of convergence.

4. Skin friction Coefficient

The skin friction coefficient at the sheet is defined as

$$C_f=\frac{2\tau_w}{\rho U_w^2}=2\,\mathrm{Re}^{-1/2}\; f''(0), \qquad ...(23)$$

where $\tau_w = \mu_0 \left(\frac{\partial u}{\partial y}\right)_{y=0}$ is the shear stress at the sheet and $\mathrm{Re} = \frac{xU_w}{\upsilon^*}$ is the Reynolds number.

5. Nusselt Number

The rate of heat transfer in terms of Nusselt number at the surface of sheet is given by

$$Nu_x = \frac{x}{\kappa_0} \frac{q_w}{(T_w - T_\infty)} = -\mathrm{Re}^{1/2}\, \theta'(0)\,, \quad \text{...(24)}$$

where $q_w = -\kappa_0 \left(\frac{\partial T}{\partial y}\right)_{y=0}$, is the rate of heat transfer at the sheet.

6. Sherwood Number

The rate of mass transfer in terms of Sherwood number at the surface of sheet is given by

$$Sh_x = \frac{x}{D} \frac{m_w}{(C_w - C_\infty)} = -\mathrm{Re}^{1/2}\, \phi'(0)\,, \quad \text{...(25)}$$

where $m_w = -D\left(\frac{\partial C}{\partial y}\right)_{y=0}$, is the rate of mass transfer at the sheet.

7. Results and Discussion

In order to investigate the behavior of velocity, temperature, species concentration, skin-friction coefficient at the sheet, rate of heat transfer in terms of Nusselt Number at the sheet and rate of mass transfer in terms of Sherwood Number at the sheet, a comprehensive numerical computation is carried out for various values of parameters that describe the flow characteristics and the results are reported in terms of graphs and tables, discussed numerically and explained physically.

Figures 1 and 2, respectively represent that fluid velocity increase due to increase in Grashof number or modified Grashof number. It is noted from figure 3 that fluid velocity decreases with increase in Hartmann number. Figure 4 illustrates that fluid velocity decreases near the plate with an increase in temperature dependent viscosity parameter but the reverse behavior is seen away from the plate. It is observed from figure 5 that fluid velocity increases due to increase in unsteadiness parameter. Figure 6 depicts that fluid velocity decreases with an increase in cross flow velocity of fluid. Figures 7 reveals that fluid velocity increases due to increase in parameter d. It is observed from Figure 8 that fluid temperature decreases due to increase in Prandtl number. Figure 9 represents that fluid temperature increases due to increase in parameter d. Figures 10 and 11, respectively show that fluid temperature decreases due to increase in unsteadiness parameter or cross flow velocity of fluid. Figure 12 illustrates that fluid temperature increases due to increase in temperature dependent viscosity.It is seen from figure 13 that fluid temperature increases with heat source while decreases with heat sink. Figure 14 that species concentration decreases due to increase in Schmidt number. Figures 15 and 16 respectively illustrates that species concentration decreases due to increase in unsteadiness parameters or cross flow velocity of fluid. Figure 17 shows that concentration profiles increase near the plate with an increase in temperature dependent viscosity parameter but the reverse behavior is seen away from the plate.

Table 1 depicts that Skin-friction coefficient at the sheet increases due to increase in Grashof number, modified Grashof number or parameter d, while it decreases due to increase in Hartmann number, temperature dependent viscosity parameter, unsteadiness parameter or cross flow velocity of fluid. Table 2 shows that Nusselt number at the sheet increases due to increase in cross flow velocity of fluid, unsteadiness parameter, heat sink or Prandtl number, while it decreases due to increase in temperature dependent viscosity parameter, parameter d or heat source. Table 3 illustrates that Sherwood number at the sheet increases due to increase in cross flow velocity of fluid, unsteadiness parameter, Schimdt number while it decreases due to increase in temperature dependent viscosity parameter .

8. Conclusions

The findings of the numerical results can be summarized as follows:

1. Grashof number, modified Grashof number, Unsteadiness parameter or parameter d accelerate fluid velocity, whereas Hartmann

number or cross flow velocity of fluid retards fluid velocity.

2. Increase in temperature dependent viscosity parameter decreases fluid velocity near the sheet but increases far away from the sheet.

3. Increase in temperature dependent viscosity parameter or parameter d lead to increases in fluid temperature.

4. Prandtl number, unsteadiness parameter or cross flow velocity of fluid retard fluid temperature.

5. Heat source tends to enhance fluid temperature whereas heat sink has reverse effect on it.

6. Species concentration decreases due to increase in Schmidt number, cross flow velocity of fluid retards fluid velocity or unsteadiness parameter.

7. Increase in temperature dependent viscosity parameter increases species concentration near the sheet but have reverse effect far away from the sheet.

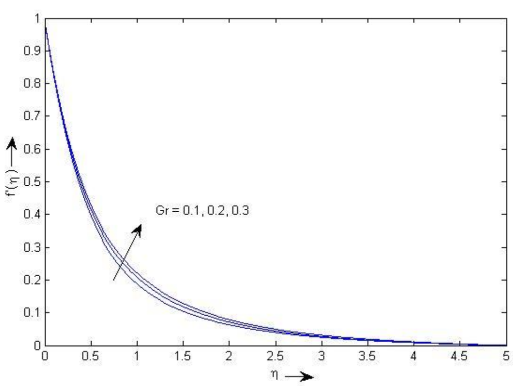

Figure 1 Velocity profiles versus η for different Valuesof Gr when $A=0.1,\ Gm=0.1,\ M=1,$ $\lambda_T=1,\ d=2,\ \Pr=3,\ S=0.5,\ Sc=0.3$ and $V_0=1$.

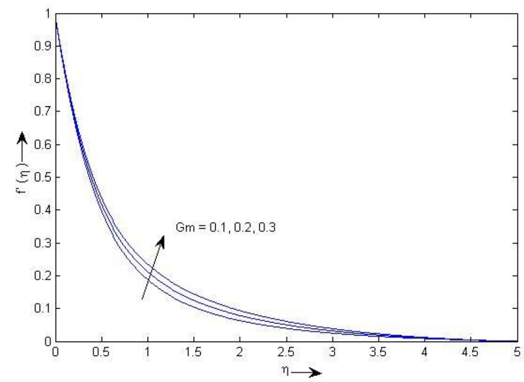

Figure 2 Velocity profiles versus η for different values of Gm when $Gr=0.1,\ A=0.1,\ M=1,$ $\lambda_T=1,\ d=2,\ \Pr=3,\ S=0.5, Sc=0.3,$ and $V_0=1$.

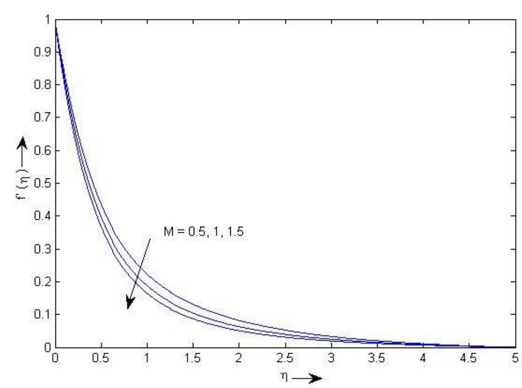

Figure 3 Velocity profiles versus η for different valuesof M when $Gr=0.1,\ Gm=0.1, A=0.1,$ $\lambda_T=1,\ d=2,\ \Pr=3,\ S=0.5, Sc=0.3$ and $V_0=1$.

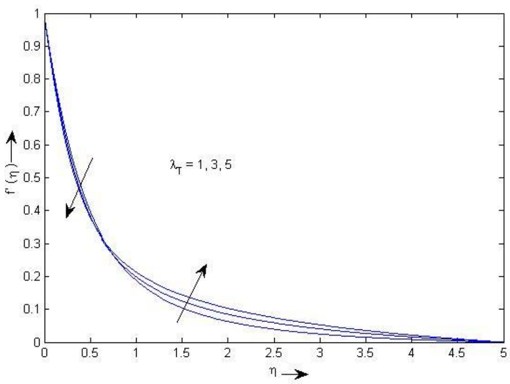

Figure 4 Velocity profiles versus η for different valuesof λ_T when $Gr = 0.1,\ Gm = 0.1,\ A = 0.1,$ $M = 1,\ d = 2, \Pr = 3,\ S = 0.5,\ Sc = 0.3$ *and* $V_0 = 1$.

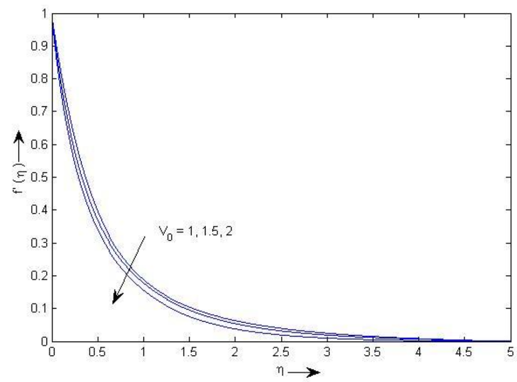

Figure 6 Velocity profiles versus η for different valuesof V_0 when $Gr = 0.1,\ Gm = 0.1,\ A = 0.1,$ $M = 1,\ d = 2, \Pr = 3,\ S = 0.5,\ Sc = 0.3$ *and* $\lambda_T = 1$.

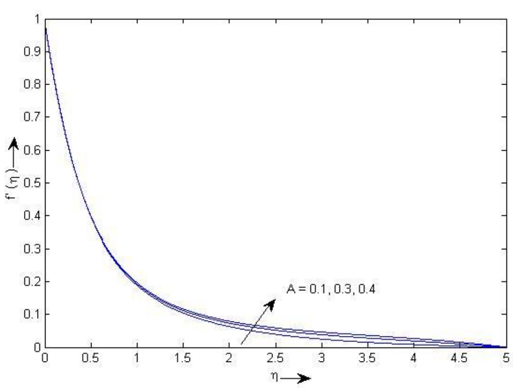

Figure 5 Velocity profiles versus η for different valuesof A when $Gr = 0.1,\ Gm = 0.1,\ \lambda_T = 1,$ $M = 1,\ d = 2, \Pr = 3,\ S = 0.5,\ Sc = 0.3$ *and* $V_0 = 1$.

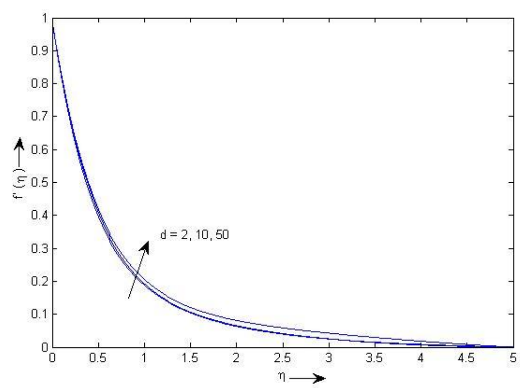

Figure7 Velocity profiles versus η for different values of d when $Gr = 0.1,\ Gm = 0.1,\ A = 0.1,$ $M = 1,\ \lambda_T = 1, \Pr = 3,\ S = 0.5,\ Sc = 0.3$ *and* $V_0 = 1$.

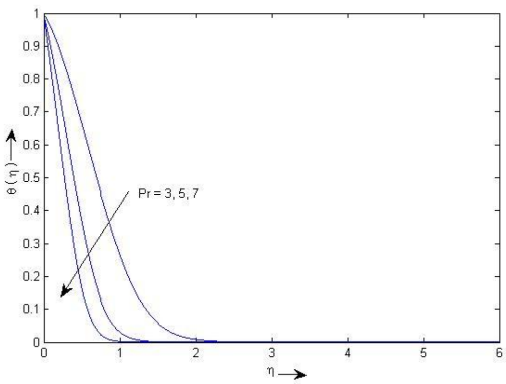

Figure8 Temperature profiles versus η for different values of $\Pr$ when $Gr = 0.1,\ Gm = 0.1,\ A = 0.1,$ $M = 1,\ d = 2, \lambda_T = 1,\ S = 0.5,\ Sc = 0.3$ *and* $V_0 = 1$.

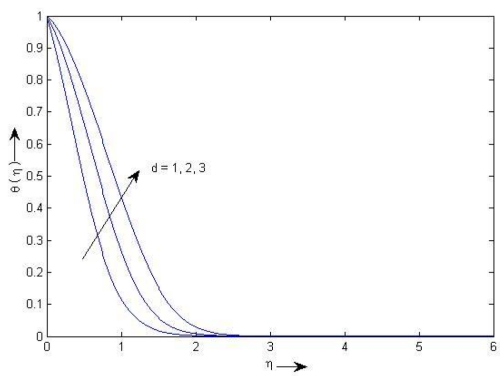

Figure 9 Temperature profiles versus η for different values of d when $Gr = 0.1,\ Gm = 0.1,\ A = 0.1,$ $M = 1,\ \lambda_T = 1, \Pr = 3,\ S = 0.5,\ Sc = 0.3$ *and* $V_0 = 1$.

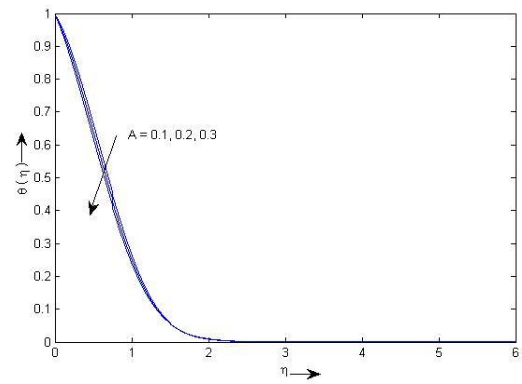

Figure10 Temperature profiles versus η for different values of A when $Gr = 0.1,\ Gm = 0.1,\ d = 2,$ $M = 1,\ \lambda_T = 1, \Pr = 3,\ S = 0.5,\ Sc = 0.3$ *and* $V_0 = 1$.

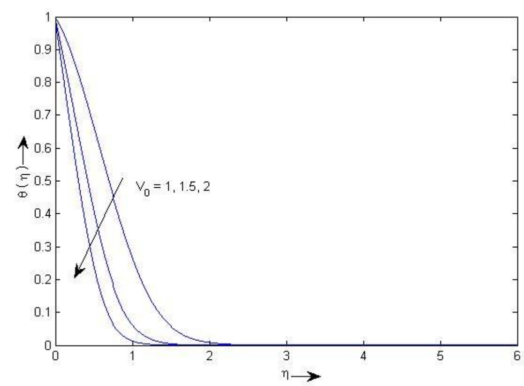

Figure 11 Temperature profiles versus η for different values of V_0 when $Gr = 0.1,\ Gm = 0.1,\ A = 0.1,$ $M = 1,\ \lambda_T = 1, \Pr = 3,\ S = 0.5,\ Sc = 0.3$ *and* $d = 2$.

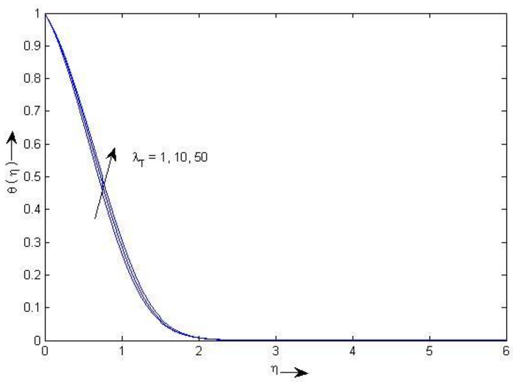

Figure12 Temperature profiles versus η for different values of λ_T when $Gr = 0.1,\ Gm = 0.1,\ A = 0.1,$ $M = 1,\ d = 2,\ \Pr = 3,\ S = 0.5,\ Sc = 0.3$ *and* $V_0 = 1$.

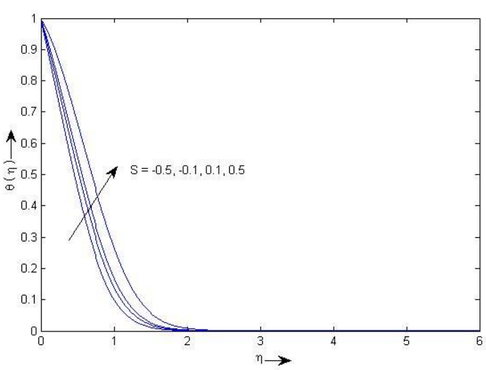

Figure13 Temperature profiles versus η for different values of S when $Gr = 0.1,\ Gm = 0.1,\ A = 0.1,$ $M = 1,\ \lambda_T = 1,\ \Pr = 3,\ d = 2,\ Sc = 0.3$ *and* $V_0 = 1$.

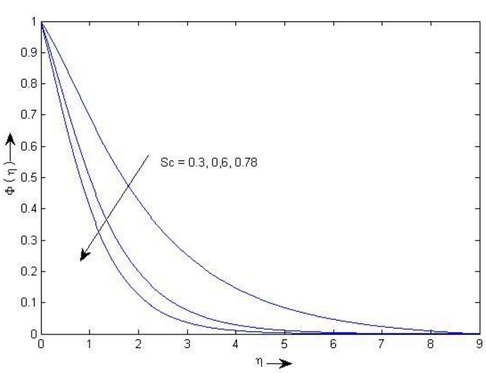

Figure14 Concentration profiles versus η for different values of Sc when $Gr = 0.1,\ Gm = 0.1,\ A = 0.1,$ $M = 1,\ \lambda_T = 1,\ \Pr = 3,\ S = 0.5,\ d = 2$ *and* $V_0 = 1$.

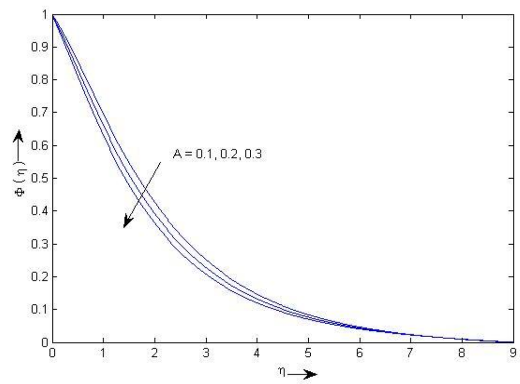

Figure 15 Concentration profiles versus η for different values of A when $Gr = 0.1,\ Gm = 0.1,\ Sc = 0.3,$ $M = 1,\ \lambda_T = 1,\ \Pr = 3,\ S = 0.5,\ d = 2$ *and* $V_0 = 1$.

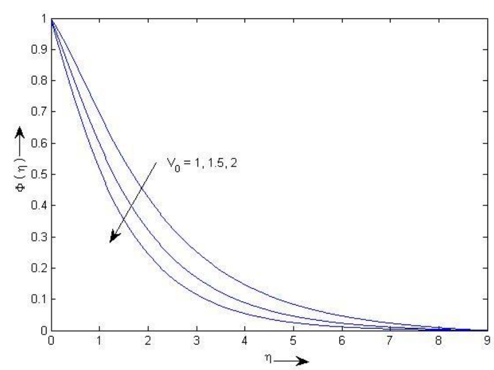

Figure 16 Concentration profiles versus η for different values of V_0 when $Gr = 0.1,\ Gm = 0.1,\ A = 0.1,$ $M = 1,\ \lambda_T = 1,\ \Pr = 3,\ S = 0.5,\ d = 2$ *and* $Sc = 0.3$.

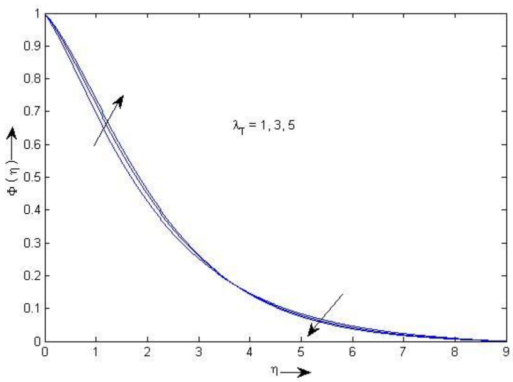

Figure 17 Concentration profiles versus η for different values of λ_T when $Gr = 0.1, Gm = 0.1, A = 0.1, M = 1, Sc = 0.3, \Pr = 3, S = 0.5, d = 2$ and $V_0 = 1$.

Table-1: Numerical values of skin friction coefficient at the sheet for various values of physical parameters.

Gr	Gm	M	λ_T	A	V_0	d	$f''(0)$
0.1	0.1	1	1	0.1	1	2	-1.90412
0.2	0.1	1	1	0.1	1	2	-1.89548
0.3	0.1	1	1	0.1	1	2	-1.84452
0.1	0.2	1	1	0.1	1	2	-1.858405
0.1	0.3	1	1	0.1	1	2	-1.80758
0.1	0.1	1.5	1	0.1	1	2	-2.0912
0.1	0.1	1	3	0.1	1	2	-1.96254
0.1	0.1	1	5	0.1	1	2	-2.08461
0.1	0.1	1	1	0.3	1	2	-1.93367
0.1	0.1	1	1	0.1	1.5	2	-2.25673
0.1	0.1	1	1	0.1	1	10	-1.85928

Table -2: Numerical values of Nusselt number at the sheet for various values of physical Parameters

V_0	λ_T	d	A	S	$\Pr$	$-\theta'(0)$
1	1	2	0.1	0.5	3	0.33663
1.5	1	2	0.1	0.5	3	1.05796
2	1	2	0.1	0.5	3	1.66872
1	10	2	0.1	0.5	3	0.30613
1	50	2	0.1	0.5	3	0.30353
1	1	3	0.1	0.5	3	0.15622
1	1	1	0.1	0.5	3	0.71854
1	1	2	0.2	0.5	3	0.42873
1	1	2	0.3	0.5	3	0.51255
1	1	2	0.1	0.1	3	0.67383
1	1	2	0.1	-0.1	3	0.80292
1	1	2	0.1	0.5	7	1.57693

Table -3: Numerical values of Sherwood number at the sheet for various values of physical parameters.

V_0	A	Sc	λ_T	$-\phi'(0)$
1	0.1	0.3	1	0.21214
1.5	0.1	0.3	1	0.37502
2	0.1	0.3	1	0.53159
1	0.2	0.3	1	0.27714
1	0.3	0.3	1	0.33518
1	0.1	0.6	1	0.41414
1	0.1	0.78	1	0.54814
1	0.1	0.3	3	0.17375
1	0.1	0.3	5	0.1519

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