

TOPOLOGY OPTIMIZATION AND STRUCTURAL ANALYSIS OF SIMPLE COLUMN AND SHORT PRESSURIZED BEAMS USING OPTIMALITY CRITERION APPROACH IN ANSYS

Kishan Anand¹, Anadi Misra²

¹P.G. Student, Department of Mechanical Engineering, G.B.P.U.A.T., Pantnagar, U.S.Nagar, India

²Professor, Department of Mechanical Engineering, G.B.P.U.A.T., Pantnagar, U.S.Nagar, Uttarakhand, India

Abstract –The optimization technique for structural topology can be done by two methods, analytically and by using numerical technique. These days, FEA software packages are used for various analysis of designs which are purely based on numerical methods. In this paper ANSYS is used for structural analysis and topological optimization of a simple column fixed at bottom edge and central point load applied on upper edge, two point and three points supported pressurized short beams. This paper presents compliance, optimized shape, deformed and undeformed shape, displacements and stresses of the linearly elastic isotropic structures by using ANSYS software based on optimality criterion.

Key Words: Topological Optimization, Simple Column, Short Pressurized Beams, ANSYS, Compliance, Optimality Criterion Approach, NAND SIMP solution etc...

1. INTRODUCTION

The paper presents the optimal design of a simple column, a two point supported pressurized short beam and three point supported short beam. The plane state of stress is assumed for the given structures. The optimal design is performed by ANSYS software which gives the optimum topology of the structures mainly it is achieved by reducing material in the design domain. For the optimization, the finite element method is used to discretize the structures and topology is performed by removing parts of elements to get a continuum design with holes. The models are considered to be linearly elastic isotropic structures whose analysis has not been done so far by using optimality criterion approach in ANSYS. The work presented in the paper is obtaining compliance value, optimal topology, deformed shape, displacements of optimized shapes with deformed and undeformed edges, stress distribution in the optimized topology and von-Mises stresses variation of the structures.

Bendsoe and Kikuchi 1988, developed and applied homogenization scheme to structural optimization. Here a small cell structure was designed using a fixed grid

finiteelement representation and then homogenization was used to calculate the effectiveproperties of a material composed of the individual cells.

Suzuki and Kikuchi 1991, applied the homogenization method of Bendsoe and Kikuchi to extra problems in order to validate it.

Giles and Thompson 1973, noted that a process of optimization leads almost inevitably to designs which exhibit the notorious failure characteristics often associated with the buckling of thin elastic shells". Thus removing material deemed unnecessary based on a given set of loading and boundary conditions may make the structure subject to failure or collapse under loads.

This has led to engineers wanting to impose extra constraints on the optimization problem in order to find optimal structures which are not unstable. This constraint is an eigenvalue constraint which is similar in mathematical structure to a constraint on the harmonic (or resonant) modes of the structure.

Haftka and Gurdal 1991, published their book on elements of structural optimization. They prescribe the derivative of an eigenvalue constraint for the case in which the eigenvalue is simple. However they completely neglect to give an expression for the derivative of the stress stiffness matrix, Neves et al. 1995, maximize the minimum buckling load of a continuum structure subject to a volume constraint in an optimal reinforcement sense. They do find spurious buckling modes in which the buckling of the structure is concerned to the regions which are supposed to represent voids. Their solution to eradicate such modes was to set the stress contributions of low density elements in the stress stiffness matrix to zero.

Also Neves et al. 2002, considered the problem of minimizing a linear combination of the homogenized elastic properties of the structure subject to volume and buckling constraints applied to periodic microstructures. They do not use the SIMP method to penalize intermediate densities but instead add a penalty term to the objective function considered. They also make some assumptions that all the eigenvalues of the buckling problem are positive which significantly simplifies the calculations. They note that the appearance of low-density regions may result in non-physical localized modes in the low-density regions, which are an artefact of the

inclusion of these low-density regions that represent void material in the analysis".

Eschenauer and Olhoff 2001, described that the topology optimization is based on the usage of a SIMP material model. This paper presents short pressurized beams subjected to static pressure. The presentation of the material is given in a 2D context.

Bendsoe and Sigmund 2003, described that the domain of high density then defines the shape of the mechanical element. For intermediate densities, material parameters given by an artificial material law can be used.

According to loads conditions, the structural topology optimization can be classified into two types: the topology optimization with fixed loads and that with design-dependent loads. Most of researchers focused on the continuum structures design subjected to the fixed loads. That is, the optimal material distribution is found upon the given design domain with the loads and the constraints specified. This class of optimization problem can be solved efficiently and satisfactorily now by the present methods.

Hammer and Olhoff 2000, the other class of topology optimization problem, that external loads depend on the shape and topology of the structure itself, has hardly been studied and is still open to explore.

Rozvany and Prager (1979), The main difficulty of this kind of problem, compared with the fixed load problem, is to identify the load surfaces of the structures during the optimization.

Zheng and Gea (2005), a similar method by using other physical fields to identify the load surfaces.

Hui Zhang & Xiong Zhang & Shutian Liu In worked for a new simple element-based search scheme is introduced to identify the load surface and the formulation of topology optimization problems stays with the context of the classical topology optimization formulation based on the Solid Isotropic Material with Penalization (SIMP) material model. The load surfaces are formed by the connection of the real boundary of elements and the pressures are transferred directly to corresponding element nodes.

Philip D. Browne 2013, worked for the minimization of compliance subject to maximum volume problem which is included for comparison with results.

One advantage of the present method is the linear density-stiffness relationship which has advantage for self weight or Eigen frequency problem. The topology optimization problem is solved through derived Optimality criterion method (OC), which is also introduced in the paper. Gunwant et al. obtained topologically optimal configuration of sheet metal brackets using Optimality Criterion approach through commercially available finite element solver ANSYS and obtained compliance versus iterations plots for various aspect ratio structures (brackets) under different boundary conditions.

Chaudhuri, worked on stress concentration around a part through hole weakening a laminated plate by finite element method. Peterson has developed good theory and charts on the basis of mathematical analysis and presented excellent

methodology in graphical form for evaluation of stress concentration factors in isotropic plates under in-plane loading with different types of abrupt change, but no results are presented for transverse loading. Patle et al. determined stress concentration factors in plate with oblique hole using FEM. Various angle of holes have been considered to evaluate stress concentration factors at such holes. The stress concentration factors are based on gross area of the plate.

The goal of topological optimization is to find the best use of material for a body such that an objective criterion (i.e. global stiffness, natural frequency, etc.) attains a maximum or minimum value subject to given constraints (i.e. volume reduction).

In this work, maximization of static stiffness has been considered. This can also be stated as the problem of minimization of compliance of the structure. Compliance is a form of work done on the structure by the applied load. Lesser compliance means lesser work is done by the load on the structure, which results in lesser energy is stored in the structure which in turn, means that the structure is stiffer. ANSYS employs gradient based methods of topology optimization, in which the design variables are continuous in nature and not discrete. These types of methods require a penalization scheme for evolving true, material and void topologies. SIMP (Solid Isotropic Material with Penalization) is a most commonly penalization scheme, and is explained in the next section.

2. MATERIALS AND METHODS

Topology optimization aims to answer the question, what is the best domain in which to distribute material in order to optimize a given objective function subject to some constraints?

Topology optimization is an incredibly powerful tool in many areas of design such as optics, electronics and structural mechanics. The field emerged from structural design and so topology optimization applied in this context is also known as structural optimization.

Applying topology optimization to structural design typically involves considering quantities such as weight, stresses, stiffness, displacements, buckling loads and resonant frequencies, with some measure of these defining the objective function and others constraining the system. For other applications aerodynamic performance, optical performance or conductance may be of interest, in which case the underlying state equations are very different to those considered in the structural case.

In structural design, topology optimization can be regarded as an extension of methods for size optimization and shape optimization. Size optimization considers a structure which can be decomposed into a finite number of members. Each member is then parameterized so that, for example, the thickness of the member is the only variable defining the member. Size optimization then seeks to find the optimal values of the parameters defining the members.

Shape optimization is an extension of size optimization in that it allows extra freedoms in the configuration of the structure such as the location of connections between members. The designs allowed are restricted to a fixed topology and thus can be written using a limited number of optimization variables. The topology optimization is performed using optimality criteria method through ANSYS software. There are many approaches derived to solve pressure load problems in topology optimization. Structural analysis is used to assess the behaviour of engineering structures under the application of various loading conditions. Commonly used structural analysis method includes analytical methods, experimental methods and numerical methods.

Analytical method provides accurate solutions with applications limited to simple geometries. Experimental methods are used to test prototypes or full scale models. However they are costly and may not be feasible in certain cases. Numerical methods are most sought-after technique for engineering analysis which can treat complex geometries also. Among many numerical methods, finite element analysis (FEM) is the most versatile and comprehensive numerical technique in the hands of engineers today. This process leads to a set of linear algebraic simultaneous equations for the entire system that can be solved to yield the required field variable (e.g., strains and stresses). As the actual model is replaced by a set of finite elements, this method gives an approximate solution rather than exact solution. However the solution can be improved by using more elements to represent the model.

2.1 The Optimality Criterion approach

The discrete topology optimization problem is characterized by a large number of design variables, N in this case. It is therefore common to use iterative optimization techniques to solve this problem, e.g. the method of moving asymptotes, optimality criteria (OC) method, to name two. Here we choose the latter. At each iteration of the OC method, the design variables are updated using a heuristic scheme.

Optimality criteria (OC) method was analytically formulated by Prager and co-workers in 1960. It was later developed numerically and become a widely accepted structural optimization method (Venkaya et al. 1968). OC methods can be divided into two types. One type is rigorous mathematical statements such as the Kuhn-Tucker conditions. The other is algorithms used to resize the structure for satisfying the optimality criterion. Different optimization problems require different forms of optimality criterion.

This paper considers the maximization of static stiffness through the inbuilt topological optimisation capabilities of the commercially available FEA software to search for the optimum material distribution in two plane stress structures.

The optimum material distribution depends upon the configuration of the initial design space and the boundary conditions (loads and constraints).

The goal of the paper is to minimize the compliance of the structure while satisfying the constraint on the volume of the material reduction.

Minimizing the compliance means a proportional increase in the stiffness of the material. A volume constraint is applied to the optimisation problem, which acts as an opposing constraint.

2.1.1 Element Type

Selection of element type is one of the most important features in topology optimization through ANSYS. Topological optimization in ANSYS supports 2-D and 3-D solid elements. By this technique the model can be discretized into following element type:

- (a). 2-D Solids: Plane 82
- (b). 3-D Solids: Plane 95

Plane 82: This is an 8-node element and is defined by eight nodes having two degree of freedom at each node. Translations in the nodal x and y directions (Figure 1). The element may be used as a plane element or as an axisymmetric element. The element has plasticity, creep, swelling, stress stiffening, large deflection, and large strain capabilities.

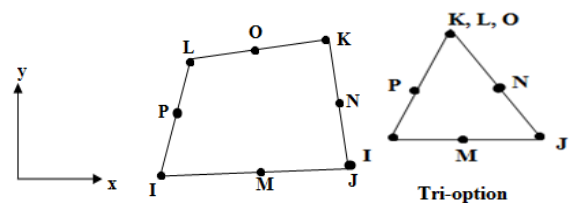


Fig- 1:Plane82 element with quad and tri options

Solid95: This element type has a quadratic displacement behavior and is well suited to model irregular meshes (such as produced from various CAD/CAM systems). The elements are defined by 20 nodes having three degrees of freedom at each node: translations in the nodal x , y , and z directions (Figure 2). The elements also have plasticity, creep, swelling, stress stiffening, large deflection and large strain capabilities. In the present paper structures are considered to be 2 D, so here plane 82 8-node type element is taken for discretization.

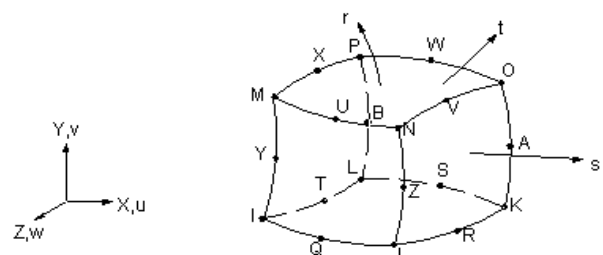


Fig- 2:Rectangular domain meshing

To visualize, more the volume of material, lower will be the compliance of the structure and higher will be the structural stiffness of the structure. For implementation of this, APDL codes for various beam modelling and topological optimisation were written and run in ANSYS.

2.2 Specimen Geometry and Boundary Conditions

In the present investigation, three specimen geometries and boundary conditions applied have been used as shown in the figures below. The specimen 1 is taken from the research work of Philip Anthony Browne. Specimen 2 and specimen 3 are taken from the research work of H. Zhang et al. All the three models are under plane state of stress.

2.2.1 Centrally loaded column (Model 1): Example 1 is a stiffness topology optimization problem for a simple column structure. Here is presented a somewhat trivial optimization problem which is included for comparison with results of ANSYS based and another method. The design domain is square and a unit load is applied vertically downwards at the centre of the top of the design domain and the base is fixed, as shown in Figure-3. The properties for given problem are $E=1 \text{ Pa}$ and Poisson's ratio is 0.3.

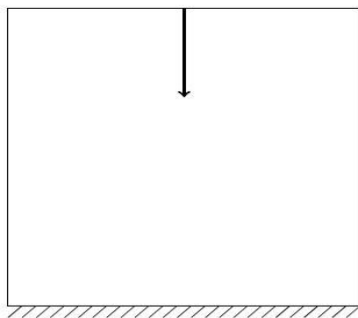


Fig-3: Design domain of model column problem. This is a square domain with a unit load acting vertically at the midpoint of the upper boundary of the space.



Fig-4: NAND SIMP solution to centrally loaded column problem on a 750 x 750 mesh and $V_{frac} = 0.2$

2.2.2 Model 2: In this example, a simply supported short beam subjected to the surface loading on the top is optimized. The admissible design domain, boundary conditions, and initial load conditions are shown in Fig. 8a. The pressure is set to 1.0. The design domain is discretized

by 800 (40x20) square elements. The material properties are $E=100$ and $\nu=0.3$. The volume fraction of the solid material is 0.5.

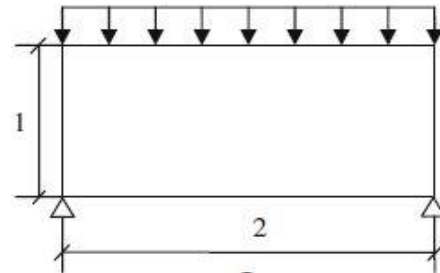


Fig-5: Geometry and boundary conditions of two point pressurized short beam

The structure with the fixed load is optimized first as a reference. The optimized topology is seen in Fig. 6 to be a bridge-like result. The optimal topology result for the structure with movable pressure is shown in Fig. 8c. An arch-like structure is obtained and the compliance of the structure is 0.157.

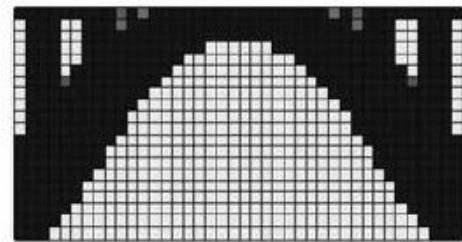


Fig-6: Optimal topology by using a new kind of element based search scheme of load surfaces method

2.2.3 Model 3: A general case of more than two support points is considered. As shown in Fig. 7, another support point is added in the middle bottom of the beam. The elastic properties and volume fraction of solid material are the unchanged. Eight hundred square elements are used to discretize the design domain. The optimal topology of structure is shown in Fig. 8. The structural compliance is 0.060, and it is much less than that of two-point supported structure.

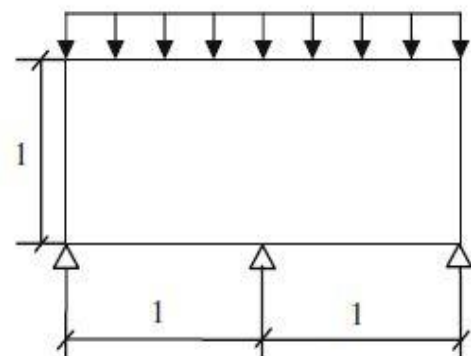


Fig-7: A three-point supported short beam design, a Pressurized short beam sketch.

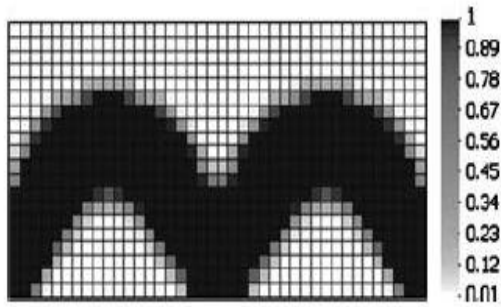


Fig-8: The optimized topology with a new kind of element based search scheme of load surfaces method

Fig-9: Optimal design for Model 1 using optimality criteria approach

The topologically optimized shape as obtained for the two point supported pressurized short beam (model 2) under the given boundary conditions is obtained by using optimality criteria using ANSYS. Figure 10 shows the topologically optimized shape through ANSYS.

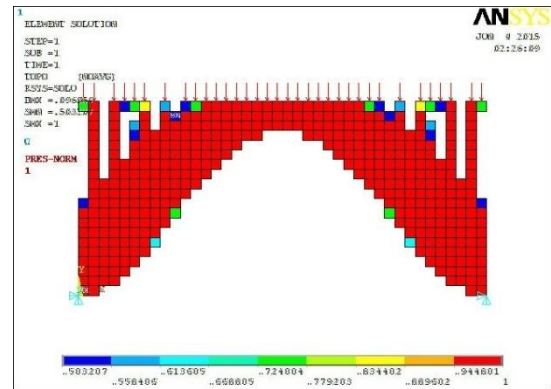


Fig-10: Optimal design for Model 2 using optimality criteria approach

The topologically optimized shape as obtained for the three point supported pressurized short beam (model 3) under the given boundary conditions is obtained by using optimality criteria using ANSYS. Figure 11 shows the topologically optimized shape through ANSYS.

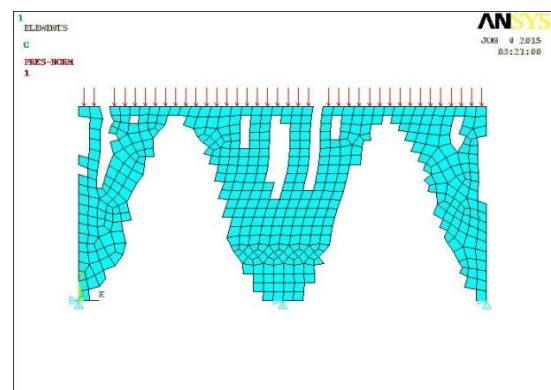


Fig-11: Optimal design for Model 3 using optimality criteria approach

The optimal topology through ANSYS for all the three structures are nearly same as obtained in the research paper from which the problems are taken for validation. The topologies obtained by ANSYS shows the applied boundary conditions at bottom and pressure load on the top from figures of two point and three point pressurized short beams.

3.3 Compliance:

For structure 1, the initial value of compliance was 77.9263Nmm and the final value as obtained after 16

3. RESULTS

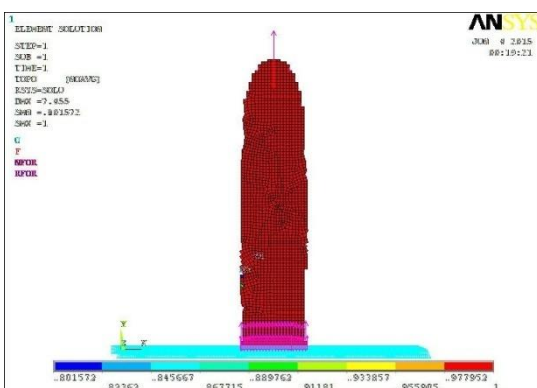
In this section the optimal topology of structures are shown obtained from the Optimality Criteria Approach through ANSYS. Further the iteration versus values of compliances for all the structures are shown in the charts [1, 2 & 3]. Chart shows the graph between Compliance and iterations.

3.1 Structure Compared:

In this section, final compliance and optimal shape of the models obtained with the help of ANSYS based Optimality Criterion. Model 1 (simple column) has been compared with the research paper of P.D. Browne (Minimization of Compliance Subjected to Maximum Volume) and Model 2 & Model 3 are compared with the research paper of Hui Zhang & Xiong Zhang & Shutian Liu (A new boundary search scheme for topology optimization of continuum structures with design-dependent loads, 2008).

3.2 Optimized Shape:

Figure 6, Shows the topology optimization through NAND SIMP Solution Method which is nearly same as the topologically optimized shape as obtained for the beam structure under the given boundary conditions which is obtained by using optimality criteria using ANSYS. Figure 9, shows the topologically optimized shape through ANSYS for a Simple Column (Model 1).



iterations is 7.50157Nmm for mesh size of 200. A reduction of 70.42473Nmm from its initial value. Variation of compliance with iteration is shown in the graph 1 below. Vertical axis represents the compliance and the horizontal axis represents the iteration.

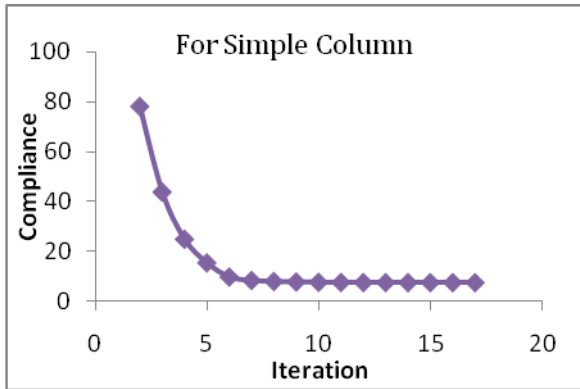


Chart -1: Compliance and iteration plot for Simple Column structure (Model 1)

For structure 2, the initial value of compliance was 0.45281Nmm and the final value as obtained after 36 iterations is 0.12743Nmm. A reduction of 0.32538Nmm from its initial value. Variation of compliance with iteration is shown in the graph 2 below. Vertical axis represents the compliance and the horizontal axis represents the iteration.

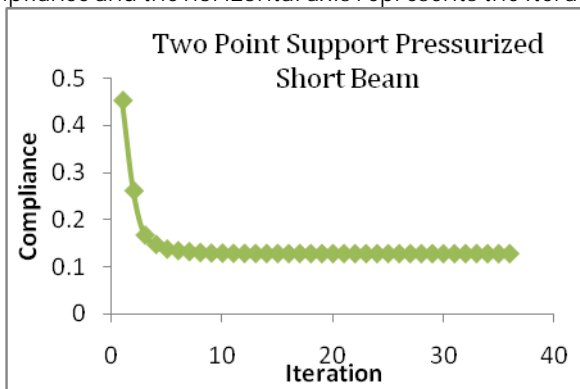


Chart -2: Compliance and iteration plot for Two Point Supported Pressurized Short beam (Model 2)

For structure 3, the initial value of compliance was 0.20737Nmm and the final value as obtained after 24 iterations is 0.0715953Nmm. A reduction of 0.1357747Nmm from its initial value. Variation of compliance with iteration is shown in the graph 3 below. Vertical axis represents the compliance and the horizontal axis represents the iteration.

The compliance obtained by ANSYS is nearly same as that obtained by BESO method.

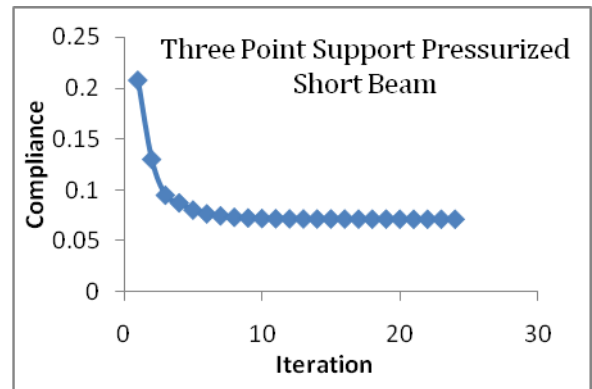


Chart -3: Compliance and iteration plot for Three Point Supported Pressurized Short beam (Model 3)

3.3.1 For structure 1:

Compliance obtained by NAND SIMP Solution method= 8.2047 Nmm.and iterations= 104
 Compliance obtained by ANSYS using optimality criteria method = 7.5017Nmm and iterations= 16
 Variation in two results= 8.5683%

The optimized shape obtained by optimality criteria using ANSYS is nearly same as that by Nested Analysis and Design (NAND) SIMP solution method.

3.3.2 For structure 2:

Compliance obtained by a new kind of element based search scheme of load surfaces method= 0.157 Nmm. and iterations= 92
 Compliance obtained by ANSYS using optimality criteria method = 0.127428Nmm and iterations= 36
 Percent Reduction= 18.835%

3.3.3 For structure 3:

Compliance obtained by a new kind of element based search scheme of load surfaces method= 0.060 Nmm. and iterations= 68
 Compliance obtained by ANSYS using optimality criteria method = 0.0715953Nmm and iterations= 24
 Percent Reduction= 19.325%

As we have seen from the above problems that the optimized shape obtained for the linearly elastic isotropic structures with ANSYS are nearly same and comparable with the methods mentioned in the research papers taken for validation.

3.3.4 Structural Analysis (Nodal Solution using ANSYS)

The structural analysis has been also done for the above mentioned structures. The table-1 given below shows the vector sum of displacement and von-Mises stress for all the three structures. From the figures 12, 13 and 14 we can see the deformed shapes with undeformed edge for the above mentioned problems and given boundary conditions.

(a). Deformed Shape for structure 1:

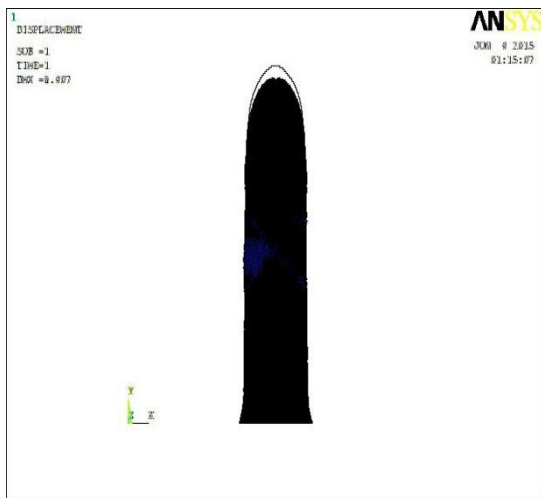


Fig-12: Deformed shape with undeformed edge (Model 1)

(b). Deformed Shape for Structure 2:

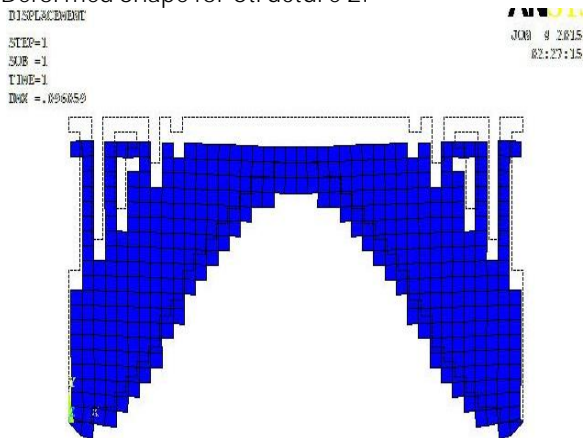


Fig-13: Deformed shape with undeformed edge (Model 2)

(c). Deformed Shape for Structure 3:

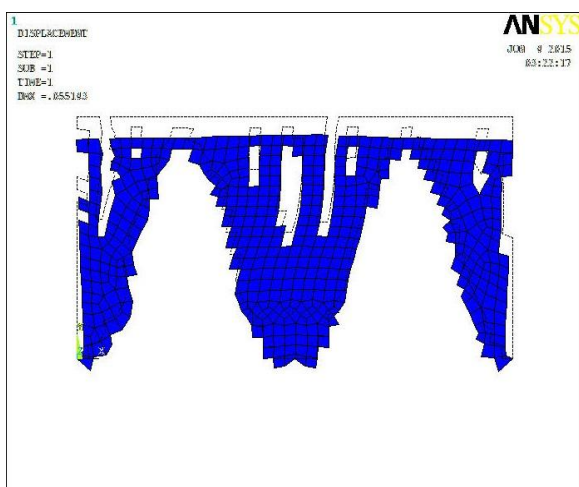


Fig-14: Deformed shape with undeformed edge (Model 3)

Table -1: Material Properties of Structures and Nodal Solutions (displacements and von-Mises stress)

S.N.	Structure	E	ν	Displacement	Stress
				Vector Sum	von-Mises
1.	Simple Column	1 Pa	0.3	8.407	813.94
2.	Two Point Supported pressurized Short beam	100	0.3	0.096059	99.23
3.	Three Point Supported pressurized Short beam	100	0.3	0.55143	49.166

3. CONCLUSIONS

The optimized shape of model 1 using optimality criteria in ANSYS is nearly the same as that by the Nested Analysis and Design (NAND) SIMP solution method of topological optimization. Further the variation in compliance is very small for model 2. Also the compliance obtained from optimality criteria using ANSYS is comparable than that obtained by a new kind of element based search scheme of load surfaces method for model 3, which is our basic objective of topological optimization. Thus ANSYS is an effective tool for topological optimization and the results obtained by ANSYS are more effective than the result obtained by the other method taken for comparison in this paper but in some cases it is comparable. For further work structural analysis has been done for the above mentioned structures.

ACKNOWLEDGEMENT

The present paper is written under the guidance of Dr. Anadi Misra. I am very thankful to him and Dheeraj Gunwant, Ph. D. scholar from GBPUA&T for supervising me through this work and for kindly supporting me in ANSYS software and helping enormously with their technical knowledge.

REFERENCES

- [1]. Bendsoe, M. P. and Kikuchi, N. 1988. Generating optimal topologies in structure design using a homogenization method, computer methods in applied mechanics and engineering 71: 197-224
- [2]. Bendsoe, M.P. and Sigmund, O. 1999. Material interpolation schemes in topology

- optimization, *Archive of Applied Mechanics* 69: 635-654
- [3]. Bendsoe, M.P. 1989. Optimal shape design as a material distribution problem, *Structural Optimization* 1: 193-202
- [4]. Hammer and Olhoff, N. 2000. Topology optimization of continuum structures subjected to pressure loading. *Struct. Multidisc. Optim* 19:85-92
- [5]. Suzuki, K. and Kikuchi, N. 1991. A homogenization method for shape and topology optimization. *Comput. Mech. Appl. Mech. Eng.* 93:291-318
- [6]. Zhou, M. and Rozvany, G.I.N. 1992. DCOC: an optimality criteria method for large systems part I: theory, *Structure Optimization* 5:12-25
- [7]. Bendsoe MP (1989) optimal shape design as a material distribution problem. *StructOptim* 1:193-202
- [8]. Rozvany GIN, Zhou M, Sigmund O (1994) Optimization of topology. In: Adeli H (ed) *Advances in design optimization*. Chapman & Hall, London
- [9]. Bendsøe MP, Sigmund O (2003) *Topology optimization: theory, methods and applications*. Springer-Verlag, Berlin Heidelberg
- [10]. Katsuyuki Suzuki and Noboru Kikuchi. A homogenization method for shape and topology optimization. *Computer Methods in Applied Mechanics and Engineering*, 93:291{318, 1991.
- [11]. M.M. Neves, O. Sigmund, and M.P. Bendsoe. Topology optimization of periodic microstructures with a penalization of highly localized buckling modes. *International Journal for Numerical Methods in Engineering*, 54(6):809{834, 2002.
- [12]. M.M. Neves, H. Rodrigues, and J.M. Guedes. Generalized topology design of structures with a buckling load criterion. *Structural and Multidisciplinary Optimization*, 10(2):71{78, 1995.
- [13]. G.W. Hunt and J.M.T. Thompson. *A General Theory of Elastic Stability*. Wiley-Interscience, 1973.
- [14]. Raphael T. Haftka and Zafer Gurdal. *Elements of Structural Optimization*. Kluwer Academic Publishers, third edition, 1991.
- [15]. Martin Philip Bendsoe and Noboru Kikuchi. Generating optimal topologies in structural design using a homogenization method. *Computer Methods in Applied Mechanics and Engineering*, 71(2):197{224, November 1988.

BIOGRAPHIES



Kishan Anand obtained his bachelor's degree (B.Tech.) in Mechanical Engineering from Kumaon Engineering college Dwarahat (Almora), Uttarakhand, in the year 2011 and M. Tech. in Design and Production Engineering from G. B. Pant University of Agriculture and Technology, Pantnagar, Uttarakhand in the year 2015. His areas of interest are optimization and finite element analysis.



Anadi Misra obtained his Bachelor's, Master's and doctoral degrees in Mechanical Engineering from G. B. Pant University of Agriculture and Technology, Pantnagar, Uttarakhand, with a specialization in Design and Production Engineering. He has a total research and teaching experience of 28 years. He is currently working as professor in the Mechanical Engineering department of College of Technology, G. B. Pant University of Agriculture and Technology, Pantnagar and has a vast experience of guiding M. Tech. and Ph. D. students.