TOPOLOGICAL OPTIMIZATION TECHNIQUES FOR LINEAR ISOTROPIC STRUCTURES SUBJECTED TO STATIC AND SELF-WEIGHT LOADING CONDITIONS

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Abstract- ***This paper represents the optimal criteria method for topological optimization of isotropic material under different loads and boundary conditions with the objective to reduce mass of an existing material and study the different shape obtained. Topological optimization mainly comprises of a mathematical approach that optimizes the layout within a given design constraints, for a given set of loads and boundary condition such that the performance matches with the prescribed set of performance targets. Topological optimization solve the problem of distributing a given amount of material in a design domain subjected to load and supports conditions, such that the compliance of the structure is minimized while the stiffness of structure is maximized. For material distribution system solid isotropic with penalization approach is used. In all the structures objective function is compliance, design variable is pseudo density and state variables are the response of structures that is deflection. Objected function is subjected to volume constraint and by minimize the compliance stiffness of structures are maximize. Different numerical examples are taken to study the optimal criteria approach and validate the results obtained with SA-SIMP and BESO method. This paper work represents topological optimization for static and self-weight loading using finite element solver ANSYS. APDL (ANSYS Parametric Design Language) has been employed for utilizing the topological optimization capabilities of commonly used finite element solver ANSYS. 8 node 82 elements are used to model and mesh the isotropic structures in ANSYS.***

Keywords- Optimality Criterion, SIMP, Topology Optimization, Pseudo-densities and Compliance minimization

1 INTRODUCTION

Topology optimization is a useful tool for a designer which generates the optimal conceptual shape of a mechanical structure. The structural shape is generated within a predefined design space. In addition, the user defines structural supports and loads. Without any further decision and guidance of the user, the method will give the structural shape thus provides a first idea of an optimum geometry. A desired property of the structure is maximized by changing the shape of the given material. Usually this maximized property is stiffness. Another usage of topology optimization is minimizing the weight, subjected to a given constraint (such as stress). Topology optimization method is a technique to find out optimal material distribution within predefined design domain. It can give the best conceptual design that can satisfy all design requirements. Topology optimization problem includes objective function, design domain and design constraints. Objective function represents the goal of the optimization method which is to be minimized or maximized.

With the exception of a few early landmark results [3 12], the historical development of the field of structural optimization seems to have followed an opposite route to the actual structural design process [2 20]. Since its inception, research in numerical optimal structural design went from element stiffness design, through geometric and shape optimization to topology optimization design. It is also clear that the major impact on the structural efficiency, in the sense of stiffness/volume or stress/volume ratio, is determined at the conceptual stage by the topology and shape of the structure. No amount of fine-tuning of the cross-sections and thicknesses of the elements will compensate for a conceptual error in the topology or the structural shape [13]. With the development of high-speed computer, the topology optimization method using numerical approach has been growing quickly [1 5 16]. In the present work we will be studying the topology optimization of continuum structures with the help of Optimality criteria method using ANSYS, also ANSYS use SIMP method for penalization of intermediate densities. The finite element based continuum topology optimization as a generalized shape optimization problem has experienced tremendous progress since the influential work of Bendsoe and Kikuchi [2]. They presented a homogenization based optimization approach of topological optimization. They assumed that the structure is formed by a set of non-homogenous elements which are composed of solid and void regions. They obtained optimal design under volume constraint through optimization process. In their method, the regions with dense cells are defined as structural shape, and those with void cells are areas of unnecessary material. It has also been demonstrated that the optimal material distribution can be considerably simplified by employing a density dependent isotropic material. In both the approaches, remeshing of the structural domain and the evaluation of shape density are avoided. This problem had a discrete nature, since the material distribution consisted of solid or void regions.

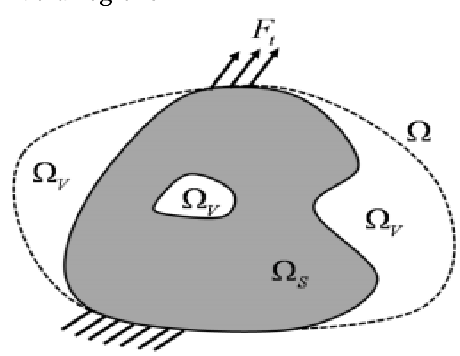


Fig. 1: Design domain of typical topology optimization problem [10]

A scheme of design domain is shown in Figure 3.1, where F_t is the external force, Ω is the design domain, Ω_s denotes a solid domain and Ω_v represents a sub-domain without material. Topology optimization methods are based on FEM and sensitivity analysis. In FEM each finite element is assigned a design variable which is the material density of the element. By updating material density of each element, structure design can be improved to optimal design.

2 SIMP METHOD

The SIMP stands for Solid Isotropic Material with Penalization method. It is also known as the power-law approach, in which the material properties can be expressed in terms of the design variable material density using a simple "power-law" interpolation as an explicit means to suppress intermediate values of the bulk density. This method has been presented by Bendsoe [3]. The SIMP, material model where material properties are assumed constant within each finite element, discretizes the design domain with the design variables being the element densities. At each point of the design domain, the material properties are modeled as the relative material density raised to some power times the material properties of solid material. The common choice of design parameterization is to take *xi* as the design variable by convention, *xi* = 1 at a point signifies a material region while *xi* = 0 represents void. Each finite element (formed due to meshing in ANSYS) is given an additional property of pseudo-density, *xi* where 0≤*xi*≥1, which alters the stiffness properties of the material.

$$x_i = \frac{\rho_i}{\rho_o} \tag{1}$$

ρi= Density of the ith element
ρ0= Density of the base material
xi= Pseudo-density of the ith element

This Pseudo-density of each finite element serves as the design variables for the topology optimization problem and the intermediate values are penalized according to the following scheme:

$$E_i = x_i^p E_o \tag{2}$$

Here E_i is the material young modulus of the i^{th} element while E_0 denotes the young modulus of the solid phase material. The stiffness of intermediate densities is penalized through the power law relation, so they are not favoured. As a result, the final design consists primarily of solid and void regions.

3 MATERIAL AND METHOD

3.1 Optimal Criteria Approach

Optimality criteria are necessary conditions for minimality of the objective function and these can be derived by using either variational methods or extremum principles of mechanics. Optimality criteria (OC) method was analytically formulated by Prager and co-workers in 1960. It was later developed numerically and become a widely accepted structural optimization method.OC methods can be divided into two types. One type is rigorous mathematical statements such as the Kuhn-Tucker conditions. The other is algorithms used to resize the structure for satisfying the optimality criterion. Different optimization problems require different forms of optimality criterion. In Kuhn-Tucker conditions, the inequality constraints can be transformed into equality constraints by adding slack variables. Here the optimization in its most general form may be expressed as follows

$$\text{Minimize } f(x)$$
$$\text{Such that } h_j(x) = 0 \quad j=1,2, \ldots n_j \qquad (3)$$
$$g_k(x) \leq 0 \quad k=1,2,\ldots n_k$$

Where h_j and g_k are constraints, j and k are the number of equality of constraints and inequality constraints, respectively

The Lagrangian function of the optimization can be defined as

$$L(x,t,\lambda,\zeta) = f(x) + \sum_{j=1}^{n_k} \zeta_j h_j(x) + \sum_{k=1}^{n_j} \lambda_k (g_k(x) \qquad (5)$$

Where ζ_j *and* λ_k are Lagrangian multipliers

Differentiating the Lagrangian function (5) with respect to x, t, λ, ζ we obtain

$$\frac{\partial L}{\partial x_i} = \frac{\partial f}{\partial x_i} + \sum_{j=1}^{n_k} \zeta_j \frac{h_j(x)}{\delta x_i} + \sum_{k=1}^{n_g} \lambda_k \frac{g_k(x)}{\partial x_i} = 0 \qquad (6)$$

$$\frac{\partial L}{\partial \zeta_j} = h_j = 0 \qquad (7)$$

$$\frac{\partial \zeta}{\partial \lambda_k} = g_k + t_k^2 = 0 \qquad (8)$$

$$\frac{\partial L}{\partial t_k} = 2\lambda_k t_k = 0 \qquad (9)$$

From equation 7 & 8

$$g_k(x) \leq 0 \qquad (10)$$
$$\lambda_k g_k = 0 \qquad (11)$$

This implies that when an inequality constraint is not active, the Lagrangian multiplier associated with the constraint is zero. By using Kuhn-Tucker conditions, the optimality conditions for the optimization problem can be stated as

$$\frac{\partial L}{\partial x_i} = \frac{\partial f}{\partial x_i} + \sum_{j=1}^{n_g} \zeta_j \frac{h_j(x)}{\partial x_i} + \sum_{k=1}^{n_g} \lambda_k \frac{g_k(x)}{\partial x_i} = 0 \qquad (12)$$

$$\frac{\partial L}{\partial \zeta_j} = h_j = 0 \qquad (13)$$

$$g_k(x) \leq 0 \qquad (14)$$

$$\lambda_k g_k = 0 \qquad (15)$$

$$\lambda_k \geq 0 \qquad (16)$$

The optimal criteria method is one of the best-established and widely accepted optimization techniques.

3.2 Numerical Examples

Three numerical examples are taken to demonstrate the validity and efficiency of the proposed approach. The specimens are taken from the work of Garicia-Lopez *et al.* [8] and Huang and Xie [9]. All the models are under plane state of stress.

Model 1: Cantilevered beam under static loading

A cantilever beam of thickness 1mm is considered in this case. The cantilever is under the state of plane stress and supports a concentrated load of magnitude 1N at the bottom right corner. The left edge is fixed as shown in Figure 2. The meshing is done with 8-node quadrilateral elements by giving element edge length one for each line. The results were compared with combining simulated annealing and SIMP approach [8]. Table 4 shows the final compliance obtained with ANSYS (OC) and combining simulated annealing and SIMP approach. Material properties for Model 1 are shown in Table 1.

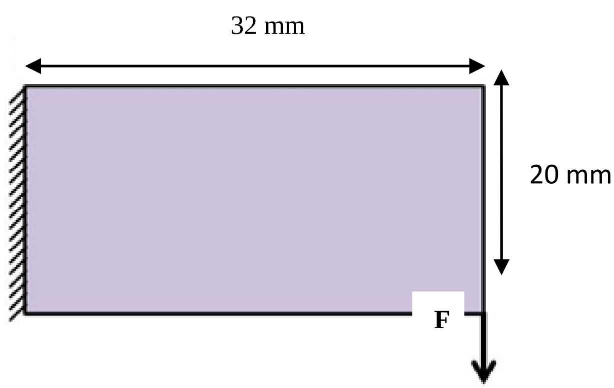


Fig. 2: Geometry and boundary conditions for Model 1

Table 1: Material properties, Load, Elements and Volume usage fraction for Model 1

Young's modulus	Poisson's ratio	Load	Elements	Volume Fraction
1 N/m^2	0.3	1N	640(32*20)	0.4

Model 2: Messerschmitt Bolkow Bolhm beam under static loading

The beam is in the state of plane stress with a thickness of 1 mm. The beam is optimized for minimum compliance. Due to symmetry of the model, only half of the model is considered with symmetry boundary conditions as it is symmetric about the vertical axis. The beam is supported by a roller support at the bottom right corner and symmetric boundary conditions are applied on the left edge as shown in Figure 3. The meshing is done with 8 nodes quadrilateral elements by giving element edge length one for each line. The results are compared with combining simulated annealing and SIMP approach [8]. Table 5 shows the final compliance obtained with ANSYS (OC) and combining simulated annealing and SIMP approach. Material properties for model 2 are shown in Table 2.

Table 2: Material properties, Load, Elements and Volume usage fraction for Model 2

Young's Modulus (E)	Poisson's ratio (ν)	Load (N)	Elements	Volume Usage Fraction
1 N/m²	0.3	1	1200(60*20)	0.5

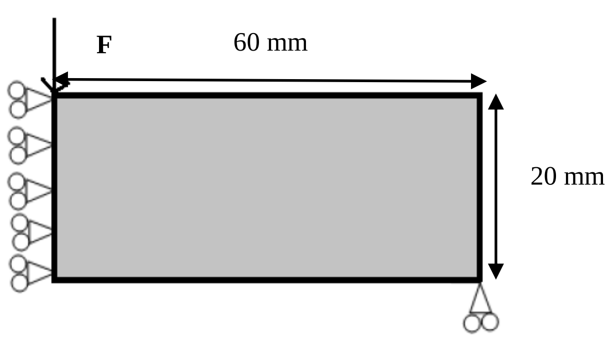


Fig. 3: Geometry and boundary conditions for Model 2

Model 3: Messerschmitt Bolkow Bolhm beam under self-weight

The beam is in the state of plane stress with a thickness of 1 mm. Here the classic MBB beam subjected to a concentrated load and its self-weight is to be optimized. The dimensions and support conditions of the design domain are shown in Figure 4. Due to the symmetry, only half of the design domain is discretized with 100x50 8-node plane stress elements. The results are compared with the results of X.Huang et al. [9] who utilized BESO method for topological optimization. The material volume constraint is set to be 40% of the whole design domain. Material properties for model 2 are shown in Table 3.

Table 3: Material Properties and Density Used for MBB Beam (Model 4)

Young's modulus	Poisson's ratio	Density	Volume fraction
200 N/ mm^2	0.3	78 kg/m^3	0.4

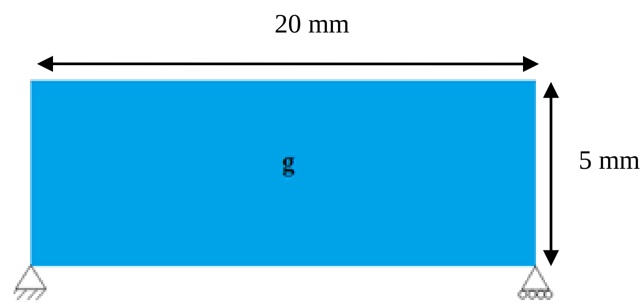


Fig. 4: Geometry and boundary conditions for Model 3

4 RESULT AND DISCUSSIONS

This section presents the detailed results of FE analysis and optimization of the above structures. Final compliance and optimal shape of the models obtained with the help of gradient based ANSYS based Optimality Criterion have been compared with SA-SIMP and BESO method [8 9].

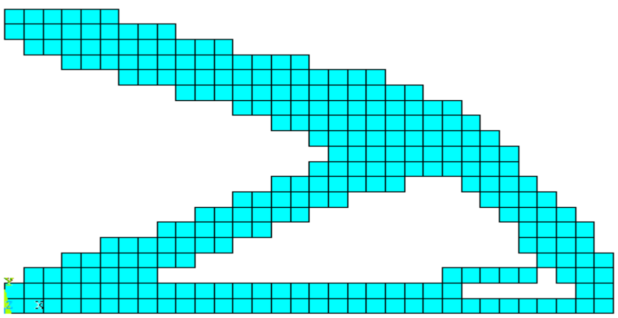
Model 1: Cantilevered beam under static loading

The optimal shape of the cantilever beam has been obtained through ANSYS (OC) as shown in Figure 5 (a). The shapes obtained through different methods are almost same. The final value of compliance after topological optimization is presented in Table 4 comprising of the optimal compliance values ANSYS (OC) method give lowest value. As it has been observed that, final compliance value obtained through ANSYS is 2.084% lower than PS-RoA method, 1.814% lower than RS-RoA method. From the table it has been observed that number of iterations required by ANSYS based OC is 39. In ANSYS (OC) method convergence criteria is 0.0001 given Mesh density is same in all the method. Above result show that ANSYS (OC) can use for topological optimization and on comparison ANSYS (OC) is more effective.

Table 4: Comparison between ANSYS OC, PS-RoA and RS-RoA for Model 1

Method	ANSYS OC	PS–RoA	RS–RoA
Compliance (Nmm)	52.224	53.3123	53.1714
Iterations	39	*	*

*Not available



(a)

(b)

(c)

Fig. 5: Optimal shapes obtained using (a) ANSYS OC (b) PS-RoA (c) RS-RoA

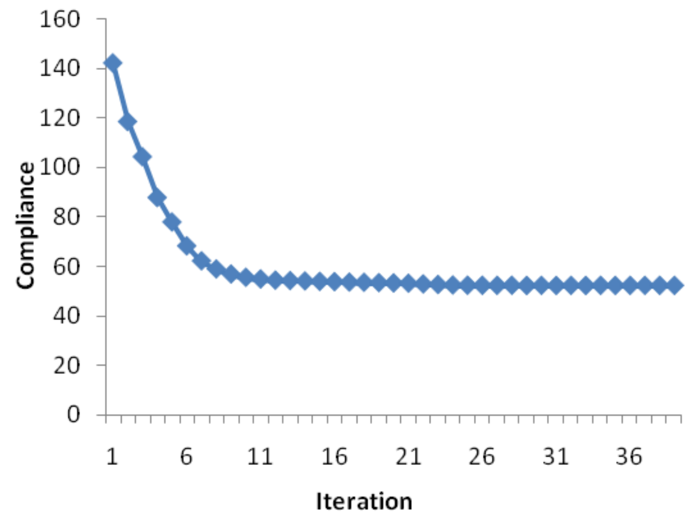
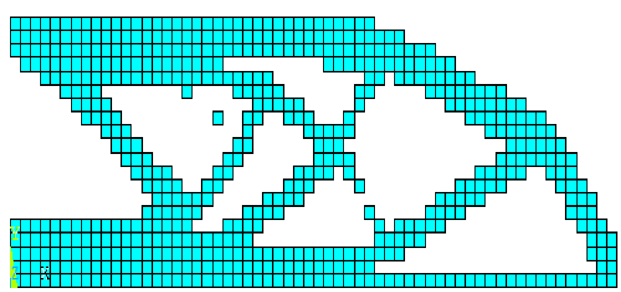


Fig. 6: Convergence of compliance values for cantilever beam

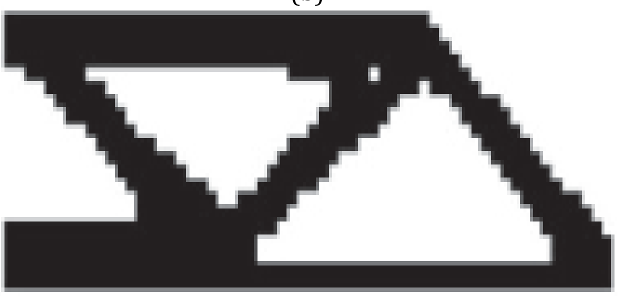
Model 2: Messerschmitt Bolkow Bolhm beam under static loading

The optimal shape of the MBB beam has been obtained through ANSYS b(OC) as shown in Figure 7 (a). The shapes obtained through different methods are almost same. The final value of compliance after topological optimization is presented in Table 5. On comparison of optimal compliance value ANSYS (OC) method give lowest value. As it has been observed that, final compliance values obtained through ANSYS is 3.499% lower than PS-RoA method, 3.44% lower than RS-RoA method. From the Table 5 it has been observed that number of iterations required by ANSYS (OC) is 32. In ANSYS (OC) method convergence criteria is 0.0001 given. Mesh density is same in all the method. From above result we conclude that ANSYS can used for topological optimization and on comparison ANSYS based OC method is more effective.



(a)

(b)



(c)

Fig. 7: Optimal shapes obtained using (a) ANSYS OC (b) PS-RoA (c) RS-RoA

Table 4: Comparison between ANSYS OC, PS-RoA and RS-RoA for Model 2

Method	ANSYS OC	PS–RoA	RS–RoA
Compliance (Nmm)	183.345	189.7603	189.6530
Iterations	32	*	*

*Not available

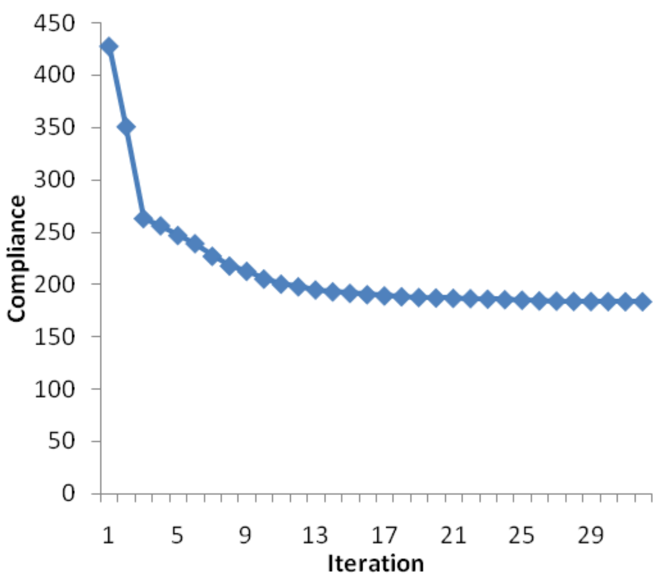
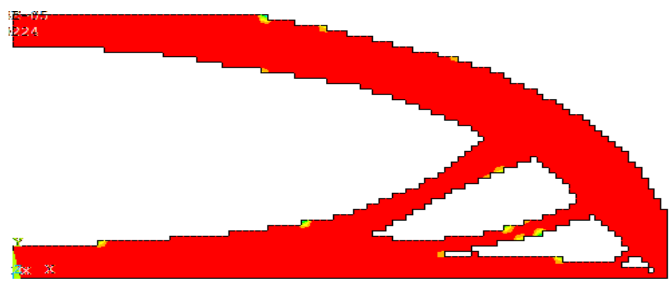


Fig. 8: Convergence of compliance values by ANSYS for MBB beam under static loading

Model 3: Messerschmitt Bolkow Bolhm beam under self-weight loading

The optimal shape of the cantilever beam has been obtained through ANSYS (OC) as shown in figure 9 (a). The shapes obtained through both methods are almost same. The final value of compliance after topological optimization is presented in Table 6 comprising of the optimal compliance value ANSYS (OC) method give higher value. As it has been observed that, final compliance value obtained through ANSYS is 5.88% higher than BESO method. From the Table 6 it has been observed that number of iterations required by ANSYS (OC) is 17 while for BESO method is 76. Figure 10 show the convergence of compliance values (OC). In ANSYS (OC) convergence criteria is 0.0001 given. Mesh density is same in both the method. From above result we conclude that ANSYS can use for topological optimization and on comparison ANSYS (OC) is more effective on the basis of number of iterations.



(a)

(b)

Fig. 9: Optimal Shapes Obtained by (a) ANSYS (OC) and (b) BESO Method

Table 6: Final compliance value for self weight for MMB beam Model 3

Method	Compliance	Iterations
BESO	0.034	76
ANSYS based OC	0.036	17

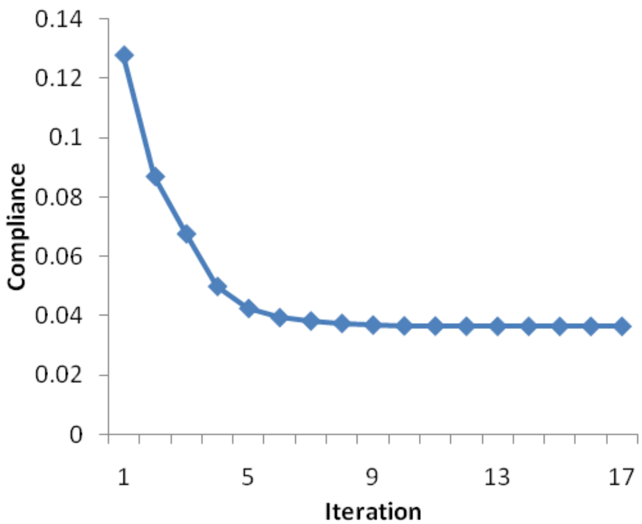


Fig. 10: Convergence of compliance values by ANSYS for MBB beam under static loading

5 CONLUSIONS

The results of Optimality Criteria method using ANSYS when compared with results obtained by SA-SIMP and BESO method for the linear elastic isotropic structures taken for study are better. Compliance value obtained through ANSYS for cantilever beam under static loading is 2.084% lower than PS-RoA method and 1.814% lower than RS-RoA method and for MBB beam under static loading is 3.499% lower than PS-RoA method and 3.44% lower than RS-RoA method also it takes lesser number of iterations to reach the optimal solution it is found that the Optimality Criteria using ANSYS converges very fast in comparison to SA-SIMP method. The optimal topologies obtained by both the methods are almost same. Compliance value obtained by ANSYS for MBB beam under self-weight is 5.88% higher than BESO method where as number of iterations required by ANSYS is 17 while for BESO method is 76. It is found that the Optimality Criteria approach using ANSYS converges very fast in comparison to BESO method. The optimal topologies obtained by both the methods are almost same.

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