Spectral Collocation Solution of Thermosolutal Free Convective MHD Flow Past through low-heat resistance medium with internal heat source

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Abstract: In this paper attempt has been taken to understand the variation of prandtl number for MHD natural convective fluid flow past a low heat resistance sheet with internal heating. The boundary layer flow in viscous media is presented in terms of physical model which is transformed in the set of Coupled Ordinary differential equation using similarity transformation. The set of differential equation solved numerically. The main emphasis is given on the Varity of prandtl number with quantitative change of internal heating. The velocity , temperature and stream function is plotted w.r.t to different physical parameter .

Key Words: MHD, Spectral collocation, Free Convection, Thermosolutal

1. INTRODUCTION

Thermosolutal convection in a porous media occurs in many systems, and this problem has attracted considerable interest during the past few decades because of its wide range of applications, from the solidification of binary mixtures to the migration of solutes in water saturated soils. The other examples include geophysical system, electrochemistry and migration of moisture through air contained in fibrous insulation. Extensive reviews on this subject can be found in the books by Ingham and Pop [1] , Vafai [2,3] , Nield and Bejan [4], Vadasz [5]

There are large number of practical situations in which convection is driven by internal heat source. Due to the internal heating of the earth that there exists a thermal gradient between the interior and exterior of the earth's crust, saturated by multi components fluids, which helps convective flow. Further, internal heat source is the main energy source of celestial bodies which is generated by radioactive decay and nuclear reaction. Therefore, the effect of internal heat generation is very important in several applications that include reactor safety analyses, geophysics, metal waste form development for spent nuclear fuel, fire and combustion modeling , and storage of radioactive materials. Research articles related to internal heat source in porous media are provided by Khalili et al.[6], Hill [7], Capone et al. [8], Bhadauria [9], Gaikwad and Dhanraj [10]

Acharya et al [11] studied heat and mass transfer over an accelerating surface with heat source in presence of suction and injection. Atul Kumar Singh [12] investigated the effects of mass transfer on free convection in MHD flow of a viscous fluid. Cortell [13] studied flow and heat transfer of an electrically conducting fluid of second grade over a stretching sheet subject to suction and to a transverse magnetic field. Gebhart and Pera [14] made extensive studies to such a combined heat and mass transfer flow to highlight the insight of the flow phenomena. Helmy[15] has studied MHD unsteady free convection flow past a vertical porous plate. Kandasamy [16] studied the effects of heat and mass transfer along a wedge with heat source and convection in the presence of suction or injection. Kumari and Nath ,[17] have studied development of two-dimensional boundary layer with an applied magnetic field due to an impulsive motion. Muthukumaraswamy and Ganesan [18] have studied unsteady flow past an impulsively started vertical plate with heat and mass transfer. Kim [19] presented an analysis of an unsteady MHD convection flow past a vertical moving plate embedded in a porous medium in the presence of transverse magnetic field.

Double-diffusive mixed convection in a vertical pipe under local thermal non-equilibrium state has been investigated by P. Bera et al[20]). The non-Darcy Brinkman–Forchheimer-extended model has been used and solved numerically by spectral collocation method. Special attention is given to understand the effect of buoyancy ratio (N) and thermal non-equilibrium parameters: inter phase heat transfer coefficient (H) as well as porosity scaled thermal conductivity ratio (c) on the flow profiles as well as on rates of heat and solute transfer. S.Kapoor et al [21] , to understand the influence of prandtl number for MHD natural convective fluid flow past a low heat resistance sheet , which is physically modeled as a boundary layer flow in viscous media and Solve it using finite difference methods not only this in his [22,23] he contributed in the direction of numerical method also to find solution of MHD problems , keeping in view of this

In the present paper, we consider the magneto hydrodynamic laminar boundary layer heat transfer of allow heat resistance sheet embedded in viscous regime solve using finite difference methods. Such study has not appeared so far in the technical literature and constitutes a more realistic attempt to simulate the complex flows encountered in industrial engineering systems. presented graphically.

2. MATHEMATICAL MODEL

Let us consider the problem of cooling of a low-heat-resistance sheet that moves downwards in a viscous fluid when the velocity of the fluid far away from the plate is equal to zero. The variation of surface temperature are linear. The flow configuration and coordinate system is shown in Fig.1. All the fluid properties are assumed to be constant except for the density variations in the buoyancy force term of linear momentum. The magnetic Reynolds number is assumed to be small, so that the induced magnetic field is neglected. Electric field is assumed to exists and both viscous and magnetic dissipation are neglected. The Hall Effect, viscous dissipation and the joule heating term are neglected. Under these assumption along with the Bousineque approximation, the boundary layer equation for the problem

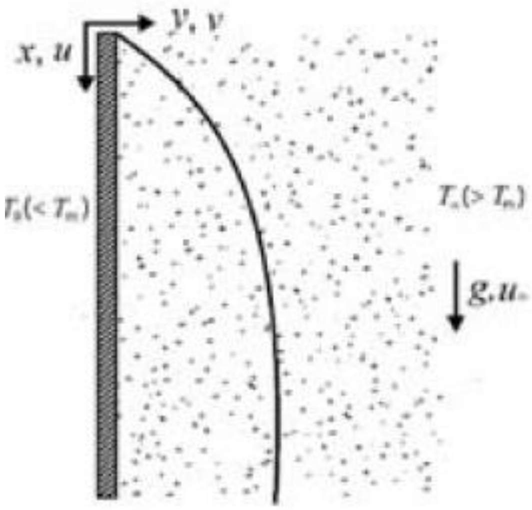

Figure 1 Physical Model of the problem

3. GOVERNING EQUATIONS

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\left(\frac{\partial^2 u}{\partial y^2}\right)+g\beta(T-T_\infty)+J\times B \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\kappa\frac{\partial^2 T}{\partial y^2}+Q_0(T-T_\infty) \tag{3}$$

$$u\frac{\partial s}{\partial x}+v\frac{\partial s}{\partial y}=\sigma\frac{\partial^2 s}{\partial y^2} \tag{4}$$

Where J is Current density

Neglecting the displacement current , the Maxwell equation and Ohm's law becomes

$$div\ B=0, Curl\ B=\mu_e J\ , Curl\ E=-\frac{\partial B}{\partial t} \tag{5}$$

Where B is magnetic field strength

$$J=\sigma\,(E+V\times B) \tag{6}$$

Where σ is electrical conductivity

and μ_e is the magnetic permeability

E is the electric field

The imposed and induced electric field are assume to be negligible under the assumption of low magnetic Reynolds number

$$J\times B=-\sigma\,\mu_e^2 H_0^2 u \tag{7}$$

i.e equation reduce to

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \tag{8}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\left(\frac{\partial^2 u}{\partial y^2}\right)+g\beta(T-T_\infty)-\sigma\,\mu_e^2 H_0^2 u \tag{9}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\kappa\frac{\partial^2 T}{\partial y^2}+Q_0(T-T_\infty) \tag{10}$$

$$u\frac{\partial s}{\partial x}+v\frac{\partial s}{\partial y}=\sigma\frac{\partial^2 s}{\partial y^2} \tag{11}$$

Subject to the boundary conditions

$$u=0,\quad v=0,\quad T=T_0\ ,S=S_0 \text{ at } y=0 \tag{12}$$

$$u \to 0,\quad T \to \infty\ ,S \to \infty \text{as}\ \ y \to \infty, \tag{13}$$

Using the similarity transformation parameters we have

$$\psi = [g\beta(T - T_\infty)v^2 x_0^3]^{\frac{1}{4}} f(\eta), \tag{14}$$

$$T = T_\infty + (T - T_\infty)\left[\frac{x_0}{x_0 - x}\right]^3 \theta(\eta), \tag{15}$$

$$S = S_\infty + (S - S_\infty)\left[\frac{x_0}{x_0 - x}\right]^3 \emptyset(\zeta), \tag{16}$$

$$\eta = \left[\frac{g\beta(T - T_\infty)x_0^3}{v^2}\right]\frac{y}{(x_0 - x)}, \tag{17}$$

$$\zeta = \left[\frac{g\beta(S - S_\infty)x_0^3}{v^2}\right]\frac{y}{(x_0 - x)} \tag{18}$$

$$f''' - (f' + M)f' + \theta = 0, \tag{19}$$

$$\frac{1}{Pr}\theta'' - 3f'\theta + Q\theta = 0. \tag{20}$$

$$\frac{1}{Sc}\emptyset'' - 3f'\emptyset = 0 \tag{21}$$

The corresponding boundary conditions are

$$f(0) = 0,\ f'(0) = 0,\ f'(\infty) \to 0, \tag{22}$$

$$\theta(0) = 1,\ \theta'(0) = 0\ \ \theta(\infty) \to 0, \tag{23}$$

$$\emptyset(0) = 1, \emptyset'(0) = 0\ \ \emptyset(\infty) \to 0 \tag{24}$$

4. NUMERICAL COMPUTATION

The Spectral collocation method is adopted to find the numerical solution of the nonlinear coupled differential Equations (19)-(21) under the boundary condition (22)-(24) The comparison is also made with the finite difference technique which is available in literature.

The equation (19)-(21) may be written as

$$\frac{d^3 f}{d\eta^3} - \left(\frac{df}{d\eta} + M\right)\frac{df}{d\eta} + \theta = 0 \tag{25}$$

$$\frac{1}{Pr}\frac{d^2\theta}{d\eta^2} - 3\frac{df}{d\eta}\theta + Q\theta = 0 \tag{26}$$

$$\frac{1}{Sc}\frac{d^2\emptyset}{d\eta^2} - 3\frac{df}{d\eta}\emptyset = 0 \tag{27}$$

To approximate the field variables Suppose f and θ by Chebyshev polynomials the range [0,4] of the independent variable , η is mapped in to [-1,1] by using the function

$$2 - 2\xi = \eta \tag{28}$$

Here the maximum value of η is fixed to be 4 for the sake of connivance of the solution of flow dynamics

The governing equation (25)-(27) and the corresponding boundary condition in terms of Chebyshev ξ

$$\frac{d^3 f}{-2d\xi^3} + \frac{1}{2}\left(\frac{df}{-2d\xi} + M\right)\frac{df}{d\xi} + \theta = 0 \tag{29}$$

$$-\frac{1}{Pr}\frac{d^3\theta}{2d\xi^2} + \frac{3}{2}\frac{df}{d\xi}\theta + Q\theta = 0 \tag{30}$$

$$-\frac{1}{2\,Sc}\frac{d^3\emptyset}{d\xi^2} + \frac{3}{2}\frac{df}{d\xi}\emptyset = 0 \tag{31}$$

The boundary conditions (22)-(24) becomes

$$\theta = f = \emptyset = 0\ \ at\ \xi = 1\ and\ \ \frac{df}{d\xi} = \frac{d\theta}{d\xi} = \frac{d\emptyset}{d\xi} = 0\ at\ \xi = -1 \tag{32}$$

Now the equations becomes

$$-\frac{1}{2}\ \Sigma_{k=0}^{n} C_{jk} f_k\ + \frac{1}{2}(-\frac{1}{2}\Sigma_{k=0}^{n} A_{jk} f_k\ + M)\,\Sigma_{k=0}^{n} A_{jk} f_k\ + \theta = 0 \tag{33}$$

$$-\frac{1}{2Pr}\Sigma_{k=0}^{n} B_{jk}\theta_k\ + \frac{3}{2}\Sigma_{k=0}^{n} A_{jk} f_k\ \ \theta + Q\theta = 0 \tag{28}$$

$$-\frac{1}{2\,Sc}\Sigma_{k=0}^{n} B_{jk}\emptyset_k\ + \frac{3}{2}\Sigma_{k=0}^{n} A_{jk} f_k\ \ \emptyset = 0 \tag{29}$$

Where j=1,2,3…n-1

$$A_{jk} = \begin{cases} \frac{c_j(-1)^{k+j}}{c_k(\xi_j-\xi_k)} , & j \neq k \\ \frac{\xi_j}{2(1-\xi_j^2)} , & 1 \le j = k \le n-1 \\ \frac{2n^2+1}{6} , & j = k = 0 \\ -\frac{(2n^2+1)}{6} , & j = k = n \end{cases}$$

And

$$B_{jk} = A_{jm} A_{mk}$$

$$C_{jk} = B_{jm} A_{mk}$$

In the above

$$c_j = \begin{cases} 2 , & j = 0, n \\ 1, & 1 \le j \le n-1 \end{cases}$$

And $\xi_j = \frac{\cos\pi j}{n}, for\ 0 \le j \le n$ are chebyshev collocation points

5. RESULT AND DISCUSSION

However the effect of Schmidt number over transverse velocity is different from magnetic field effect. The effect of Sc is to enhance the transverse velocity. The different value of M=0.5, 1.0, 1.5, 2.5 . The effect of S for each value of Sc remains uniform as we move away from the wall and this effect is elucidated through Fig. 2

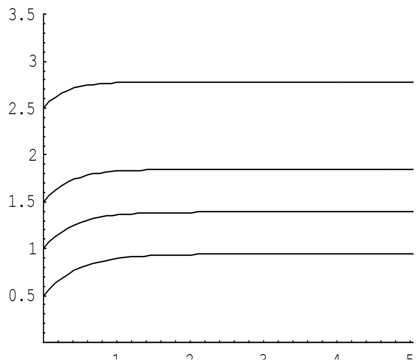

Fig.2 Dimensionless Transverse Velocity profiles for various values of Sc

The effect of Sc over non dimensional longitudinal velocity is depicted in Fig. 3. For M=0.5, 1.0, 1.5, 2.5 . It is observed that a steady decrease in longitudinal velocity accompanies a rise in S, with all profiles tending asymptotically to the horizontal axis. The non dimensional longitudinal velocity is observed to be maximum in all cases at the wall. It is also noted that the boundary layer thickness is reduced due to the effect of hear source.

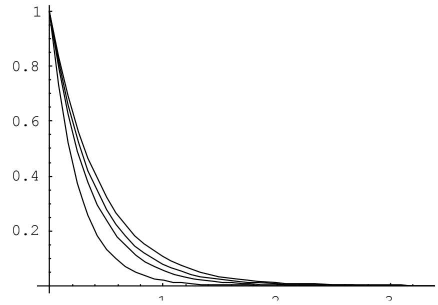

Fig. 3 Dimensionless longitudinal velocity profiles for various values of Sc

Variation in dimensionless temperature due to the variation of parameter Sc is visualized through Fig. 4. As the suction parameter Sc increases, the temperature reduces.Non-dimensional temperature profiles for various values of the parameter Q are depicted through Fig. 5. Where Q= -.75,0.5,0.1 It is noted that the temperature field decreases as Q increases and the increased temperatures are seen only near the wall

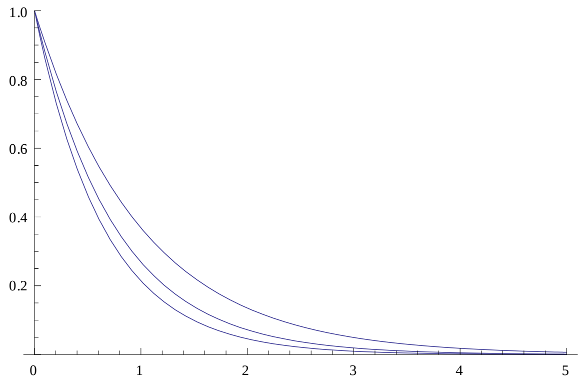

Fig. 4 Effect of Sc over Temperature distribution

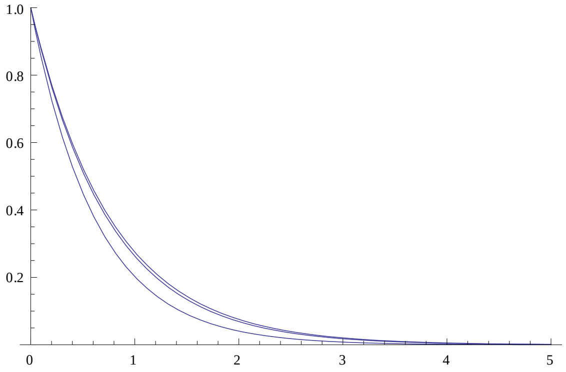

Fig.5 Influence of Q over Temperature profiles

The effect of Prandtl number over the temperature distribution is disclosed in Fig.6.The different values of Pr=0.71,1,1.5. The increasing values of Prandtl number decrease the temperature. The effect of Prandtl number is also to suppress the thermal boundary layer thickness. The dimensionless concentration profiles for various values of Schmidt number is shown in Fig.7.

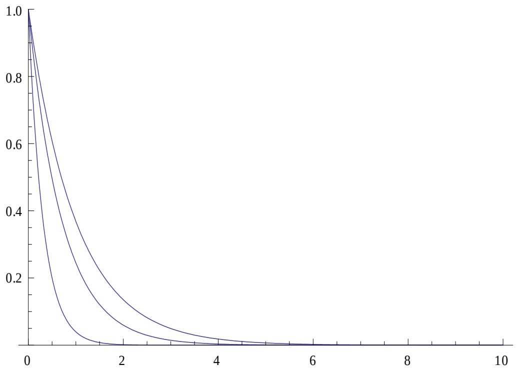

Fig. 6 Temperature profiles for various values of Pr

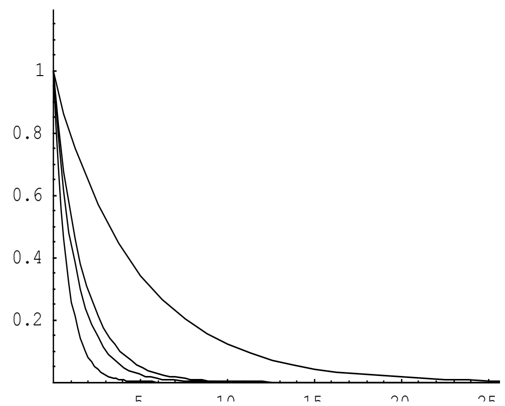

Fig.7 Effect of Sc over dimensionless concentration distribution

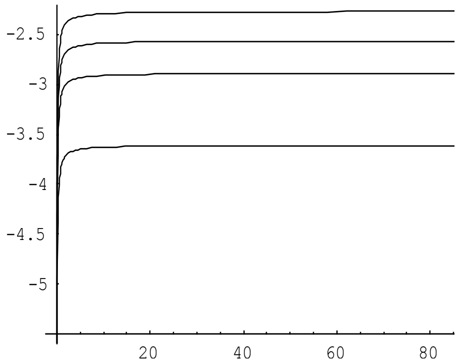

Fig.8 Skin friction coefficient for various values of S

Values of Sc = 0.22, 0.62, 0.78, 1.3.As Sc increases, the concentration decreases. Species concentration reaches higher values within the boundary layer.Variation in skin friction coefficient for different values of porosity parameter is studied using Fig- 8 The different values of Sc= 0.5,1.0,1.5,2.5. The effect its to decrease the skin friction coefficient

5. CONCLUSIONS

From the above study we conclude the following

i) The Spectral collocation method gives much similar result as we obtained in Finite difference method

ii) Every profile (Velocity or temperature) has shows an asymptomatic behavior and converges for the large value of η

iii) The velocity profiles are found to increase to a certain maximum point and then reduce asymptotically to zero. As Prandtl number increases, the temperature profile and the thermal boundary layer thickness decrease.

iv) The internal heat source is effected in case of low heat resistance sheet in thermosoultal case

The magnetic number help in dragging the flow mechanism.

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