Deformations due to periodically varying heat sources in a reference temperature dependent thermoelastic porous material with a time-fractional heat conduction law

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Abstract - ***A one-dimensional problem for a homogeneous and isotropic thermoelastic infinite porous material under the dependence of modulus of elasticity and thermal conductivity on reference temperature subjected to a periodically varying heat sources is considered in the context of the fractional order generalized thermoelasticity with one relaxation time parameter. The Laplace transform together with an eigenvalue approach technique is used to find the solution for the physical variables in the transform domain. The transformed solutions are inverted using the Zakian algorithm. The effect of the fractional parameter, the dependence of modulus of elasticity and the time on the temperature field, the volume fraction field, the displacement field and the stress field have been evaluated and presented graphically and the results obtained are analyzed. A comparison is made with the results obtained in case of reference temperature independent modulus of elasticity.***

Key Words: *Riemann–Liouville fractional integral operator, Lord & Shulman model, Porous material, Laplace transform, Eigenvalue approach.*

1. INTRODUCTION

At high temperature the material characteristics such as the modulus of elasticity, the Poisson's ratio and the thermal conductivity are no longer constants [1]. In the literatures [2–8], one can find some investigations with varying material properties using various generalized thermoelastic model.

The formation of one-dimensional porous material will help researchers in the field of material chemistry for developing one-dimensional nano-composites for applications in pharmaceutical technology and also in environmental chemistry. A nanosized highly luminescent LaPo4 : Ce3+, T b3+ is nowadays one of the important material for biomedical applications such as fluorescence resonance energy-transfer assays, optical imaging, etc. A new mesoporous hybrid titanium (IV) phosphonate nanomaterial has been synthesized by using Benzene-1,3,5-triphosphonic acid as the organophosphorus source in the absence of any template molecule [4].

In the purpose of the mathematical study of the mechanical behavior of porous materials, Cowin and Nunziato [9] established a theory of linear elastic materials with voids. Later, Iesan [10] formulated a linear theory for thermoelastic materials with voids. He derived the basic equations and studied the uniqueness of solution, reciprocity theorem and the variational principle of this theory. A large number of problems for one-dimensional thermoelastic porus material were studied by many authors and they have been appeared in the literatures [11–22].

During recent years, several interesting models have been developed by using fractional calculus to study the physical processes particularly in the area of heat conduction, diffusion, mechanics of solids, electricity, etc. Fractional calculus has been used successfully to modify many existing models of physical processes. Various type definitions and approaches of fractional order derivatives have become the main aim of many researchers. Youssef [23] established the fractional order generalized thermoelasticity in the context of generalized thermoelasticity with one relaxation time. Ezzat and Karamany [24, 25] established a new model of fractional heat equation based on a Taylor's seies expansion of time-fractional order. Sherief et al. [26] also established a new model by using Lord-Shulaman model [31] of generalized thermoelasticity. Recently, a new formula of heat conduction has been considered in the context of the Riemann-Liouville fractional integral operator and Green-Lindsay model of generalized thermoelasticity for porous materials by Bachher et al. [21]. This new consideration generated the fractional order generalized thermoelasticity for porous material. Bachher et al. [22] also studied fractional order thermoelastic interactions in an infinite voids material due to distributed time-dependent heat sources. Abbas [27] applied eigenvalue approach to study fractional order generalized magnetothermoelastic interactions due to a moving heat source. Among the other works devoted to applications of fractional calculus to thermoelasticity, we can refer to the works of Povstenko [28, 29], who introduced a fractional heat conduction law, found the associated thermal stresses. In the present research, we consider a problem for a homogeneous and isotropic thermoelastic infinite porous material under the dependence of modulus of elasticity and thermal conductivity on reference temperature subjected to a periodically varying heat sources, distributed over the plane x = 0 in the context of the fractional order model of generalized thermoelasticity with one relation time parameter. The Laplace transform together with eigenvalue approach technique [21] is applied to find the solution for the physical variables in the Laplace transform domain. The transformed solutions are inverted using the Zakian algorithm [30] for the inversion of Laplace transform. The effect of the fractional parameter, the dependence of modulus of elasticity and the time on the field variables have been presented graphically and the results obtained are analyzed. A comparison is made with the results obtained in case of reference temperature independent modulus of elasticity.

2. BASIC EQUATIONS AND FORMULATION OF THE PROBLEM

Following Iesan [10] and Lord & Shulman [31], the governing equations for a homogeneous isotropic thermoelastic porous material can be written in the following form:

The constitutive equations are:

$$\sigma_{ij} = 2\mu e_{ij} + (\lambda e_{kk} + b\phi - \beta\theta)\delta_{ij}, \tag{1}$$

$$h_i = \alpha\phi_{,i}, \tag{2}$$

$$g = -bu_{k,k} - \xi\phi + m\theta, \tag{3}$$

$$\rho T_0 \eta = \rho C_E \theta + \beta T_0 u_{k,k} + mT_0\phi. \tag{4}$$

Fourier's law in the theory of generalized fractional heat conduction is taken from [25] as:

$$q_i + \frac{\tau_0^{\nu}}{\nu!}\frac{\partial^{\nu} q_i}{\partial t^{\nu}} = -k\theta_{,i}, 0 < \nu \le 1. \tag{5}$$

The energy equation in the presence of heat sources has the form:

$$\rho T_0 \dot{\eta} = -q_{i,i} + \rho Q, \tag{6}$$

The equations of motion in the absence of body forces:

$$\sigma_{ij,j} = \rho \ddot{u}_i, \tag{7}$$

The equations of equilibrated forces are:

$$h_{i,i} + g + \rho l = \rho\chi\ddot{\phi}, \tag{8}$$

where σ_{ij} are the components of the stress tensor, e_{ij} are the components of strain tensor, h_i are the components of equilibrated stress tensor, ϕ is the volume fraction field, ρ is the density, η is the entropy per unit mass, g is the intrinsic equilibrated body force, b is the measure of diffusion effects, α, m, ξ are void material parameters, q_i are the components of heat flux vector, k is the coefficient of thermal conductivity, $\theta = T - T_0, T$ is the absolute temperature, T_0 is the temperature of the medium in its natural state assumed to be such that $|\theta / T_0| << 1$, l is the extrinsic equilibrated force, χ is the equilibrated inertia, λ, μ are Lame's constants, $\beta = (3\lambda + 2\mu)\alpha_t$, α_t is the coefficient of linear thermal expansion, δ_{ij} is the Kronecker delta, u_i are the components of the displacement vector, C_E is the specific heat at constant strain, τ_0 is the relaxation time parameters, Q is the intensity of the applied heat sources, and

$$\frac{\partial^{\nu}}{\partial t^{\nu}} f(x,t) := \begin{cases} f(x,t) - f(x,0) & \text{when } \nu \to 0, \\ I^{1-\nu}\dfrac{\partial f}{\partial t} & \text{when } 0 < \nu < 1, \\ \dfrac{\partial f}{\partial t} & \text{when } \nu = 1. \end{cases}$$

In the above definition, I^{ν} is the *Riemann–Liouville fractional integral operator* defined as

$$I^{\nu}[f(x,t)] := \frac{1}{\Gamma(\nu)}\int_0^t (t-s)^{\nu-1} f(x,s)ds,$$

where $\Gamma(...)$ is the well-known *Gamma function*. A superposed dot represents differentiation with respect to time variable t, and a *comma* followed by a *suffix* denotes material derivative and $i, j = x, y, z$ referee to a general coordinates.

For a homogeneous isotropic generalized thermoelastic porous material, the governing field equations in terms of the displacement, the volume fraction field and the temperature field subjected to a distributed time-dependent continuous heat sources in the absence of body forces and the extrinsic equilibrated body forces can be obtained from Eqs. (1)-(9) as:

$$\mu u_{i,jj} + (\lambda + \mu)u_{j,ij} + b\phi_{,i} - \beta\theta_{,i} = \rho\ddot{u}_i, \tag{9}$$

$$k\theta_{,ii} = \left(1 + \frac{\tau_0^{\nu}}{\nu!}\frac{\partial^{\nu}}{\partial t^{\nu}}\right)\left(\rho C_E\dot{\theta} + \beta T_0 \dot{u}_{k,k} + mT\dot{\phi} - \rho Q\right), \tag{10}$$

$$\alpha\phi_{,ii} - bu_{k,k} - \xi\phi + m\theta = \rho\chi\ddot{\phi}, \tag{11}$$

$$\sigma_{ij} = 2\mu e_{ij} + (\lambda u_{k,k} + b\phi - \beta\theta)\delta_{ij}. \tag{12}$$

The homogeneous isotropic infinite porous thermoelastic solid body is unstrained and unstressed initially but has a uniform temperature distribution T_0. Let $x=0$ represents the plane area over which the applied heat sources Q is situated and the solid occupies the infinite space $-\infty < x < +\infty$. Due to the symmetry of the problem, all the field quantities depend only on x and t and thus for one-dimensional case, the displacement vector u_i will have the components $u_1 = u(x,t), u_2 = 0, u_3 = 0$. Thus from Eqs. (9)-(12) we can write:

$$(\lambda+2\mu)u_{,xx} + b\phi_{,x} - \beta\theta_{,x} = \rho\ddot{u}, \tag{13}$$

$$k\theta_{,xx} = \left(1+\frac{\tau_0^\nu}{\nu!}\frac{\partial^\nu}{\partial t^\nu}\right)\left(\rho C_E\dot{\theta} + \beta T_0\dot{u}_{,x} + mT_0\dot{\phi} - \rho Q\right), \tag{14}$$

$$\alpha\phi_{,xx} - bu_{,x} - \xi\phi + m\theta = \rho\chi\ddot{\phi}, \tag{15}$$

$$\sigma_{xx} = (\lambda+2\mu)u_{,x} + b\phi - \beta\theta. \tag{16}$$

For temperature dependent material properties we may suppose that [32]

$$[\lambda,\mu,\alpha,\beta,\chi,\xi,b,k,m] = [\lambda_0,\mu_0,\alpha_0,\beta_0,\chi_0,\xi_0,b_0,k_0,m_0]\,f(T), \tag{17}$$

where $\lambda_0,\mu_0,\alpha_0,\beta_0,\chi_0,\xi_0,b_0,K_0,m_0$ are constants and $f(T)$ is a given non-dimensional function of temperature. In case of a temperature independent modulus of elasticity, $f(T)=1$. In generalized thermoelasticity as well as in the coupled theory only the infinitesimal temperature deviations from the reference temperature T_0 are considered. Therefore we may consider $f(T)$ in the form $f(T) = (1-\alpha^* T_0)$ [32] where $\alpha^*[K^{-1}]$ is an empirical material constant. In the case of the reference temperature independent modulus of elasticity and thermal conductivity, $\alpha^* = 0$.

Under the above assumption, Eqs. (13)-(16) become

$$f(T)\left[(\lambda_0+2\mu_0)u_{,xx} + b_0\phi_{,x} - \beta_0\theta_{,x}\right] = \rho\ddot{u}, \tag{18}$$

$$k_0 f(T)\theta_{,xx} = \left(1+\frac{\tau_0^\nu}{\nu!}\frac{\partial^\nu}{\partial t^\nu}\right)\begin{pmatrix}\rho C_E\dot{\theta} + \beta_0 T_0\dot{u}_{,x} f(T)\\ +m_0T_0\dot{\phi} f(T) - \rho Q\end{pmatrix}, \tag{19}$$

$$\alpha_0\phi_{,xx} - b_0u_{,x} - \xi_0\phi + m_0\theta = \rho\chi_0\ddot{\phi}, \tag{20}$$

$$\sigma_{xx} = f(T)\left[(\lambda_0+2\mu_0)u_{,x} + b_0\phi - \beta_0\theta\right]. \tag{21}$$

To transform Eqs. (18)-(21) in non-dimensional forms we will use the following non-dimensional variables

$$(x',u') = \frac{\tilde{\omega}}{c_1}(x,u), (t',\tau_0') = \tilde{\omega}(t,\tau_0), \sigma'_{xx} = \frac{\sigma_{xx}}{\beta_0 T_0}, \phi' = \frac{\tilde{\omega}^2\chi_0}{c_1^2}\phi,$$

$$\alpha'_0 = \frac{\alpha_0}{\rho\chi_0 c_1^2}, \theta' = \frac{\theta}{T_0}, Q' = \frac{k_0 Q}{\rho T_0 c_1^2 C_E^2}, \tilde{\omega} = \frac{\rho_0 c_1^2 C_E}{k_0}, c_1^2 = \frac{\lambda_0+2\mu_0}{\rho}.$$

After using these non-dimensional variables, Eqs. (18)-(21) take the following forms (omitting the primes for convenience):

$$u_{,xx} + g_1\phi_{,x} - g_2\theta_{,x} = \gamma^*\ddot{u}, \tag{22}$$

$$\phi_{,xx} - g_3u_{,x} - g_4\phi + g_5\theta = g_6\ddot{\phi}, \tag{23}$$

$$\theta_{,xx} = \left(1+\frac{\tau_0^\nu}{\nu!}\frac{\partial^\nu}{\partial t^\nu}\right)\left(\gamma^*\dot{\theta} + \varepsilon\dot{u}_{,x} + g_7\dot{\phi} - \gamma^* Q\right), \tag{24}$$

$$\sigma_{xx} = (1-\alpha^* T_0)\left(g_8u_{,x} + g_9\phi - \theta\right), \tag{25}$$

where $\gamma^* = 1/(1-\alpha^* T_0)$ and

$$g_1 = \frac{b_0}{\rho\chi_0\tilde{\omega}^2}, g_2 = \frac{\beta_0 T_0}{\rho c_1^2}, g_3 = \alpha_0, g_4 = \frac{b_0}{\rho c_1^2}, g_5 = \frac{\xi_0}{\rho\chi_0\tilde{\omega}^2},$$

$$g_6 = \frac{m_0 T_0}{\rho c_1^2}, g_7 = \frac{m_0 c_1^4}{k_0\chi_0\tilde{\omega}^3}, g_8 = \frac{\rho c_1^2}{\beta_0 T}, g_9 = \frac{b_0 c_1^2}{\beta_0 T_0\chi_0\tilde{\omega}^2}, \varepsilon = \frac{\beta_0 c_1^2}{k_0\tilde{\omega}}.$$

For periodically varying heat sources distributed over the plane area x = 0 we may represent it as

$$Q(x,t) := \begin{cases} Q_0\delta(x)\sin\left(\dfrac{\pi t}{\eta}\right), & 0 \le t \le \eta; \\ 0, & t > \eta; \end{cases},$$

where Q_0 is a constant and $\delta(x)$ is the *Dirac's delta function* defined by:

$$\delta(x) := \begin{cases} +\infty, & x = 0; \\ 0, & x \ne 0; \end{cases}, \text{ and } \int_{-\infty}^{+\infty}\delta(x-a)f(x)dx = f(a).$$

3. SOLUTION IN THE LAPLACE TRANSFORM DOMAIN: EIGENVALUE APPROACH

Taking the Laplace transform of parameter s defined by

$$L\{f(x,t)\} := \int_0^\infty e^{-st}f(x,t)dt = \bar{f}(x,s), \quad \mathrm{Re}(s) > 0 \tag{26}$$

on both sides of the Eqs. (22)-(25) (assuming the homogeneous initial conditions) we get

$$D^2\bar{u} = s^2\gamma^*\bar{u} - g_1D\bar{\phi} + g_2D\bar{\theta}, \tag{27}$$

$$D^2\bar{\phi} = g_3D\bar{u} + (g_4+s^2)\bar{\phi} - g_5\bar{\theta}, \tag{28}$$

$$D^2\theta = \left(1+\frac{s^\nu\tau_0^\nu}{\nu!}\right)\left(s\gamma^*\bar{\theta} + s\varepsilon D\bar{u} + sg_7\bar{\phi} - \frac{Q_0\pi\eta\gamma^*}{\pi^2+s^2\eta^2}\delta(x)\right), \tag{29}$$

$$\bar{\sigma}_{xx} = (1-\alpha^* T_0)\left(g_8D\bar{u} + g_9\bar{\phi} - \bar{\theta}\right). \tag{30}$$

Following Bachher et al. [21], Eqs. (27)-(29) can be written in a vector-matrix differential equation as follows:

$$D\tilde{v}(x,s) = \mathrm{A}(s)\tilde{v}(x,s) + \tilde{f}(x,s), \tag{31}$$

where

$$D \equiv \frac{d}{dx}, \tilde{v}(x,s) = \begin{pmatrix} \bar{u} & \bar{\phi} & \bar{\theta} & D\bar{u} & D\bar{\phi} & D\bar{\theta} \end{pmatrix}^T,$$

$$\tilde{f}(x,s) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & f_6 \end{pmatrix}^T,$$

$$\mathrm{A}(s) = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ C_{41} & 0 & 0 & 0 & C_{45} & C_{46} \\ 0 & C_{52} & C_{53} & C_{54} & 0 & 0 \\ 0 & C_{62} & C_{63} & C_{64} & 0 & 0 \end{pmatrix},$$

$$C_{41} = s^2\gamma^*, C_{45} = -g_1, C_{46} = g_2, C_{52} = (g_4 + s^2), C_{53} = -g_5,$$

$$C_{54} = g_3, C_{62} = g_7\left(s + \frac{s^{1+\nu}\tau_0^\nu}{\nu!}\right), C_{63} = \gamma^*\left(s + \frac{s^{1+\nu}\tau_0^\nu}{\nu!}\right),$$

$$C_{64} = \varepsilon\left(s + \frac{s^{1+\nu}\tau_0^\nu}{\nu!}\right), f_6 = -Q^*\,\delta(x), Q^* = \frac{Q_0\pi\eta\gamma^*\left(1 + \dfrac{s^\nu\tau_0^\nu}{\nu!}\right)}{\pi^2 + s^2\eta^2}.$$

Using the solution methodology through eigenvalue approach discussed in Bachher et al. [22] the solutions for $\bar{u}(x,s), \bar{\phi}(x,s)$ and $\bar{\theta}(x.s)$ bounded as $x \to +\infty$ in the Laplace transform domain can be obtained as:

$$\bar{u}(x,s) = Q^*\begin{bmatrix} k_1\left\{C_{45}C_{53} - C_{46}\left(C_{52} - k_1^2\right)\right\} w_{26}e^{-k_1x} \\ +k_2\left\{C_{45}C_{53} - C_{46}\left(C_{52} - k_2^2\right)\right\} w_{46}e^{-k_2x} \\ +k_3\left\{C_{45}C_{53} - C_{46}\left(C_{52} - k_3^2\right)\right\} w_{66}e^{-k_3x} \end{bmatrix}, \tag{32}$$

$$\bar{\phi}(x,s) = -Q^*\begin{bmatrix} \left\{k_1^2C_{46}C_{54} - C_{53}\left(C_{41} - k_1^2\right)\right\} w_{26}e^{-k_1x} \\ +\left\{k_2^2C_{46}C_{54} - C_{53}\left(C_{41} - k_2^2\right)\right\} w_{46}e^{-k_2x} \\ +\left\{k_3^2C_{46}C_{54} - C_{53}\left(C_{41} - k_3^2\right)\right\} w_{66}e^{-k_3x} \end{bmatrix}, \tag{33}$$

$$\bar{\theta}(x,s) = -Q^*\begin{bmatrix} \left\{\left(C_{41} - k_1^2\right)\left(C_{52} - k_1^2\right) - k_1^2C_{45}C_{54}\right\} w_{26}e^{-k_1x} \\ +\left\{\left(C_{41} - k_2^2\right)\left(C_{52} - k_2^2\right) - k_2^2C_{45}C_{54}\right\} w_{46}e^{-k_2x} \\ +\left\{\left(C_{41} - k_3^2\right)\left(C_{52} - k_3^2\right) - k_3^2C_{45}C_{54}\right\} w_{66}e^{-k_3x} \end{bmatrix}, \tag{34}$$

where

$$k_1^2 = \frac{1}{3}\left[2p\sin q + G_1\right], k_2^2 = -\frac{1}{3}\left[p\left(\sqrt{3}\cos q + \sin q\right) - G_1\right],$$

$$k_3^2 = \frac{1}{3}\left[p\left(\sqrt{3}\cos q - \sin q\right) + G_1\right], p = \sqrt{G_1^2 - 3G_2}, q = \frac{\sin^{-1} r}{3},$$

$$r = \frac{9G_1G_2 - 2G_1^3 - 27G_3}{2p^3}, G_1 = C_{41} + C_{52} + C_{63} + C_{45}C_{54} + C_{46}C_{64},$$

$$G_2 = C_{41}\left(C_{52} + C_{63}\right) + C_{52}C_{63} - C_{53}C_{62} + C_{46}\left(C_{52}C_{64} - C_{54}C_{62}\right) - C_{45}\left(C_{53}C_{64} - C_{54}C_{63}\right),$$

$$G_3 = C_{41}\left(C_{52}C_{63} - C_{53}C_{62}\right).$$

The stress component $\bar{\sigma}_{xx}(x,s)$ can now be determined using the Eqs. (32)-(34) in the Eq. (30).

4. NUMERICAL RESULTS AND DISSCUSSIONS

To illustrate and compare the theoretical results obtained in Section 3, we now present some numerical results which depict the variations of the temperature, the volume fraction field, the displacement and the stress component. For this purpose, we choose magnesium crystal as the hypothetical material for which the values of the different physical constants are:

$$\lambda_0 = 2.17\times10^{10}\,N.m^{-1}, \mu_0 = 3.278\times10^{10}\,N.m^{-1}, T_0 = 298K,$$
$$\rho = 2.17\times10^3\,kg.m^3, C_E = 1.04\times10^3\,J.kg^{-1}.\deg^{-1},$$
$$k_0 = 1.7\times10^2\,W.m^{-1}.\deg^{-1}, \beta_0 = 2.68\times10^6\,N.m.^{-2}.\deg^{-1}.$$

The void parameters are:

$$\chi_0 = 1.753\times10^{-15}\,m^2, \alpha_0 = 3.688N, \xi_0 = 1.475\times10^{10}\,N.m^{-2},$$
$$b_0 = 1.13849\times10^{10}\,N.m^{-2}, m_0 = 2\times10^6\,N.m.^{-2}.\deg^{-1}.$$

The non-dimensional relaxation time is $\tau_0 = 0.02$ s and other constants are taken as $Q_0 = 1, \eta = 1$ s.

Figs. 1-4 are drawn for a non-dimensional particular time $t = 0.5$ and $\alpha^* = 0.00025$. These figures exhibit the space variations of the field quantities in the context of reference temperature dependent generalized thermoelasticity for three different values of the fractional order $\nu = 0.1, 0.5, 1.0$. The case $\nu = 1.0$ represents the Lord & Shulman model of generalized thermoelasticity. All these figures display that the maximum value of all the physical quantities attains on the boundary of the half-space $x \geq 0$ for $\nu = 0.1$. We also observe that all the series approach to zero as x increases further. From Fig. 2 and 3 it is observed that the fractional parameter ν has a decreasing effect on the magnitude of θ and ϕ in the range $0 \leq x < 0.39$ and ν has increasing effect for $x \geq 0.4$ while Fig. 3 and 4 show that ν has a decreasing effect on the magnitude of u and σ_{xx} for 0 $0 \leq x < 0.6$ and then ν acts to increase the numerical value of u and σ_{xx}. This type of behavior of all the field variables can be noticed due to presence of the periodically varying heat sources distributed over the plane area $x = 0$.

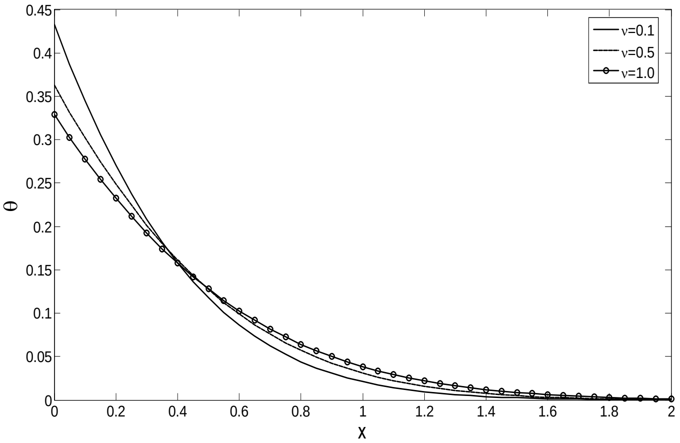

Fig-1: Temperature (θ) distribution against x at $\alpha^* = 0.00025, t = 0.5$.

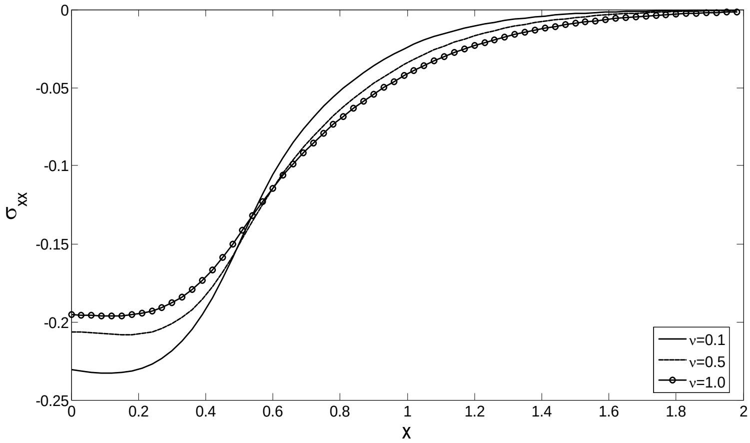

Fig-4: Stress (σ_{xx}) distribution against x at $\alpha^* = 0.00025, t = 0.5$.

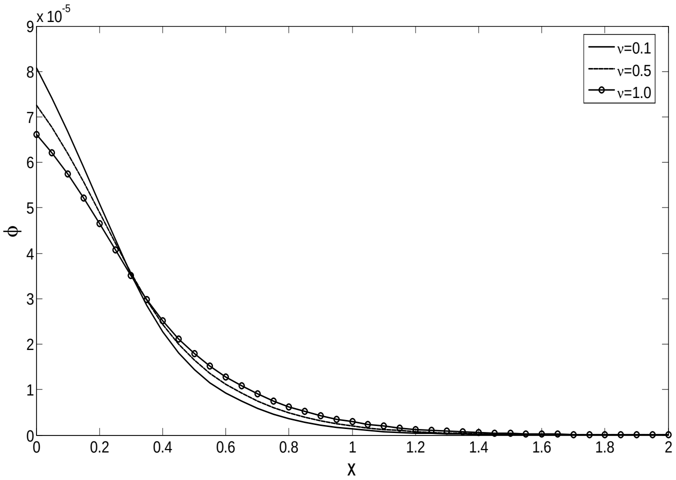

Fig -2: Volume fraction field (ϕ) distribution against x at $\alpha^* = 0.00025, t = 0.5$.

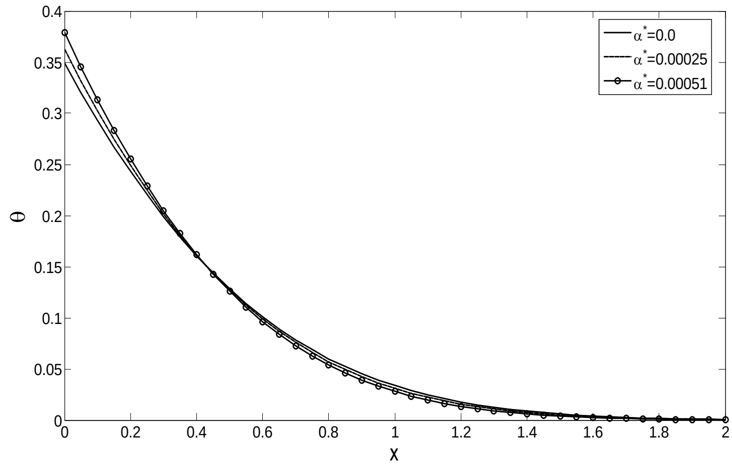

Fig-5: Temperature (θ) distribution against x at $\nu = 0.5, t = 0.5$.

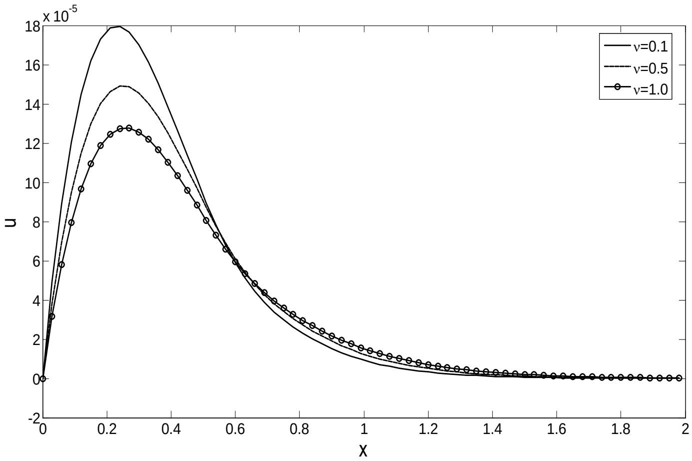

Fig-3: Displacement (u) distribution against x at $\alpha^* = 0.00025, t = 0.5$.

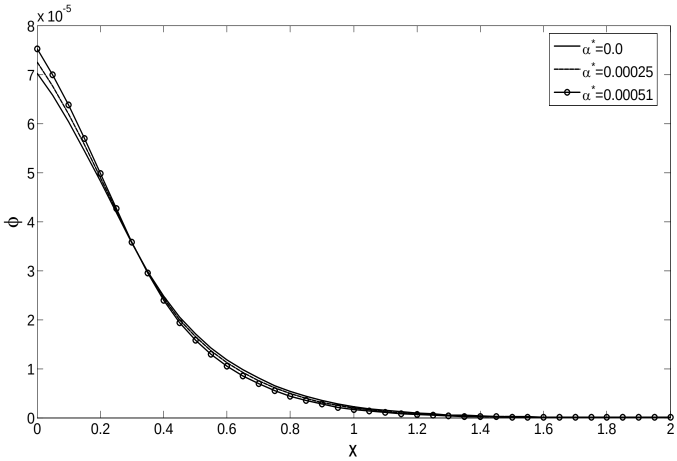

Fig -6: Volume fraction field (ϕ) distribution against x at $\nu = 0.5, t = 0.5$.

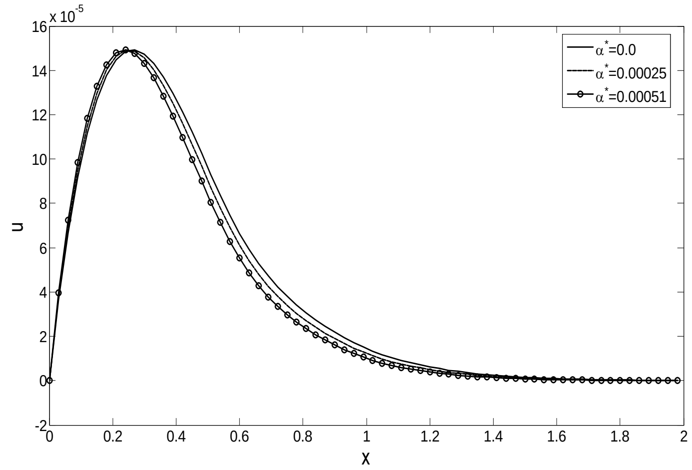

Fig-7: Displacement (u) distribution against x at $\nu = 0.5, t = 0.5$.

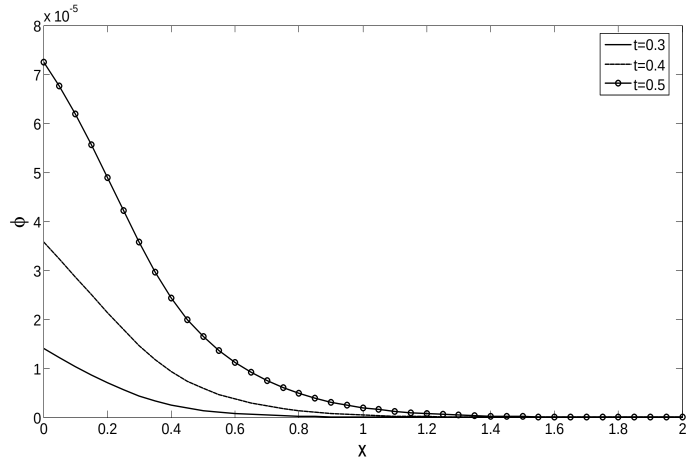

Fig -10: Volume fraction field (ϕ) distribution against x at $\nu = 0.5, \alpha^* = 0.00025$.

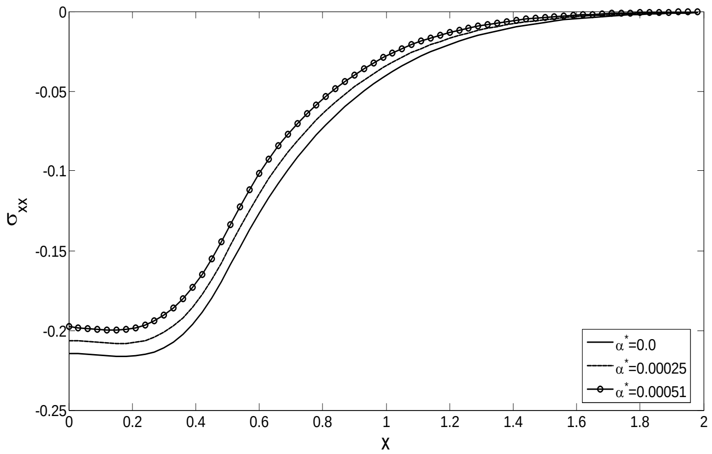

Fig-8: Stress (σ_{xx}) distribution against x at $\nu = 0.5, t = 0.5$.

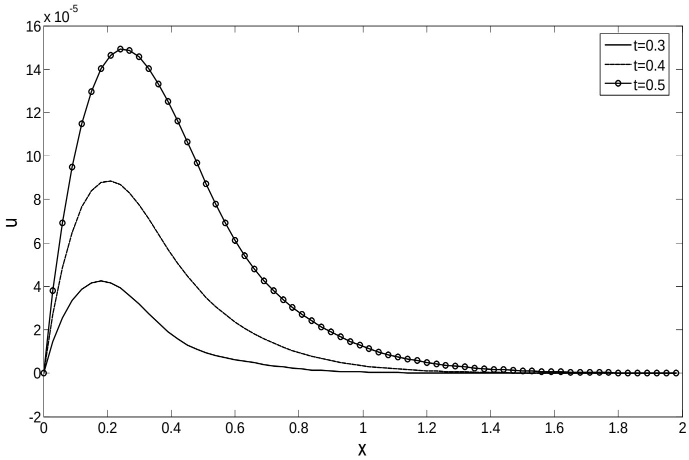

Fig-11: Displacement (u) distribution against x at $\nu = 0.5, \alpha^* = 0.00025$.

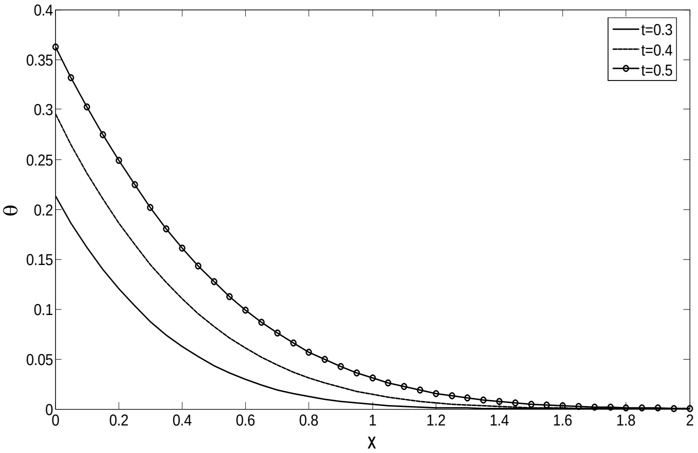

Fig-9: Temperature (θ) distribution against x at $\nu = 0.5, \alpha^* = 0.00025$.

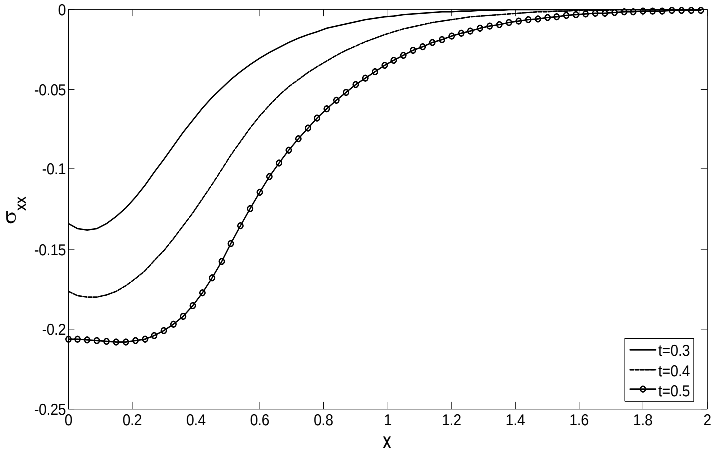

Fig-12: Stress (σ_{xx}) distribution against x at $\nu = 0.5, \alpha^* = 0.00025$.

The second set of Figs. 5-8 display the variations of θ, ϕ, u and σ_{xx} at $\nu = 0.5, t = 0.5$ for three different value of $\alpha^* = 0.0, 0.00025, 0.00051$ within $0 \le x \le 2$. The ase $\alpha^* = 0.0$ represents the temperature independent modulus of elasticity. From all of these four figures it is observed that all the field variables exhibits its greater magnitude for $\alpha^* = 0.00051$ at the boundary $x = 0$ except the stress component σ_{xx} which attains its maximum value for $\alpha^* = 0.0$ at $x = 0$.

Figs. 9-12 display the temperature, the volume fraction field, the displacement, and the stress distribution within a wide range of $x(0 \le x \le 4)$ for $\nu = 0.5, \alpha^* = 0.00025$ for different values of the time parameter $t = 0.3, 0.4, 0.5$ and we have noticed that the time parameter t play a significant role on all the studied fields. The increasing of the value of t causes increasing of the values of all the studied fields and makes the speed of the waves propagation vanishes more rapidly.

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