# A Review of Applications of Meshfree Methods in the area of Heat Transfer and Fluid Flow: MLPG method in particular

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**Abstract:** *A review is presented for the analysis of heat transfer and fluid flow problems in engineering and science, with the use of different meshfree methods. The success of the meshfree methods lay in the local nature, as well as higher order continuity, of the trial function approximations and a low cost to prepare input data for numerical analyses, since the creation of a finite element mesh is not required. There is a broad variety of meshless methods available today; however the focus is placed on the meshless local Petrov- Galerkin (MLPG) method, in this paper.*

**Key words**
Meshfree, meshless local Petrov-Galerkin method, heat transfer, fluid flow

## 1. INTRODUCTION

There are numbers of well known conventional numerical methods (Finite Element Method, Finite Volume Method and Finite Difference Method) but Finite Element Method (FEM), because of its versatility and flexibility is extensively used as an analysis tool in various engineering applications. However, on the other hand FEM suffers from drawbacks such as locking problem, element distortion, loss in accuracy and the need for remeshing. The root of these problems is the use of mesh in the formulation stage. The idea of getting rid of the meshes in the process of numerical treatments has naturally evolved and the concepts of meshfree methods have been shaped up. Meshfree methods use a set of nodes scattered within the problem domain. Many meshfree methods have been achieved remarkable progress over the past years in the areas of engineering such as solid mechanics, deformation problems, structural analysis, heat transfer and fluid flow analysis etc. Some of these are the element free Galerkin (EFG) method (Belytschko et al., 1994); the radial point interpolation method (RPIM) [GR Liu and Gu, 2001; Wang and GR Liu, 2000; 2002]; reproducing kernel particle method (RKPM) [Liu and co- workers in 1995]; general finite difference method (GFDM) [Girault, 1974; Pavlin and Perrone, 1975; Snell et al., 1981; Liszka and Orkisz, 1977; 1980; Krok and Orkisz, 1989]; meshfree collection methods [Kansa, 1990; Wu, 1992; Zhang and Song et al., 2000; Liu X et al., 2002; 2003]; the finite point method (FPM) [Onate et al., 1996; 1998; 2001]; meshfree weak-strong (MWS) form methods [Liu and Gu, 2002; 2003]; meshfree local Petrov- Galerkin (MLPG) method [Atluri and Zhu, 1998]; smooth particle hydrodynamic (SPH) method [Lucy, 1977; Gingold and Monaghan, 1977; GR Liu and Liu, 2003]; the point interpolation method (PIM) [Liu and Gu, 2001]; hp- cloud method [Durarte and Odenm, 1996], the partition of unity (PU) method [Melenk and Babuska, 1996; Babuska and Melenk, 1997]; the boundary node method (BNM) [Mukherjee and Mukherjee, 1997; Kothnur et al., 1999] and the local boundary integral equation (LBIE) method [Zhu et al., 1998a, 1998b; Sladek et al., 2002] etc.

This paper aims to highlight the applications of different meshfree methods, first in the areas of heat transfer and then the fluid flow. Also an attempt has been made to highlight the superiority of MLPG method over other in the mentioned areas.

## 2. MESHFREE METHODS IN HEAT TRANSFER APPLICATIONS

Apart from different applications meshfree methods can be applied to heat transfer problems in the manner as mentioned in the under sections:

### 2.1 Smooth Particle Hydrodynamics (SPH) method

Zhang and Batra (2004) have applied meshless SPH method to investigate 2D heat conduction problem and found SPH to be more accurate than classical FDM. Das and Cleary (2007) have applied SPH method to investigate thermal response in welding pool. The results obtained by SPH are found to be in good agreement with the established FEM and FVM respectively. Hou and Fan (2012) have applied modified SPH method to 1D and 2D transient heat conduction problems and found that the

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proposed method demonstrates the better accuracy than the conventional one. Szewc and Pozorski (2013) have employed novel SPH method, based on Hu and Adams (2006) formalism, for multiphase heat transfer modeling and found that the novel approach is more accurate than the standard SPH formulation.

### 2.2 Reproducing Kernel Particle Method (RKPM)

Rong-Jun and Hong-Xia (2010) have applied meshfree RKPM to complex 3D steady-state heat conduction problems. The results obtained are compared with the exact solutions and found to be accurate and efficient. Xie and Wang (2014) have analyzed the coupled hydro-mechanical system with the help of meshfree RKPM. Very promising results have been demonstrated by the stated method.

### 2.3 Element Free Galerkin (EFG) Method

Singh and Tanaka (2006) have applied the meshless EFG method to obtain thermal solution of cylindrical composite system and found that the EFG results obtained using different weight functions are in good agreement with classical FEM. Sharma et al. (2012) have examined the unsteady magneto-hydrodynamic (MHD) convection heat transfer of viscous fluid over an unsteady stretching sheet placed in a porous medium by EFGM. The results obtained by the proposed method have been found to be excellent. Brar and Kumar (2012) have solved a 1D heat conduction problem with uniform heat generation by EFGM. The results obtained by EFG method have been compared with FEM results and observed that the results obtained by proposed method are as accurate as analytical or FEM results. Das et al. (2012) have employed EFGM to analyse 3D transient heat conduction during a welding process and found that the results obtained by EFGM agrees closely with FE solutions and experimental results. Zan et al. (2013) have applied IEFG (improved EFG) method (combination of improved MLS and EFGM) to 3D transient heat conduction problems. Comparison of the results demonstrates that the IEFG method is more efficient than the conventional one. Zhao and Hongping (2014) have applied an interpolating EFGM (based on interpolating moving least-squares scheme) to the 2D transient heat conduction problems. They have explored that interpolating EFGM gives better computational efficiency and accuracy than conventional EFG.

### 2.4 Radial Point Interpolation Method (RPIM)

Chen et al. (2010) have applied RPIM to solve 2D steady-state temperature field problems and found that RPIM is more advanced than the EFGM. On contrary, in the methods based on local weak-form formulation, no background cells are required and therefore they are often referred to as truly meshless methods and local radial point interpolation method (LRPIM) is among one of them. Sarabadan et al. (2014) have applied LRPI method to solve time dependent Maxwell's equation. After solving the example problem it can be demonstrated that the present approach leads to acceptable results in comparison with classical FDM.

### 2.5 Finite Point Method (FPM)

Lei et al. (2004) have applied the FPM to simulate the heat transfer and solidification in a continuous casting mold. It has been divulge by the authors that the FPM is a convenient method that can be used to accurately analyze moving boundary problems. Revealing the features of FPM, Onate et al. (1996) have employed the said method to solve convection-diffusion problems. The results obtained are compared with the well established FEM and found to be in good agreement with it.

### 2.6 Boundary Knot Method (BKM)

Chen (2001) has solved 2D inhomogeneous Helmholtz problems both by BKM and boundary point method (BPM). The experimental results show that both BKM and BPM produce very accurate solutions with a small number of nodes for inhomogeneous Helmholtz problems. Revealing the features of BKM, Hon and Chen (2002) have applied this method to solve 2D and 3D Helmholtz and convection-diffusion problems under complicated irregular geometry. Numerical experiments validated that the BKM can produce highly accurate solutions using only a small number of nodes. The completeness, stability and convergence of the BKM has also been established numerically. Chen et al. (2005) have further employed BKM to examine high-order general solutions of the Helmholtz and modified Helmholtz equations. They explored that while comparing with the BEM, the proposed method has higher accuracy and low computational cost. Wang et al. (2009) have highlighted the approach to overcome the numerical instability induced from highly dense and ill-conditioned BKM interpolation matrix. For this purpose they have considered three regularization methods and two approaches for the determination of the regularization parameter. They found that the regularization technique is excellent for solving inverse problems with noisy boundary conditions. Fu et al. (2011) have applied the BKM to analyse 2D nonlinear heat conduction problem of FGMs. Numerical demonstrations have shown that the proposed BKM is mathematically simple, easy-to-program, meshless, high accurate and integration-free. Fu et al. (2012) have further investigated steady-state and transient heat conduction problems in nonlinear FGMs by using three boundary meshless methods [the method of fundamental solution (MFS), the boundary knot method (BKM) and the collocation Trefftz method (CTM)] in conjunction with Kirchhoff transformation and various variable transformations. It is found that the CTM

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performs best among these three methods but the BKM converges much faster than the MFS.
However, numerical solutions of the BKM always perform oscillatory convergence when using a large number of boundary points in solving the Helmholtz-type problems. The main reason for this phenomenon may contribute to the severely ill-conditioned full coefficient matrix. In order to obtain admissible stable convergence results Wang and Zheng (2014) have employed three regularization techniques and two algorithms in the process of 3D Helmholtz-type simulation problems. It is found that the BKM in combination with the regularization techniques is able to produce stable numerical solutions.

### 2.7 Singular Boundary Method (SBM)

A class of singular boundary value problems modeling the heat conduction in the human head has been studied by Morgado and Lima (2009). Numerical results have been presented and suggested that second order convergence can be obtained even in this case, by introducing a variable substitution which makes the solution smooth near the origin. Htike et al. (2011) have investigated the applications of the SBM to solve 2D problems of steady-state heat conduction in isotropic bi-materials. Numerical investigations indicate that the proposed SBM technique agrees pretty well with the BEM in terms of efficiency and accuracy. Gu et al. (2012) have extended SBM to solve 2D and 3D heat conduction problems in anisotropic materials with arbitrary domains. They found that the SBM is computationally efficient, robust, accurate, stable and convergent with respect to increasing the number of boundary nodes.

### 2.8 Local Boundary Integral Equation (LBIE) Method

Sladek et al. (2002) have used LBIE method to approximate the thermostatic problems and found that the present method possesses a tremendous potential for solving nonlinear and nonhomogeneous problems. Sladek et al. (2003) have further analyzed transient heat conduction in FGM by LBIE method and found the stated method to be in good agreement with the classical methods. Sladek et al. (2004) have also analyzed heat conduction in nonhomogeneous solids by LBIE. Result demonstrates the high accuracy of proposed method.

### 2.9 Meshless Local Petrov- Galerkin (MLPG) method

Atluri and Zhu (1998) have applied MLPG method to solve 2D heat conduction problems and found the proposed method to be in good agreement with classical FEM. 2D steady and transient heat transfer problems in nonhomogeneous body with anisotropic material property has been analyzed by Sladek et al. (2004), using MLPG approach. They found that the accuracy and adaptability of the present method is higher than the classical FEM. 3D transient heat conduction in functionally graded (FG) thick plate has been analyzed by Qian and Batra (2005) by MLPG method. The results found to be in good agreement with the classical methods. XueHong et al. (2008) have applied MLPG method to analyse the linear 2D steady state heat conduction problems. Numerical results demonstrated the high accuracy of the method and found to be in good agreement with the results of classical FVM. MLPG method has also been extended to transient 3D heat conduction problems in continuously nonhomogeneous solids (Sladek et al., 2008a). The stated method seems to be more promising for the problems, which cannot be solved by conventional BEM due to unavailable fundamental solution. In addition to this Sladek et al. (2008b) have analyzed 3D transient and linear heat conduction equation in continuously non- homogeneous anisotropic functionally graded material (FGM). Results obtained by MLPG method have been found to be in good agreement with classical FEM and BEM. Baradaran and Mahmoodabadi (2009) have applied MLPG method to solve 2D steady state heat conduction problems. They have found MLPG to be in good agreement with the analytical solution in terms of convergence rate, accuracy and efficiency. Fereidoon and Saeidi (2009) have analyzed 2D steady state heat transfer analysis in FGM and Non-FGM as well by using MLPG method. They found that the results obtained are in good agreement with that of classical FEM. Thakur et al. (2010) have solved 1D solid-liquid phase change (originally solved by Voller, 1987) and nonhomogeneous heat conduction in FGM by MLPG method. The results obtained by MLPG method have been found to be in good agreement with classical FEM. Baradaran and Mahmoodabadi (2011) have analyzed 2D heat conduction problems under steady state conditions by MLPG method. A discrete parametric study has been conducted and found that MLPG shows high convergence, accuracy and efficiency with respect to the exact solution. Mahmoodabadi (2011) have also solved 3D heat conduction problems under steady state conditions by MLPG method. It is found that MLPG is more computationally efficient as compare to classical FDM. Shibahara and Atluri (2011) have investigated a transient 2D heat conduction problem due to moving heat source by MLPG method and found that the proposed MLPG approach provides sufficiently high accuracy. Techapirom and Luadsong (2013) have applied MLPG method to solve 2D transient heat conduction equations with non- local boundary conditions. The efficiency, accuracy and effectiveness of the method are reported to be significantly good. Dai et al. (2013) have applied improved MLPG method to 2D transient heat conduction problems and found that the test problems are in good agreement with that of FEM. Qi-Fang et al. (2013) have presented complex variable meshless local Petrov- Galerkin (CVMLPG) method based on complex variable moving least square (CVMLS) approximation to solve 2D transient heat conduction problems. Results obtained are found to be in good agreement with conventional MLPG method.

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Zhang et al. (2014) have used MLPG mixed collocation method to solve Cauchy inverse problems of nonlinear 2D steady- state heat transfer. It is found that the present method is simple, accurate, stable and hence suitable to solve inverse heat transfer problems.

## 3. MESHFREE METHODS IN FLUID FLOW APPLICATIONS

Apart from different applications meshfree methods can be applied to fluid flow problems in the manner as mentioned in the under sections:

### 3.1 Smooth Particle Hydrodynamics (SPH) method

Holmes et al. (2000) have analyzed the fluid flow in 2D and 3D porous media with the help of SPH method. Simulation results for friction coefficient and permeability are shown to agree well with the available benchmarks. Chaniotis et al. (2002) have employed the high order remeshing scheme to the classical SPH method and solved the low Mach number compressible viscous conductive flow problems. It has been found that the proposed methodology is in good agreement with the established methods. Also, the accuracy of the method comes with a minimal additional computational cost. Muller et al. (2003) have proposed an interactive method based on SPH to simulate fluids with free surfaces. The authors have found the promising results by this method. Monaghan et al. (2005) have shown that how SPH method can be used to simulate the freezing of one and two-component (binary alloy) systems. The method found simple to be used. More over the results are found to be in good agreement with the available exact solution. Jafary and Khandekar (2011) have simulated the fluid droplets by using SPH technique and found that the SPH simulations are in excellent agreement with the experimental data. Noutcheuwa and Owens (2012) have developed a truly incompressible SPH method and applied it to discretize incompressible N-S equations in time and found that the improved version of SPH is in very good agreement with both exact solutions. Zou and Jing (2013) have simulated the fluid flow in single as well as intersected rock fractures by solving the N-S equations by using SPH method. The results show that SPH models are effective and have acceptable accuracy with the benchmarked tests. Terissa et al. (2013) have provided a basic method of SPH to simulate liquid droplet with surface tension in 3D. The result show that the droplet tries to keep its shape of a sphere as expected. Hou et al. (2014) have solved the problem of flow separation at bends with various leg ratios and turning angles by the SPH particle method. It has been revealed that the proposed SPH solver appears to be a powerful tool to deal with flow separation problems in channels.

### 3.2 Reproducing Kernel Particle method (RKPM)

Xai and Wang (2014) have developed a novel iterative coupling scheme to solve coupled hydro-mechanical problems by using RKPM. The accuracy and convergence of the proposed numerical scheme are demonstrated through extensive parametric studies of 1D and 2D consolidation and showing its great promise for use in solving practical problems.

### 3.3 Element Free Galerkin (EFG) method

Vidal and Huerta (2003) have proposed a novel mesh-free approach, so called Pseudo Divergence-Free EFG (PDF EFG) obtained by modifying only the interpolating polynomials of standard EFG for incompressible flow which is based on defining a Pseudo Divergence-Free interpolation space. The accuracy of PDF EFG is found to be better than the standard EFG. Singh (2004) has examined 2D transient and steady state fluid flow problems by using meshless EFG method. It is found that the EFG results are well converged and in good agreement with FE and exact methods. Fries and Metthies (2004) have applied a novel method (combination of meshfree EFG and classical FEM) to solve 2D incompressible N-S equations. They concluded that coupled FEM/EFG approximation is a very promising tool for the simulation of complex flow problems. Vlastelica et al. (2008) have explored the applicability of EFG method to model incompressible fluid flow. They concluded that an increased number of free points per cell with increased number of integration points lead to more accurate results. Bhargava and Singh (2012) have investigated unsteady MHD (magneto-hydrodynamic) flow and heat transfer of a non-Newtonian second grade viscoelastic fluid over an oscillatory stretching sheet. It has been found that the effect of viscoelastic parameter is to increase the velocity distribution and that of magnetic parameter to decrease the velocity distribution while temperature increases with the increased magnetic parameter.

### 3.4 Radial Point Interpolation Method (RPIM)

Liu et al. (2003) have adopted a novel iterative coupling scheme for solving coupled hydro-mechanical problems by using RPIM. The accuracy and convergence of the proposed numerical scheme are demonstrated through extensive parametric studies of 1D and 2D consolidation simulations. The results are found to be in good agreement with the classical methods.

### 3.5 Finite Point Method (FPM)

Onate et al. (1996) have used FPM for solving 1D and 2D convection- diffusion and fluid flow type problems. Excellent results have been obtained in all cases. Lohner et al. (1999) have applied a weighted least squares FPM to study compressible flow problems. The results obtained show accuracy comparable to equivalent mesh-based FVM or FEM and makes the proposed FPM competitive. Fang

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and Parriaux (2008) have presented FPM for the numerical simulation of incompressible viscous flows. The results demonstrate that the proposed method is able to perform accurate and stable simulations of incompressible viscous flows.

**3.6 Singular Boundary Method (SBM)**

Qu and Chen (2014) have applied the SBM to analyse 2D Stokes flow problems. The numerical solutions obtained with the proposed method agree well with the exact solutions also the numerical results exhibit a stable convergence trend in all tested examples.

**3.7 Boundary Element Method (BEM)**

Grilli et al. (1989) has presented a computational model for highly nonlinear 2D water waves with the help of BEM. Problems of wave generation and absorption are investigated. The results found to be accurate with respect to the existing data. Kikani (1989) has used BEM to analyse streamline generation in odd shaped reservoirs with multiple walls. Pressure and pressure derivative behavior is also studied. Numerical features of BEM such as accuracy, consistency and the optimum number of nodal points are investigated. The results are found to be in good agreement with the classical methods. Sueyoshi et al. (2007) have investigated the applicability of particle methods to various practical problems. The results are found to be accurate and in good agreement with the existing solutions. Gu and Wang (2008) have proposed a coupled numerical approach to assess the nonlinear dynamic responses of near-bed submarine pipeline by combining the meshless technique and the BEM. It has been demonstrated that the present approach is very effective to obtain numerical solutions for the stated problem.

**3.8 Local Boundary Integral Equation (LBIE) Method**

Pavlova et al. (2010) have applied the LBIE method for the solution of 2D incompressible fluid flow problems governed by the N-S equations. Numerical examples illustrate the proposed methodology and demonstrate its accuracy.

**3.9 Meshless Local Petrov- Galerkin (MLPG) method**

Lin and Atluri (2000) have solved 1D and 2D steady state convection-diffusion problems. The presence of convection term causes serious numerical difficulties, such as oscillatory solution, which can be solved by upwinding. It is found that the proposed method is computationally efficient and cost effective. The study has been further extended to solve steady-state 1D incompressible N-S equation using MLPG method (Lin and Atluri, 2001). The numerical results have shown that MLPG method with upwinding schemes gives better performance at low cost for high Reynolds number flows than MLPG method without upwinding. Arefmanesh et al. (2007) have studied steady, non- isothermal fluid flow problem by using MLPG method. The results found to in close agreement with classical numerical methods. Avila and Atluri (2009) have applied MLPG method to solve non-study 2D incompressible N-S equation for different flow field calculations. It has been found that MLPG method can be used to solve a variety of fluid flow problems where certain surfaces in the flow domain are in arbitrary motion. 2D steady state Stokes equation, for elliptical shaped domain has been solved by using novel MLPG-Mixed FE approach (Avila et al., 2011). It is found that the results provided by the novel mixed method for the stated problem is excellent. Arefmanesh and Tavakoli (2012) have used MLPG method in investigating the effect of Rayleigh number and the volume fraction of the different nano-particles ($Al_2O_3$-water, cu-water, $TiO_2$-water) on the characteristics of the natural convection inside the investigated control volume. Investigations show that the average Nusselt number (Nu), in general increases with increasing the volume fraction of the nano-particles for the different considered nano-fluids. Satapharm and Luadsong (2013) have applied MLPG method to simulate unsteady incompressible fluid flow problems. It has been found that LSWF with classical Gaussian weight order two gives comparatively accurate results than the improved Gaussian weight.

**4. CRITICAL ANALYSIS OF MESHFREE METHODS**

The peer study of meshfree methods have proved their worth in solving complex engineering problems, still some of them has certain shortcomings that they use background cells to integrate a weak form over the problem domain or boundary, which may distort during large deformations. LBIE and MLPG methods are exceptional because these methods do not require the background cells for the interpolation of the trial and test functions for the solution variables. All pertinent integrals can be easily evaluated over over-lapping or regularly shaped domains and their boundaries. In fact, the LBIE approach can be treated simply as a special case of the MLPG approach (Atluri, Kim & Cho, 1999), hence MLPG is found left to be the only exceptional truly meshfree method which has been so designed to overcome these shortcomings and hence can be a point of attraction for the researchers. Also, it demonstrates flexibility to formulate all weak forms locally; choose various trial and test functions and combined them together for solving one problem. Overlapping local sub-domains provides accessibility of complicated structures. All these factors can motivate the researchers to take MLPG as an analytical tool, in the further carried research activities.

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## 5. MLPG APPROACH

MLPG method operates on Petrov-Galerkin formulation i.e. it picks up test and trial functions from different function spaces. The original formulation of Atluri and Zhu (1998a, 1998b) has subsequently evolved in various versions either by changing the meshfree approximation scheme or by selecting a new test function. Hence, the MLPG method provides a rational basis for constructing meshfree methods with a greater degree of flexibility. It is hereby provided an overview of various aspects of MLPG formulation and relevant numerical algorithms.

### 5.1 Moving least square (MLS) interpolation scheme

There are lot of local interpolation schemes, such as MLS, PUM, RKPM, hp- clouds and shepard function etc. However, moving least square (MLS) method is generally considered as one of the schemes to interpolate the data with a reasonable accuracy. Therefore MLS scheme is presented in the current work.

Consider an arbitrary point of interest x located in the problem domain. The moving least squares approximant $u^h(\mathbf{x})$ of $u(\mathbf{x})$ is given as

$$u^h(\mathbf{x}) = \sum_{j=1}^{m} p_j(\mathbf{x}) a_j(\mathbf{x}) \equiv \mathbf{p}^T(\mathbf{x})\mathbf{a}(\mathbf{x}) \quad (1)$$

Where $\mathbf{p}^T(\mathbf{x}) = (p_1(\mathbf{x}), p_2(\mathbf{x}), \ldots\ldots, p_m(\mathbf{x}))$ is a complete monomial basis and $m$ is the number of terms in the basis. For example, in 2D space the basis can be chosen as

$$\text{Linear basis: } \mathbf{p}^T(\mathbf{x}) = \{1, x_1, x_2\}, \quad m = 3$$
$$\text{Quadratic basis: } \mathbf{p}^T(\mathbf{x}) = \{1, x_1, x_2, x_1^2, x_1x_2, x_2^2\}, \quad m = 6 \quad (2)$$

The coefficient vector $\mathbf{a}(\mathbf{x})$ is determined by minimizing a weighted discrete $L_2$ norm defined as

$$J = \sum_{i=1}^{n} w(\mathbf{x}, \mathbf{x}_i)[u^h(\mathbf{x}) - u(\mathbf{x}_i)]^2 = \sum_{i=1}^{n} w(\mathbf{x}, \mathbf{x}_i)[\mathbf{p}^\mathbf{T}(\mathbf{x_i})\mathbf{a}(\mathbf{x}) - u(\mathbf{x}_i)]^2 \quad (3)$$

Where $w(\mathrm{x}, \mathrm{x_i})$ is a weight function, $u(\mathrm{x_i}) = u_i$ is the nodal parameter of the field variable at node $\mathbf{x_i}$ and $n$ is the number of nodes in the support domain of $\mathbf{x}$ for which the weight function, $w(x, x_i) \neq 0$.

The stationarity of $J$ with respect to a(x) results in the following linear system:

$$\mathbf{A(x)a(x) = B(x)u} \quad (4)$$

The above equation can be written as $\mathbf{a}(\mathbf{x}) = \mathbf{A}^{-1}(\mathbf{x})\mathbf{B}(\mathbf{x})\mathbf{u}$ (5)

Where matrices A, B and u are defined as

$$\mathbf{A}(\mathbf{x}) = \sum_{i=1}^{n} w(\mathbf{x}, \mathbf{x}_i)\mathbf{p}(\mathbf{x_i})\mathbf{p}^\mathbf{T}(\mathbf{x_i}) \quad (6)$$

$$\mathbf{B}(\mathbf{x}) = [w(\mathbf{x}, \mathbf{x}_1)\mathbf{p}(\mathbf{x}_1), w(\mathbf{x}, \mathbf{x}_2)\mathbf{p}(\mathbf{x}_2), \ldots\ldots, w(\mathbf{x}, \mathbf{x}_n)\mathbf{p}(\mathbf{x}_n)] \quad (7)$$

$$\mathbf{u} = [u_1, u_2, \ldots u_n]^T \quad (8)$$

System of equations (5) for a(x) is solvable if A is a nonsingular matrix. The requirement of the non-singularity of A is n > m. Hence, the support domain of point x must cover number of nodes which is higher than the number of terms in the monomial basis. Substituting Eq. (5) in Eq. (1), the MLS approximant is obtained as

$$u^h(\mathbf{x}) = \sum_{i=1}^{n} \Phi_i u_i = \mathbf{\Phi}(\mathbf{x})\mathbf{u} \quad (9)$$

Where meshless shape function $\Phi_i(\mathbf{x})$ is defined as

$$\Phi_i(\mathbf{x}) = \sum_{j=1}^{m} p_j(\mathbf{x})\left(\mathbf{A}^{-1}(\mathbf{x})\mathbf{B}(\mathbf{x})\right)_{ji} \quad (10)$$

The partial derivatives of $\Phi_i$ $(\mathbf{x})$ are obtained as

$$\Phi_{i,k}(\mathbf{x}) = \sum_{j=1}^{m} [p_{j,k}(\mathbf{A}^{-1}\mathbf{B})_{ji} + p_j(\mathbf{A}^{-1}\mathbf{B}_{,k} + \mathbf{A}_{,k}{}^{-1}\mathbf{B})_{ji}] \quad (11)$$

in which $(\ )_{,k}$ denotes $\partial(\ )/\partial x_k$ and $\mathbf{A}_{,k}{}^{-1}$ represents the derivative of the inverse of **A** given by

$$\mathbf{A}_{,k}{}^{-1} = -\mathbf{A}^{-1}\mathbf{A}_{,k}\mathbf{A}^{-1} \quad (12)$$

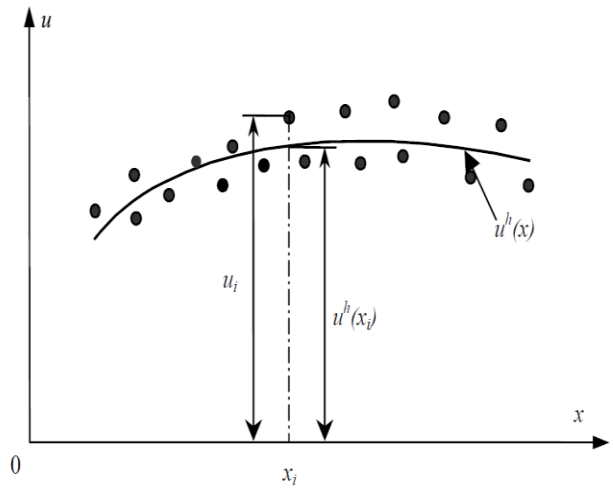


Figure 1: The approximate function $u^h(\mathrm{x})$ and fictitious nodal parameters $u_i$ (refer Thakur et al., 2010)

In implementing the MLS approximation for present LSWF, the basis functions and weight functions should be chosen at first. Fourth order spline function is considered in the present work. It can be expressed as

$$w(\mathbf{x}, \mathbf{x_i}) = \begin{cases} 1 - 6d^2 + 8d^3 - 3d^4 & \text{if } 0 \le d \le 1 \\ 0 & \text{if } d > 1 \end{cases} \quad (13)$$

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Where $d = \left\| \mathbf{x} - \mathbf{x_i} \right\|$, is the distance from node $x_i$ to point $x$.

## 6. FURTHER RESEARCH PERSPECTIVE

Referring above sections it can be found that MLPG has been used in variety of heat transfer and fluid flow applications. Still some areas are left unexplored as:

- Heat transfer enhancement or further improvement in the existing system.
- The surface structure of most of the experimental setups is large enough. It declines the intensity of heat transfer from its originated point to the end point. This phenomenon should also be taken into consideration.
- Fluid flow problems can be investigated for variable channel flows and variable obstacle structures.

## 7. CONCLUDING REMARK

In this paper, an effort has been made to present different meshfree methods and their applications in the field of heat transfer and fluid flow and also highlight the features and applications of meshless local Petrov-Galerkin (MLPG) method, as it is an only truly meshless method and provides the flexibility in choosing the trial and test functions as well as the size and shape of local sub-domains. Therefore, MLPG is characterized as more flexible and capable to handle in easier way the problems from which the conventional mesh-based methods suffer. Some unexplored areas in the field of heat transfer and fluid flow have been exposed for the purpose that these should be treated by the researchers in their further research activities.

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