Effects of variable viscosity and thermal conductivity in indirect natural convection

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Abstract: Study is carried out on the effects of variable viscosity and thermal conductivity on flow, heat and mass transfer due to indirect natural convection of a viscous incompressible fluid mixture which is at rest over a semi infinite uniformly heated horizontal plate facing upward. The governing equations of continuity, momentum, energy, and species conservation are reduced to non-linear coupled ordinary differential equations by using similarity transformations, solved numerically by using Matlab built in solver bvp4c and are presented graphically.

KEYWORDS: *Indirect natural convection, Variable viscosity, Thermal conductivity, Horizontal plate.*

1. INTRODUCTION

When ambient fluid flows over a horizontal heated semi-infinite plate facing upward heat is absorbed by the fluid thus inducing a horizontal temperature gradient within the fluid, which give a pressure gradient that drives the flow and then a boundary layer flow generates. Since horizontal plate is heated so there is a temperature gradient along it then a boundary layer flow develops at the surface of the plate which give the induced pressure gradient that leads to indirect natural convection provided that Grashof number is large.

Viscosity and thermal conductivity are two physical properties which affect the behavior of fluid flow and temperature field. It is known from the work of Herwig and Gersten [4] that these properties may change with temperature especially the fluid viscosity when the viscosity and thermal conductivity of a working fluid are sensitive to the variation of temperature. In order to have a clear picture of the flow and heat transfer it is necessary to take into account the variation of viscosity and thermal conductivity. Such analysis provides a more accurate picture of the momentum and thermal transport than the usual analysis with constant properties. Gebhart and Pera [2] have investigated the natural convection flow resulting from the combined buoyancy effects of thermal and mass diffusion. Conjugate effects of heat and mass transfer on natural convection flow along an isothermal sphere with radiation heat loss have been studied by Aktar et al. [9]. Hossain and Rees [1] have investigated the combined effect of thermal and mass diffusion in natural convection flow along a vertical wavy surface. Goldstein and Lau [3] have studied laminar natural convection from a horizontal plate and the influence of plate edge extension. Huang and Chen [5] have considered the problem of natural convection flow from lower stagnation point to upper stagnation point of a horizontal circular cylinder immersed in a micropolar fluid. Hye et al. [7] have considered conjugate effects of heat and mass transfer on natural convection flow across an isothermal circular cylinder in presence of chemical reaction. Effects of pressure work and viscous dissipation on natural convection flow around a sphere with radiation heat loss have been discussed by Akther [8]. Sharma and Konwar [10] studied the problem of flow, heat and mass transfer due to indirect natural convection by taking physical properties of the fluid to be costant.

Here, a study is carried out on the effects of variable viscosity and thermal conductivity on flow, heat and mass transfer due to indirect natural convection of a viscous incompressible fluid mixture which is at rest over a semi infinite uniformly heated horizontal plate facing upward.

2. FORMULATION OF THE PROBLEM

Consider a semi infinite horizontal uniformly heated plate facing upward which is at rest, there is steady incompressible viscous fluid. When the quiescent ambient fluid is maintained at a lower temperature T_{∞}, The plate is maintained at uniform temperature T_w. We take origin at one end of the plate, X-axis along the surface of the plate and Y-axis normal to it. The fluid properties viscosity and thermal conductivity are assumed to be inverse linear functions of temperature as given by Lai and Kulacki [6],

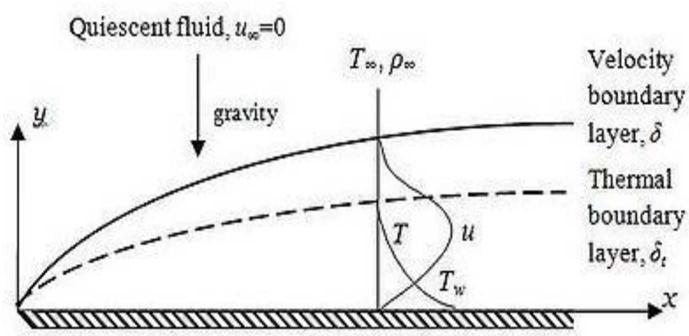

Figure 1: Schematic Representation and Coordinate System of the Problem

$\frac{1}{\mu} = \frac{1}{\mu_\infty}[1+\gamma(T - T_\infty)]$ (1)

or, $\mu = \frac{\mu_\infty}{1+\gamma(T-T_\infty)} = \frac{\mu_\infty}{1+\gamma\theta(T_w-T_\infty)} = \frac{\mu_\infty}{1+\epsilon\theta}$

where $\epsilon = \gamma(T_w - T_\infty)$

and

$\frac{1}{k} = \frac{1}{k_\infty}[1+\beta(T - T_\infty)]$ (2)

or, $k = \frac{k_\infty}{1+\beta(T-T_\infty)} = \frac{k_\infty}{1+\beta\theta(T_w-T_\infty)} = \frac{k_\infty}{1+\omega\theta}$

where $\omega = \beta(T_w - T_\infty)$

Here μ is the coefficient of viscosity, μ_∞ the reference viscosity, γ, β are constants, k is the thermal conductivity, k_∞ the reference thermal conductivity, T and T_∞ are respectively the temperature of the fluids near and far away from the plate. The governing equations of continuity, momentum, energy, and species conservation in the boundary layer region under Boussinesq approximation can be written as

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$ (3)

$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{1}{\rho}\frac{\partial}{\partial y}(\mu\frac{\partial u}{\partial y}),$ (4)

$0 = -\frac{\partial p}{\partial y} + \bar{g}\beta_T(T - T_\infty) + \bar{g}\beta_C(C - C_\infty),$ (5)

$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\rho c_P}\frac{\partial}{\partial y}(k\frac{\partial T}{\partial y}) + \frac{D_m k_T}{c_s c_p}\frac{\partial^2 C}{\partial y^2},$ (6)

$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_m\frac{\partial^2 C}{\partial y^2} + \frac{D_m k_T}{T_m}\frac{\partial^2 T}{\partial y^2},$ (7)

where u, v are fluid velocity components along X-axis and Y-axis, p is the pressure, $\bar{g}$ is the acceleration due to gravity, ρ is the density of the fluid, β_T, β_C are the coefficient of thermal expansion and concentration expansion, α is the thermal diffusivity, T_m is the mean fluid temperature , k_T thermal diffusivity ratio, c_s is the concentration susceptibility, c_p is the specific heat at constant pressure, D_m is the molecular diffusivity.

The corresponding boundary conditions are:

u=0, v=0, T=T_w, C=C_w at y=0 (8)

and

u→0, T→ T_∞, C→ C_∞, P→ P_∞ at y→ ∞. (9)

3. METHOD OF SOLUTION

The stream function ψ(x, y) is such that

$u = \frac{\partial\psi}{\partial y}, v = -\frac{\partial\psi}{\partial x}$

where $\psi = \{x^3 \nu_\infty^3 \beta g(T_w - T_\infty)\}^{1/5} f(\eta),$

$\eta = \frac{y}{x^{2/5}}\left\{\frac{(T_w-T_\infty)\beta g}{\nu_\infty^2}\right\}^{1/5},$

$P = P_\infty + \rho_\infty\{\nu_\infty\beta^2 g^2 x(T_w - T_\infty)^2\}^{2/5} g(\eta),$

$T = T_\infty + (T_w - T_\infty)\theta(\eta)$ and

$C = C_\infty + (C_w - C_\infty)\phi(\eta).$

The continuity equation (3) is satisfied identically for the stream function ψ(x, y). Substituting these transformation into the equations (4) to (9), we get the following nonlinear ordinary differential equations.

$$f''' = (1+\epsilon\theta)\left\{\begin{array}{c}\frac{1}{5}(f')^2 - \frac{3}{5}ff'' + \\ \frac{2}{5}(g - \eta g')\frac{\epsilon\theta'}{(1+\epsilon\theta)^2}f''\end{array}\right\}, \quad (10)$$

$$g' = \theta + N\varphi, \quad (11)$$

$$\theta'' = (1+\omega\theta)\left\{\begin{array}{c}-\frac{3}{5}Prf\theta' + \frac{\omega(\theta')^2}{(1+\omega\theta)^2} \\ -Pr\varphi''Du\end{array}\right\} \quad (12)$$

and

$$\varphi'' = -SrSc\theta'' - \frac{3}{5}Scf\varphi'. \quad (13)$$

The boundary conditions (8) and (9) reduces to

$f(0) = f'(0) = 0, \theta(0) = 1, \varphi(0) = 1$ at $\eta = 0$ (14) and

$g \to 0,\ f' \to 0, \theta \to 0$, at $\eta \to \infty$. (15)

where Pr = $\frac{\nu_\infty}{\alpha_\infty}$ is the Prandtl number , Sc = $\frac{\nu_\infty}{D_m}$ is Schmidt number , Du = $\frac{D_m k_T (C_w - C_\infty)}{c_s c_p \nu_\infty (T_w - T_\infty)}$ is the Dufour number, Sr = $\frac{D_m k_T (T_w - T_\infty)}{T_m \nu_\infty (C_w - C_\infty)}$ is the Soret number, N = $\frac{\beta_C (C_w - C_\infty)}{\beta_T (T_w - T_\infty)}$ measures the relative expansions of mass and thermal diffusion of the fluid.

4. RESULTS AND DISCUSSION

Since the solutions of the set of non-linear coupled ordinary differential equations (10)–(13) under the boundary conditions (14) , (15) cannot be obtained in a closed form therefore we have solved these equations numerically with MATLAB's built in solver bvp4c. Graphical representation are shown below for ε, ω. For figs. 2-4 we have taken Pr=0.71, N=1, ε=0.2, Sr=0.6, Du=0.6, Sc=0.5, for figs. 5-7 we have taken Pr=0.71, N=1, ω=0.25, Sr=0.6, Du=0.6, Sc=0.5.

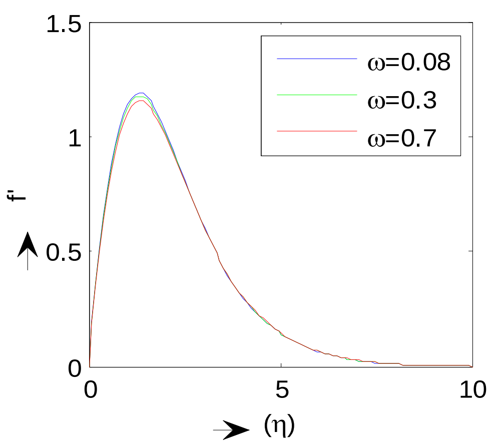

Figure 2. Velocity profile against η

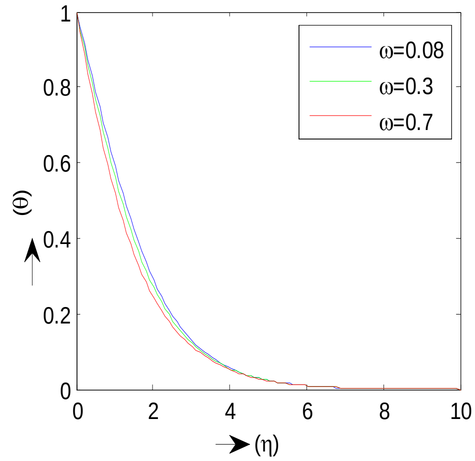

Figure 3. Temperature profile against η

Figure 2. shows that the velocity of the fluid decreases when the value of variable thermal conductivity parameter ω increases, Figure 3. shows that the temperature of the fluid decreases when the value of variable thermal conductivity parameter ω increases and Figure 4. shows that the concentration of mass of the fluid increases when the value of variable thermal conductivity parameter ω increases.

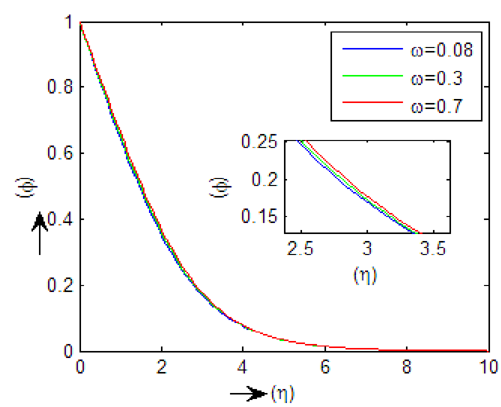

Figure 4. Concentration profile against η

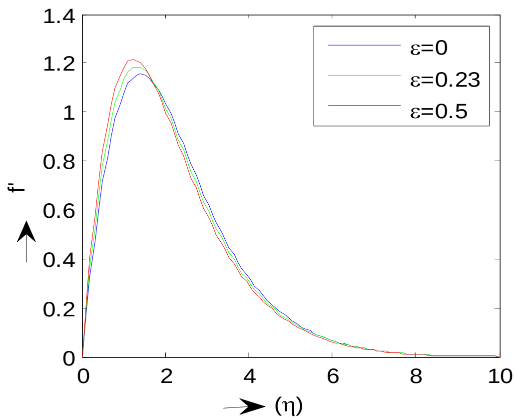

Figure 5. Velocity profile against η

Figure 5. shows that the velocity of the fluid increases when the value of variable viscosity parameter ε increases, Figure 6. shows that the temperature of the fluid decreases when the value of variable viscosity parameter ε increases and Figure 7. shows that the concentration of mass of the fluid decreases when the value of variable viscosity parameter ε increases.

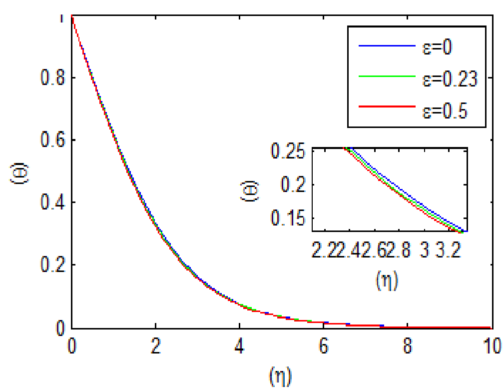

Figure 6. Temperature profile against η

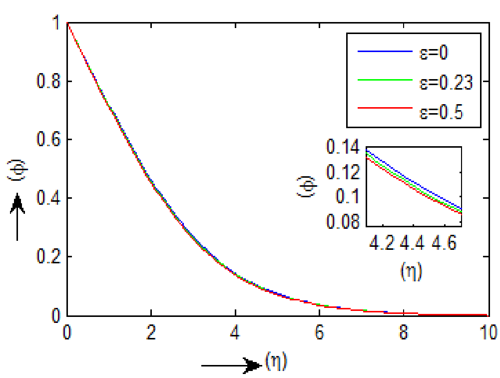

Figure 7. Concentration profile against η

Finally, the effects of the local skin friction, the Nusselt number and the Sherwood number are tabulated in Table 1. The behaviours of these parameters are self – evident from the Table 1 and hence any further discussion about them seems to be redundant.

Table 1. Numerical values of f'', $-\theta'$ and $-\varphi'$.

N	Sr	Pr	Sc	Du			f''	θ'	φ'
1	0.6	0.71	0.5	0.6	0.2	0.08	1.9820	0.4235	0.3214
1	0.6	0.71	0.5	0.6	0.2	0.3	1.9569	0.4780	0.3050
1	0.6	0.71	0.5	0.6	0	0.25	1.7614	0.4574	0.3043
1	0.6	0.71	0.5	0.6	0.23	0.25	1.9896	0.4670	0.3092
1	0.6	0.71	0.5	0.6	0.2	0.7	1.9225	0.5718	0.2770
1	0.6	0.71	0.5	0.6	0.5	0.25	2.2144	0.4759	0.3138

CONCLUSIONS

From the above discussions it is clear that values of variable viscosity and thermal conductivity play important roles in velocity, temperature and and mass transfer due to indirect natural convection of a viscous incompressible fluid mixture on a uniformly heated horizontal plate.

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