# Integration of graph theory matrix approach and analytic hierarchy process for technical institute assessment

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**Abstract** - *The evaluation and selection of technical institute involves a large number of criteria whose selection and weighting is decided in accordance with technical framework. In the present work, an integrated Graph theory matrix approach and Analytic hierarch process is adapted to deal with assessing the performance and rank the technical institutions. The criteria digraph represents the visual analysis of the criteria. The Analytic hierarchy process is used to assign the weights of the criteria. The Index score measures the performance of an alternative with respect to the criteria.*

***Key Words*: Graph theory matrix approach, Analytic hierarchy process, Technical institute, Permanent function, Index score.**

## 1.INTRODUCTION

Technical education plays a vital role in human resource development of the country by creating skilled manpower, enhancing industrial productivity and improving the quality of lives of people. In the recent years, there has been a mushroom growth of technical institutes in India. The prime concern of this day is the quality of technical education imparted at the technical institutes. The primary mission of a technical institute is to explore and transmit knowledge. Hence, this is the high time to do the performance evaluation of technical institutions. Researchers have explored various ways of evaluation of technical institutions. Ali and Bijan [1] adapted data envelopment analysis to evaluate universities as groups of decision making units. Their proposed method has the capability to calculate the efficiency of units within each group for which, linear programming process was developed. Aziz et al [2] adapted data envelopment analysis method to measure relative efficiency of academic departments in a university. Four models with input – output combinations were proposed for the analysis and the results were used to manage the resources and improve the productivity by reallocating the resources. Das et al [3] addressed the performance evaluation of technical institutions using fuzzy AHP, DEA and TOPSIS. The criteria chosen for the evaluation were faculty strength, student intake, number of PhDs awarded, number of patents applied, campus area and tuition fee based on Pareto analysis. The weights of the criteria were determined using fuzzy AHP and TOPSIS method was used to aggregate the performance scores. Das et al [4] formulated a frame work to measure the performance of technical institutions using fuzzy AHP and CORPAS method. Salah et al [5] adapted data envelopment analysis method to evaluate the efficiency of various departments in Islamic university in Gaza. In their study, operating expenses, credit hours and training resources were considered as inputs and number of graduates, promotions and public service activities are the outputs. It was found that the average efficiency score was 68.5%. Their proposed method suggests various corrective measures that should be taken for the under-performing departments. Preeti et al [6] studied the relative performance of academic departments using data envelopment analysis to evaluate the technical, pure technical and scale efficiencies of technical institutions. Sensitivity analysis was also conducted to assess overall performance, research performance and teaching performance. A DEA – TOPSIS based approach was proposed by Amrita and Shankar [7] to evaluate the performance of Indian technical institutes. In their approach, DEA was applied to shortlist the efficient institutes having the desired characteristics from the stakeholders point of view and TOPSIS method was employed to rank the institutes. Das et al [8] adapted an integrated SOWIA-MOORA method to evaluate the performance of technical institution. The findings of SOWIA-MOORA method were verified using non-parametric spearman test of relationship and the Kendall's tau test of correlation. Sarfaraz and Abdolhamid [9] studied the performance of universities using balanced scorecard, DEMATEL for research on cause and effect relations of perspectives of balanced scorecard, ANP to calculate weights of indices in perspectives and VIKOR for ranking universities.

The present study deals with a methodology to assess the evaluation and selection of technical institute by integrating Graph theory matrix approach (GTMA) and Analytic hierarchy process (AHP).

## 2. GRAPH THEORY MATRIX APPROACH AND ANALYTIC HIERARCHY PROCESS

The step by step procedure in GTMA-AHP is given below,

Step 1: The factors that influence the performance assessment of technical institute are accreditation, faculty strength, student's intake, number of PhDs, number of patents, campus area, tuition fee, operating expenses, credit hours, public service activities, location, majors, support services, campus life, safety and security, placements and training resources. The choice of criteria depends on the users requirements. In the present study, seven criteria namely accreditation (C1), faculty strength (C2), number of PhDs (C3), number of patents (C4), number of papers published (C5), number of students intake (C6) and placements (C7) are considered and the Criteria digraph is developed with criteria as nodes and the inter-dependencies among the criteria as edges. The magnitude of the directed edges indicate the relative importance of one criteria over other. The criteria digraph is shown in Fig. 1.

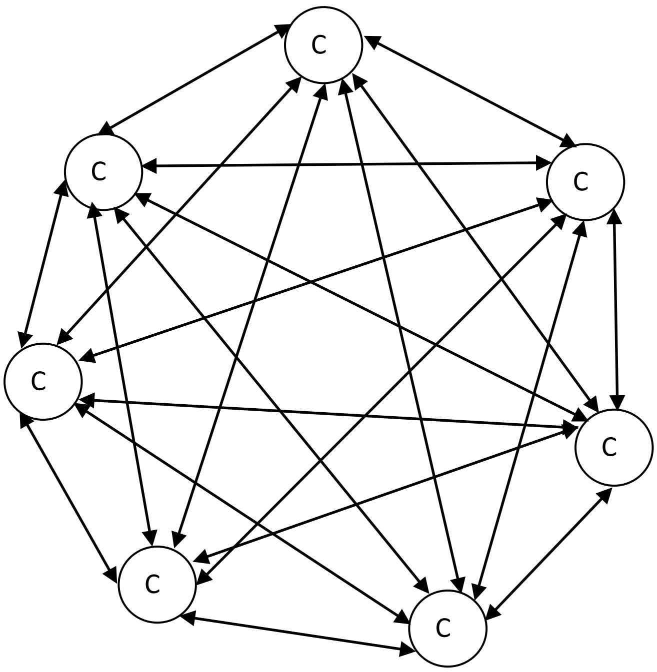


**Fig – 1**: Criteria digraph

Step 2: The relative importance matrix is composed from the criteria digraph, which is an equivalent matrix which stores the values of criteria and their relative importance. The relative importance matrix is shown in Eq. 1.

$$[D]=\begin{bmatrix} D_1 & d_{12} & d_{13} & d_{14} & d_{15} & d_{16} & d_{17} \\ d_{21} & D_2 & d_{23} & d_{24} & d_{25} & d_{26} & d_{27} \\ d_{31} & d_{32} & D_3 & d_{34} & d_{35} & d_{36} & d_{37} \\ d_{41} & d_{42} & d_{43} & D_4 & d_{45} & d_{46} & d_{47} \\ d_{51} & d_{52} & d_{53} & d_{54} & D_5 & d_{56} & d_{57} \\ d_{61} & d_{62} & d_{63} & d_{64} & d_{65} & D_6 & d_{67} \\ d_{71} & d_{72} & d_{73} & d_{74} & d_{75} & d_{76} & D_7 \end{bmatrix} \quad (1)$$

The diagonal elements Di (i.e D1 to D7) represents the weight of the criteria and the off-diagonal elements dij represents the relative importance of ith criteria over the jth criteria while dji represents the relative importance of jth criteria over the ith criteria. The values of Di may be subjective or objective. The objective values will have different units. All the values of Di are desired to be subjective. It is desirable to normalize the objective values on the same scale. i.e 0 to 1. For the criteria which has the linguistic terms, suitable scale may be adapted. In the present study, Table 1 is used for attaining the values of linguistic terms.

**Table – 1:** Values for linguistic terms

| Description | NBA | NAAC | ABET |
|---|---|---|---|
| Accreditation | 5 | 7 | 8 |

The values of relative importance of the criteria may be taken from Table 2 [10]. To maintain the accuracy of each criteria, the relative importance of one criteria over another is expressed using pairwise comparisons and the consistency is evaluated using the standard procedure [10, 11]. It is desired that the consistency ratio of the data should be less than 0.1. In the present study, the consistency obtained is 0.087.
The consistency evaluation is shown in Table 4.

Step 3: The permanent function used in Combinatorial mathematics characterizes the configurations of a system. No information is lost as there is no negative sign in the permanent function [12]. The permanent function for a standard matrix is shown in Eq. 2.

The normalized values of the criteria chosen are shown in Table 3.

**Table – 2:** Relative importance values

| S. No. | Class description | Relative importance | |
|---|---|---|---|
| | | aij | aji = 1- aij |
| 1 | Two attributes are equally important | 0.5 | 0.5 |
| 2 | One attribute (i) is slightly more important over the other (j) | 0.6 | 0.4 |
| 3 | One attribute (i) is strongly important over the other (j) | 0.7 | 0.3 |
| 4 | One attribute (i) is very strongly important over the other (j) | 0.8 | 0.2 |
| 5 | One attribute (i) is extremely important over the other (j) | 0.9 | 0.1 |
| 6 | One attribute (i) is exceptionally more important over the other (j) | 1.0 | 0.0 |

**Table – 3:** Normalized values of criteria

| S. No | Institution Name | Accreditation | faculty strength | No. of Ph.Ds. | Patents | Papers published | No. of Students | Placements |
|---|---|---|---|---|---|---|---|---|
| 1 | A 1 | 0.600 | 0.667 | 0.333 | 0.167 | 0.315 | 0.611 | 0.398 |
| 2 | A 2 | 0.250 | 0.853 | 0.583 | 0.417 | 0.420 | 0.937 | 0.682 |
| 3 | A 3 | 1.000 | 1.000 | 1.000 | 1.000 | 0.795 | 1.000 | 1.000 |
| 4 | A 4 | 0.250 | 0.867 | 0.900 | 0.833 | 0.657 | 0.787 | 0.710 |
| 5 | A 5 | 0.600 | 0.573 | 0.183 | 0.167 | 0.189 | 0.444 | 0.141 |
| 6 | A 6 | 0.600 | 0.710 | 0.300 | 0.250 | 0.361 | 0.666 | 0.481 |
| 7 | A 7 | 0.600 | 0.820 | 0.633 | 0.667 | 0.336 | 0.656 | 0.514 |
| 8 | A 8 | 0.600 | 0.947 | 0.950 | 0.500 | 0.772 | 0.992 | 0.709 |
| 9 | A 9 | 0.250 | 0.710 | 0.567 | 0.083 | 0.292 | 0.641 | 0.352 |
| 10 | A 10 | 0.250 | 0.673 | 0.750 | 0.333 | 1.000 | 0.933 | 0.688 |

**Table – 4:** Consistency evaluation

| | C 1 | C 2 | C 3 | C 4 | C 5 | C 6 | C 7 | Total | Average | Consistency Measure |
|---|---|---|---|---|---|---|---|---|---|---|
| C 1 | 0.326 | 0.514 | 0.255 | 0.197 | 0.277 | 0.308 | 0.238 | 2.115 | 0.302 | 7.806 |
| C 2 | 0.109 | 0.171 | 0.383 | 0.295 | 0.185 | 0.185 | 0.143 | 1.470 | 0.210 | 7.994 |
| C 3 | 0.163 | 0.057 | 0.128 | 0.295 | 0.185 | 0.123 | 0.095 | 1.046 | 0.149 | 7.844 |
| C 4 | 0.163 | 0.057 | 0.043 | 0.098 | 0.185 | 0.185 | 0.143 | 0.873 | 0.125 | 7.548 |
| C 5 | 0.109 | 0.086 | 0.064 | 0.049 | 0.092 | 0.123 | 0.143 | 0.666 | 0.095 | 7.568 |
| C 6 | 0.065 | 0.057 | 0.064 | 0.033 | 0.046 | 0.062 | 0.190 | 0.517 | 0.074 | 7.402 |
| C 7 | 0.065 | 0.057 | 0.064 | 0.033 | 0.031 | 0.015 | 0.048 | 0.313 | 0.045 | 7.645 |
| | | | | | | | | | Consistency Index | 0.114 |
| | | | | | | | | | Random Index | 1.32 |
| | | | | | | | | | Consistency Ratio | 0.087 |

$$
\begin{aligned}
Per(D) = {} & \\
& \prod_{i=1}^{M} D_i + \sum_{i=1}^{M-1}\sum_{j=i+1}^{M} \cdots\cdots \sum_{M=t+1}^{M} (d_{ij}d_{ji}) D_k D_l D_m D_n D_o \cdots D_t D_m \\
& + \sum_{i=1}^{M-2}\sum_{j=i+1}^{M-1}\sum_{k=j+1}^{M} \cdots\cdots \sum_{M=t+1}^{M} (d_{ij}d_{jk}d_{ki} + d_{ik}d_{kj}d_{ji}) D_l D_m D_n D_o \cdots D_t D_M \\
& + [\sum_{i=1}^{M-3}\sum_{j=i+1}^{M}\sum_{k=i+1}^{M-1}\sum_{l=i+2}^{M} \cdots\cdots \sum_{M=t+1}^{M} (d_{ij}d_{ji})(d_{kl}d_{lk}) D_m D_n D_o \cdots D_t D_M + \\
& \sum_{i=1}^{M-3}\sum_{j=i+1}^{M-1}\sum_{k=i+1}^{M}\sum_{l=j+1}^{M} \cdots \sum_{M=t+1}^{M} (d_{ij}d_{jk}d_{kl}d_{li} + d_{il}d_{lk}d_{kj}d_{ji}) D_m D_n D_o \cdots D_t D_m ] \\
& + [\sum_{i=1}^{M-2}\sum_{j=i+1}^{M-1}\sum_{k=j+1}^{M}\sum_{l=1}^{M-1}\sum_{m=l+1}^{M} \cdots\cdots \sum_{M=t+1}^{M} (d_{ij}d_{jk}d_{ki} + d_{ik}d_{kj}d_{ji})(d_{lm}d_{ml}) D_n D_o \cdots D_t D_m \\
& + \sum_{i=1}^{M-4}\sum_{j=i+1}^{M-1}\sum_{k=i+1}^{M}\sum_{l=i+1}^{M}\sum_{m=j+1}^{M} \cdots\cdots \sum_{M=t+1}^{M} (d_{ij}d_{jk}d_{kl}d_{lm}d_{mi} + d_{im}d_{ml}d_{lk}d_{kj}d_{ji}) D_n D_o \cdots D_t D_m ] \\
& + [\sum_{i=1}^{M-3}\sum_{j=i+1}^{M-1}\sum_{k=i+1}^{M}\sum_{l=j+1}^{M}\sum_{m=1}^{M-1}\sum_{n=m+1}^{M} \cdots\cdots \sum_{M=t+1}^{M} (d_{ij}d_{jk}d_{kl}d_{li} + d_{il}d_{lk}d_{kj}d_{ji})(d_{mn}d_{nm}) D_o \cdots D_t D_m \\
& + \sum_{i=1}^{M-5}\sum_{j=i+1}^{M-1}\sum_{k=j+1}^{M}\sum_{l=1}^{M-2}\sum_{m=l+1}^{M-1}\sum_{n=m+1}^{M} \cdots\cdots \sum_{M=t+1}^{M} (d_{ij}d_{jk}d_{ki} + d_{ik}d_{kj}d_{ji})(d_{lm}d_{mn}d_{nl} + d_{ln}d_{nm}d_{ml}) D_o \cdots D_t D_m \\
& + \sum_{i=1}^{M-5}\sum_{j=i+1}^{M}\sum_{k=i+1}^{M-3}\sum_{l=i+2}^{M}\sum_{m=k+1}^{M-1}\sum_{n=k+2}^{M} \cdots\cdots \sum_{M=t+1}^{M} (d_{ij}d_{ji})(d_{kl}d_{lk})(d_{mn}d_{nm}) D_o \cdots D_t D_m \qquad (2) \\
& + \sum_{i=1}^{M-5}\sum_{j=i+1}^{M-1}\sum_{k=i+1}^{M}\sum_{l=i+1}^{M}\sum_{m=i+1}^{M}\sum_{n=j+1}^{M} \cdots \sum_{M=t+1}^{M} (d_{ij}d_{jk}d_{kl}d_{lm}d_{mn}d_{ni} + d_{in}d_{nm}d_{ml}d_{lk}d_{kj}d_{ji}) D_o \cdots D_t D_m ] \\
& + \cdots\cdots
\end{aligned}
$$

Step 4: The values of Di, dij and dji are substituted in the permanent function to determine the Index score. The alternative with highest Index score represents the most preferred option for the given application and is ranked first on the preference list [10]. The index scores and ranks for the considered alternatives are shown in Table 5.

**Table – 5:** Index scores and ranks for the alternatives

| S. No | Institution Name | Index score | Rank |
|---|---|---|---|
| 1 | A 1 | 2.8045 | 8 |
| 2 | A 2 | 10.5809 | 5 |
| 3 | A 3 | 42.376 | 1 |
| 4 | A 4 | 32.1268 | 3 |
| 5 | A 5 | 0.9193 | 10 |
| 6 | A 6 | 4.0701 | 7 |
| 7 | A 7 | 10.282 | 6 |
| 8 | A 8 | 39.0427 | 2 |
| 9 | A 9 | 2.3124 | 9 |
| 10 | A 10 | 17.5035 | 4 |

## 3. Conclusion

In absolute sense, it is really a difficult task to determine performance score of technical institute as a lot of factors influence the overall performance and the measurement result is very much sensitive to the selection of the criteria. An attempt is made in this study to assess the performance and rank the technical institutions considering a few factors that influence the performance. This method enables a graphical visualization of criteria which helps to understand the complexity of the system. Also, this method offers simple and fewer computations than other multi criteria decision making methods. The use of permanent concept leads to a total objective value.

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