A Survey on Magnetic Resonance Image Denoising Methods

Snehal More[1], V.V.Hanchate[2]

[1]*M.E. Student, PES's MCOE Pune, Maharashtra, India*
[2]*Dept. of E&TC Engineering, PES's MCOE Pune, Maharashtra, India*

---***---

Abstract - *Magnetic Resonance Imaging (MRI) is one of the most effective medical imaging modality that have been used for soft tissue imaging, such as brain and muscles. Over the past several years, although the signal-to-noise ratio, resolution and acquisition speed of magnetic resonance imaging (MRI) technology have been increased, MR images are still affected by noise and artifacts. A trade-off between noise reduction and the preservation of actual detail features has to be made in such a manner that enhances the diagnostically relevant image content. Therefore, noise reduction is still a challenging task. In this paper different techniques have been presented for denoising MR images and each technique has its own advantages, assumptions and limitations.*

Key Words: **Anisotropic diffusion, Contourlet, Curvelet, Denoising, Magnetic resonance imaging, Nonlocal means, Wavelet.**

1. INTRODUCTION

Medical imaging became an integral part of disease diagnosis in present days. Various medical imaging modalities are developed for various applications since last few decades. These modalities are used to acquire the images of the anatomical structures within the body which is to be examined without opening the body. X-rays, Computed Tomography, Nuclear imaging, Magnetic resonance Imaging and Ultrasound are the popular medical imaging modalities at present to diagnose the various diseases. However these imaging modalities are suffering with a big problem called noise. Noise means, the intensity values of pixels in the image show different values instead of true pixel values i.e. noise is the undesirable effects produced in the medical images. During image acquisition or transmission, several factors are responsible for introducing noise in the imaging modality such as Quantum noise in X-rays and Nuclear imaging, speckle noise in ultrasound imaging, Rician noise in Magnetic resonance imaging etc. The noise present in the images will degrade the contrast of the image and creates problems in the diagnostic phase. So denoising is very important to remove the noise from these images [1].

Magnetic resonance imaging (MRI) is a notable imaging technique to provide highly detailed images of tissues and organs in the human body. MRI system is working on the principles of nuclear magnetic resonance (NMR), to map the spatial location and associated properties of specific nuclei or protons in a subject using the interaction between an electromagnetic field and nuclear spin. It detects and processes the signals generated when hydrogen atoms are placed in strong magnetic field and excited by a resonant magnetic excitation pulse. The human body is largely composed of fat and water molecules. Each water molecule has two hydrogen nuclei or protons. These hydrogen protons are usually imaged to demonstrate the physiological or pathological alterations of human tissues [2].

Acquisition time in MRI is limited. Therefore the signals to noise ratio (SNR) of the MR images are usually low. The qualities of the MRI images are usually degraded with several artifact and noises that is adequately modeled as Rician noise. Denoising technique that removes noise, while preserves the image details is an important step of MR image processing. MRI denoising is the focus of many researches to provide images with both good spatial resolution and high SNR. Several filtering methods have been developed in the past decades to address denoising problem in MR images.

2. NOISE IN MRI

Scanner technology has undergone tremendous improvements in spatial resolution, acquisition speed and signal-to-noise ratio (SNR), the diagnostic and visual quality of MR images are still affected by the noise in acquisition. Also, MR images contain varying amount of noise of diverse origins including noise from stochastic variation, numerous physiological processes, and eddy currents, artifacts from the magnetic susceptibilities between neighboring tissues, rigid body motion, non rigid motion and other sources. The main source of noise in MRI is thermal noise that is from the scanned object. The variance of thermal noise can be described as the sum of noise variances from independent stochastic processes representing the body, the coil and the electronics [3]. In this section, the noise distribution in MRI is explained.

2.1 Characteristic of Noise in MRI:

The raw data obtained during MRI scanning are complex values that represent the Fourier transform of a magnetization distribution of a volume of tissue. Therefore, MRI image is commonly reconstructed by computing the inverse discrete Fourier transform of the raw data. An inverse Fourier transform converts these raw data into magnitude, frequency and phase components that more directly represent the physiological and morphological features of interest in the person being scanned. The signal component of the measurements is present in both real and imaginary channels. Each of the two orthogonal channels is affected by additive white Gaussian noise. The noise in the reconstructed complex-valued data is thus complex white Gaussian noise. Therefore, noise in the k-space in MR data from each coil is assumed to be a zero mean uncorrelated Gaussian process with equal variance in both real and imaginary parts because of the linearity and orthogonality of the Fourier transform [2, 3]. Most commonly, the magnitude of the reconstructed MRI image is used for visual inspection and for automatic computer analysis. Since computation of a magnitude (or phase) image is a non-linear operation, the probability density function (PDF) of the MR data changes. Since the magnitude of the MRI signal is the square root of the sum of the squares of two independent Gaussian variables, it follows a Rician distribution i.e. in single coil MRI systems, magnitude data in spatial domain is modeled as the Rician distribution and so called Rician noise (the error between the underlying image intensities and the measurement data) is locally signal dependent. In low intensity (dark) regions of the magnitude image, the Rician distribution tends to a Rayleigh distribution and in high intensity (bright) regions it tends to a Gaussian distribution [19].

$$p_M(M|A,\sigma_n) = \frac{M}{\sigma_n^2} e^{-(M^2+A^2)/2\sigma_n^2} I_0\left(\frac{AM}{\sigma_n^2}\right)u(M) \qquad (1)$$

Where $I_0(.)$ is the modified zeroth order Bessel function of the first kind, σ_n^2 is the noise variance, A the noiseless signal level, M the MR magnitude variable and u(·) is the Heaviside step function.

In high SNR i.e., high intensity (bright) regions of the magnitude image, the Rician distribution tends to a Gaussian distribution with mean $\sqrt{A^2 + \sigma_n^2}$ and variance σ_n^2 given as,

$$p_M(M|A,\sigma_n) \approx \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-(M^2-\sqrt{A^2+\sigma_n^2})/2\sigma_n^2} u(M) \qquad (2)$$

In the image background, where SNR is zero due to the lack of water proton density in the air, the Rician PDF simplifies to a Rayleigh distribution with PDF given as,

$$p_M(M,\sigma_n) = \frac{M}{\sigma_n^2} e^{-M^2/2\sigma_n^2} u(M) \qquad (3)$$

3. DENOISING METHODS IN MRI

Denoising methods in MRI can be categorized in two groups: acquisition based and post acquisition based noise reduction methods. The method for improving the SNR during the acquisition of an image is either increasing acquisition time or decreasing spatial resolution. However, the acquisition time is limited due to patient comfort and system throughput. Therefore, in acquisition based method, there is a practical limit on the SNR of the acquired MRI data. Hence, post acquisition image denoising is an inexpensive and effective alternative. The aim of a post processing MRI denoising algorithm is reducing the noise power maintaining the original resolution of the useful features in MR images. There are different post acquisition MRI denoising methods which are mainly divided into three group i.e. filtering, transform domain and statistical approach. Here, we discuss MRI denoising methods present in filtering and transform domain group.

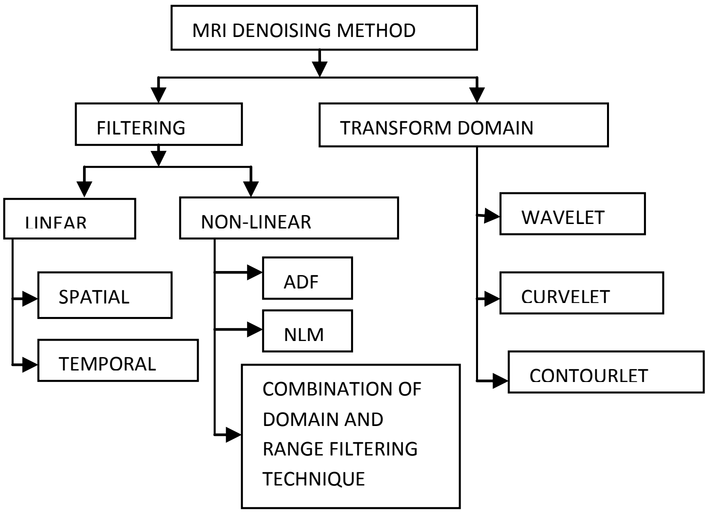

Fig-1: Classification of MRI denoising methods

3.1 Filtering Approach

3.1.1 Spatial and Temporal Filtering:

McVeigh et al. [4] proposed the spatial filter and temporal filter for denoising MR images generally to reduce Gaussian noise. In spatial filter image is convolved with filter. Spatial technique reduces the variance but blurs sharp edges by an amount of the shape of the function used in the convolution. This process is similar to reduce high spatial frequencies in the MR image. This filter smoothen the final MR image, but the signal-to-noise ratio is unaffected because both the signal and the noise are reduced by the same factor. In such type of image smoothening, there is a compromise between the reduction of noise and artifact and the loss of spatial resolution. Temporal filter is chosen in appropriate relation to the sampling interval to avoid the aliasing artifacts. A temporal filter having too narrow a frequency response diminishes the signal at the edges of the image and too broad a frequency response introduces additional noise through aliasing [19].

3.1.2 Anisotropic Diffusion Filter:

Perona and Malik [5] developed a multiscale smoothing and edge detection scheme called anisotropic diffusion filter in "Scale-space and edge detection using anisotropic diffusion". Anisotropic diffusion filter would overcome the drawback of spatial filtering and significantly improve the image quality by preserving object boundaries, removing noise in homogeneous regions and edge sharpening. This filter based on casting the problem in terms of a heat equation and this is based on second order partial differential equation (PDE) in an anisotropic medium. Smoothening is designed as a diffusive process and this is stopped or suppressed at boundaries by selecting the local gradient strengths in different directions. In this technique image u is convolved only in the direction orthogonal to the gradient of the image and so edges are preserved. The iterative denoising process of initial image u_0 can be expressed as,

$$\frac{\partial_{u(x,t)}}{\partial t} = div(c(x,t)\nabla u(x,t)) \quad (4)$$

$$u(x,0) = u_0(x) \quad (5)$$

Where, ∇u(x, t) is the image gradient at voxel x and iteration t, $\frac{\partial_{u(x,t)}}{\partial t}$ is the partial temporal derivation of u(x, t) and

$$c(x,t) = g\|\nabla u(x,t) = e^{-\|\nabla u(x,t)\|/K^2} \quad (6)$$

Where, K is the diffusivity parameter. This technique is successfully applied to 2D and 3D MR images denoising by Gerig et al. [6]. Performance of the noise filter is excellent but the underlying image model is piecewise constant or slowly varying. Therefore, edge sharpening causes a region with a constant gray value slope. Noise in image is assumed to be zero mean and Gaussian distributed.

Krissian and Aja-Fernandez [7] depicted Noise-Driven Anisotropic Diffusion Filtering of MRI. This method of filtering is presented to remove the Rician noise from magnetic resonance images. It combines Linear Minimum Mean Square Error filter (LMMSE) and anisotropic diffusion filter. Partial differential equation is further extended to use a diffusion matrix instead of a scalar, allowing a better reduction of the noise at contour locations. This new extension based on the eigen vectors of the structure tensor, combines a smoothing of isotropic, planar and linear local structures. In this each smoothing is weighted according to the corresponding LMMSE filter. It does not require user to choose a contrast parameter for the edges of the structures but relies instead on the local statistics of the image i.e. local mean and local variance of image and also on standard deviation estimation of the noise for the underlying noise model. As a result of using a Linear Minimum Mean Square Error filter and estimation of noise level at each iteration, this filter is very robust to the total diffusion time and converges fast.

3.1.3 Nonlocal Means Filter:

Nonlocal means (NLM) filter is proposed by Buades et al. [8]. There are many denoising methods which are generally considering local pixels within a small neighbour to remove the noise. Therefore, large scale structures are preserved while small structures are considered as noise and are removed. The NLM filter uses the redundancy of information contained within the images to remove the noise. There stored intensity value of the voxel is calculated as the weighted average of all the voxel intensities within the image. In the non local means Buades et al. [8], Given a discrete noisy image, u = {u(i)|i ∈ I}, the estimated value NL[u](i), for a pixel i, is computed as a weighted average of all the pixels in the image,

$$NL[u]i = \sum_{j \in I} W(i,j)u(j) \quad (7)$$

Where the family of weights {w(i, j)}j depend on the similarity between the pixels i and j, and satisfy the conditions 0≤w(i, j)≤ 1 and Σw(i, j) = 1. The similarity of two pixels i and j depends on the similarity of the intensity gray level vectors u(N_i) and u(N_j), where N_k denotes a square neighborhood of fixed size and centered at a pixel k. This similarity is measured according to decreasing function of Euclidean distance, $\|u(N_i) - u(N_j)\|_{2,a}^2$.

Where, 'a' is greater than zero and is the standard deviation of the Gaussian kernel. The application of the Euclidean distance to the noisy neighborhoods raises the following equality:

$$E\|u(N_i) - u(N_j)\|_{2,a}^2 = \|u(N_i) - u(N_j)\|_{2,a}^2 + 2a^2 \quad (8)$$

This equality shows the robustness of the algorithm since in expectation the Euclidean distance conserves the order of similarity between pixels. The pixels having a similar gray level neighborhood have larger weight in the average. These weights are defined as,

$$W(i,j) = \frac{1}{Z(i)} e^{-(\|u(N_i) - u(N_j)\|_{2,a}^2)/h^2} \quad (9)$$

Where Z(i) is the normalizing constant

$$Z(i) = \Sigma\, e^{-(\|u(N_i)-u(N_j)\|^2_{2,a})/h^2} \quad (10)$$

Where, parameter 'h' acts as a degree of filtering. It controls the decay of the exponential function and so the decay of the weights as a function of the Euclidean distances. Buades et al. [8] used Non Local Mean filter for removing Gaussian noise in 2D natural images. In natural images, the redundancy exists because they usually contain smooth regions and textured. Manjon et al. [9] modified this original NLM algorithm to denoise the multispectral MR images. In this the similarity measure can be obtained by combining information from various channels.

The main drawback of the NLM algorithm is computational burden due to its complexity mainly for 3D MRI data. Coupe et al. [10] proposed an optimized and parallelized (multi-threading) implementation of the NLM filter for denoising 3D MR images to overcome this problem. This filter decreases the computational time approximately up to the factor of 50. Later, the authors extended their work for developing fully automated and optimized block wise NLM filter [11] for denoising 3D MRI.

3.1.4 Combination of Domain and Range Filtering Techniques:

Tomasi and Manduchi [12] presented the bilateral filter as a non-iterative alternative to anisotropic diffusion filter. Both these approaches are similar in some constraint i.e. edges are preserved and images are smoothed. But, the bilateral filtering does not involve the solution of PDE and implemented in a single iteration. Bilateral filter is a combination of two filters i.e. domain and range filters which are Gaussian filter. The weights of the domain filter are proportional to the spatial distance (geometric) of a pixel around its neighborhood. Coefficients of the range filter are corresponding to the photometric (intensity) distance around the neighborhood of a pixel. The filtered image is obtained by replacing the intensity value of each pixel with an average value weighted by the geometric and photometric similarities between the pixels within a spatial window. This filter is proposed for MRI denoising by Xie et al. [13].

3.2 Transform Domain Approach

3.2.1 Wavelet Transform:

Multi-resolution analysis (MRA) of DWT analyzes the signal at different frequencies giving different resolutions. DWT splits the signal into high frequency part and low frequency parts. Most of the image information is stored in low frequency sub-bands and edge information is in high frequency sub-bands. Wavelets are obtained from mother wavelet by dilations and shifting:

$$\Psi_{a,b}(t) = \frac{1}{\sqrt{a}}\,\Psi\!\left(\frac{t-b}{a}\right) \quad (11)$$

DWT can describe local features spatially or spectrally and therefore filter out most of noise while at the same time preserving the edges and fine details. Generally, a wavelet based denoising method includes the following steps:

1. Apply DWT on noisy original image and then we get the wavelet coefficients.

2. Apply thresholding to wavelet coefficients to minimize the contribution of noise in the wavelet domain.

3. Take IDWT of the processed coefficients to produce the denoised image.

Main drawback of this filtering method was that any small structures which are similar in size to the noise are eliminated. Nowak [14] proposed the wavelet domain filter for denoising MR images mainly to minimize the Rician noise. Generally, wavelet coefficients of a noisy MR images are biased estimates of their noise free counterparts because of signal dependant mean of the Rician noise. This technique effectively overcomes this problem by filtering the squared magnitude image (sum of the squared real and imaginary components) in the wavelet domain. Data is non-central Chi-square distributed and wavelet coefficients are no longer biased estimates of their counterparts in the squared magnitude images. In the scaling coefficients bias still remains, but it is not signal dependant and it can be removed easily.

3.2.2 Curvelet Transform:

Wavelet transform based denoising methods are not suitable for describing the signals which have high dimensional singularities such as edges. To detect, represent and process high dimensional data, Cande's and Donoho depicted the concept of curvelet transform based on the theory of multiscale geometric analysis. Curvelet possess anisotropy and directionality to represent the directions of the edges in image. First curvelet transform for image denoising is used by Starck et al. [15].

There are 4 steps used in curvelet transform which are as follows:

1. Sub-band Decomposition: We define a bank of sub band filter Po, (Δ_s , s >0). The object f is filter into different subbands:

$$f \rightarrow (P_0 f, \Delta_1 f, \Delta_2 f \ldots) \quad (12)$$

This step divides the image into several resolution layers. Details of different frequencies are present in each layer.

Po - Low pass filter,

Δ_1, Δ_2 – Band- pass filter (High-pass)

So the original image can be reconstructed from the sub bands:

$$f = P_0(P_0 f) + \Sigma_s \Delta_s(\Delta_s f) \quad (13)$$

2. Smooth partitioning: It is defined as collection of smooth window $W_Q(X_1, X_2)$ localizes around dyadic squares:

$$Q_{(s,k_1,k_2)} = \left[\frac{k_1}{2^s}, \frac{k_1+1}{2^s}\right] * \left[\frac{k_2}{2^s}, \frac{k_2+1}{2^s}\right] \in Q_s \quad (14)$$

Consider W is a smooth window function with 'main' support of size $2^{-s} * 2^{-s}$ multiplying a function by the corresponding window function W_Q generate a result localized near Q. Do this for all Q at a certain scale, i.e. all Q=Q(s, K_1, K_2) with $K_1$and K_2 varying but s fixed. Apply this windowing dissection to each of the subbands in the previous stage i.e. sub-bands are shaped into squares.

$$h_Q = W_Q \Delta_s f \quad (15)$$

3. Renormalization: For a dyadic square Q,

$$T_Q f(x_1, x_2) = 2^s f(2^s x_1 - k_1, 2^s x_2 - k_2) \quad (16)$$

denote the operator which transports and renormalizes f therefore input part supported near Q becomes the output part supported near [0,1] x [0,1]. In this step, each 'square' generated in the previous stage is renormalized to unit scale:

$$g_Q = T_Q^{-1} h_Q \quad (17)$$

4. Ridgelet Analysis: The Ridgelet construction divides the frequency domain to dyadic coronae. In the angular direction, it samples the s-th corona at least 2^s times. In the radial direction, it samples using local wavelets. The ridgelet element has a formula in the frequency domain:

$$p_\lambda(\xi) = \frac{1}{2}|\xi|^{\frac{-1}{2}}(\Psi_{j,k}(|\xi|).w_{i,l}(\theta) + \Psi_{j,k}(-|\xi|).w_{i,l}(\theta+\pi)) \quad (18)$$

Each normalized square is analyzed in the Ridgelet system as,

$$\propto_{Q\lambda} = < g_Q . p_\lambda > \quad (19)$$

Following steps are involved in the denoising algorithm of curvelet transform:

1. Compute all thresholds for curvelet.

2. Compute norm of curvelet.

3. Apply curvelet transform to noisy image.

4. Apply hard thresholding to these curvelet coefficients.

5. Apply inverse curvelet transform to the result of step 4.

This method is proposed for denoising MR images by Ashamol et al. [16] and also, they combine anisotropic diffusion along with hybrid denoising techniques which involves curvelet and wavelet transforms for denoising MR images.

3.2.3 Contourlet Transform:

The wavelet transform is useful in representing images containing smooth areas which are separated with edges. But, it cannot perform effectively when the edges are smooth curves. The contourlet have the characteristics of capturing contours and fine details in the images. Do and Vetterli [17] present the Contourlet transform which is a geometrical image transform and represents images containing textures and contours. Also it provides sparse representation at both spatial and directional resolutions. It offers a flexible multiresolution and directional decomposition for images, since it allows for a different number of directions at each scale. Contourlet transform is constructed by combining two different and successive decomposition stages: a multiscale decomposition followed by a multidirectional decomposition. In the first stage Laplacian pyramid (LP) scheme is used to transform the image into one coarse version and a set of LP band pass images. The second stage applies directional filter bank (DFB) and critical sub-sampling to decompose each LP band pass image into a number of wedge shaped sub-bands and therefore capturing directional information. Finally, the image is represented as a set of directional sub-bands at multiple scales. The steps used in the denoising technique of contourlet transform are given below:

1. Determine the number of scales and directions by performing multiscale decomposition of the image using contourlet transform.

2. Apply thresholding (hard or soft thresholding) at each direction in each scale of contourlet coefficients.

3. Apply inverse contourlet transform to reconstruct the denoised image from the modified contourlet coefficients

Latha and Subramanian [18] used this contourlet transform for MRI denoising method.

4. Comparison of MRI Denoising Methods

In the previous section we study the different MRI denoising methods and there advantages. But each technique also has some disadvantages as compared to other. Existing MRI denoising methods are compared based on the quantitative performance metrics such as peak signal to noise ratio (PSNR), structural similarity index method (SSIM) and mean square error (MSE). Disadvantages of this method are given in the below table[19]:

Table -1: Comparison of MRI denoising Method

Denoising Method	Disadvantages
SPATIAL	Edges are blurred because of averaging pixels with non similar patterns (suitable only for Gaussian noise)
TEMPORAL	In order to avoid the aliasing, proper selection of frequency response for the temporal filter is important
ADF	Usually erases small feature and transforms image statistics due to its edge enchantment causes blocky (staircase) effect in the image
NLM	Computational burden due to its complexity of calculating the weight of the pixel/voxel
BILATERAL AND TRILATERAL FILTER	Use the narrow spatial window for the selection of the neighborhood pixels in order to calculate the new value of that pixel. So, large scale structures are preserved, while small structures are considered as noise and are removed
WAVELET	Local noise variance is estimated from the wavelet decomposition high frequency sub band after discarding edge pixels and therefore result in loss of information
CURVELET	It does not work well in smooth areas, produce curvelet-like artifacts
CONTOURLET	It capture the smooth contours in the image and therefore high computational complexity

5. CONCLUSION

MRI is one of the powerful diagnostic techniques in the medical application. However, the noise during image acquisition degrades its ability for the human interpretation or computer-aided analysis. Improving SNR would result in additional acquisition time and reduce temporal resolution. Therefore, denoising should be performed for more accurate diagnosis. There are several published MRI denoising methods and each method has advantages and disadvantages discussed in this paper. This paper summarized the MRI denoising techniques in the group of filtering and transform domain. These techniques are compared based on their performance metrics such as PSNR, SNR, MSE, SSIM and in terms of the visual quality of the images. Prior knowledge of the noise map is also discussed. However, no single method has shown to be superior to all others regarding noise reduction, boundary preservation, robustness, user interaction and computation cost. The aim of this survey paper is to introduce available MRI denoising techniques. This will help for the researchers who are trying to develop a new denoising technique for MR images and to develop superior technique. Also from this survey, one can choose the best denoising method for further processes such as image classification, image segmentation and image registration.

REFERENCES

[1] G. Wright, “Magnetic resonance imaging”, IEEE Signal Process. Mag. 14 (1997)56–66.

[2] R.M. Henkelman, “Measurement of signal intensities in the presence of noise in MR images”, Med. Phys. 12 (1985) 232–233.

[3] H. Zhu, Y. Li, J.G. Ibrahim, X. Shi, H. An, Y. Chen, W. Gao, W. Lin, D.B. Rowe, B.S. Peterson, “Regression models for identifying noise sources in magnetic resonance imaging”, J. Am. Stat. Assoc. 104 (2009) 623–637.

[4] E.R. McVeigh, R.M. Henkelman, M.J. Bronskill, “Noise and filtration in magnetic resonance imaging”, Med. Phys. 12 (1985) 586–591.

[5] P. Perona, J. Malik, “Scale-space and edge detection using anisotropic diffusion”, IEEE Trans. Pattern Anal. Mach. Intell. 12 (1990) 629–639.

[6] G. Gerig, O. Kubler, R. Kikinis, F.A. Jolesz, “Nonlinear anisotropic filtering of MRI data”, IEEE Trans. Med. Imaging 11 (1992) 221–232.

[7] K. Krissian, S. Aja-Fernández, “Noise driven anisotropic diffusion filtering of MRI”, IEEE Trans. Image Process. 18 (2009) 2265–2274.

[8] A. Buades, B. Coll, J.M. Morel, "A review of image denoising algorithms", with a new one, Multiscale Model. Simul. 4 (2005) 490–530.

[9] J.V. Manjon, M. Robles, N.A. Thacker, "Multispectral MRI de-noising using non-local means", Med. Image Understand. Anal. (MIUA) (2007) 41–46.

[10] P. Coupe, P. Yger, C. Barillot, "Fast non local means denoising for MR images", in:Proceedings at the 9th International Conference on Medical Image Computingand Computer assisted Intervention (MICCAI), Copenhagen, 2006, pp. 33–40.

[11] P. Coupe, P. Yger, S. Prima, P. Hellier, C. Kervrann, C. Barillot, "An opti-mized blockwise nonlocal means denoising filter for 3-D magnetic resonance images", IEEE Trans. Med. Imaging 27 (2008) 425–441.

[12] C. Tomasi, R. Manduchi, "Bilateral filtering for gray and color images", Presented at the 6th Int. Conf. Comput. Vis., Bombay, India, 1998, pp. 839–846.

[13] J. Xie, P.A. Heng, M. Shah, "Image diffusion using saliency bilateral filter", IEEE Trans. Inform. Technol. Biomed. 12 (2008) 768–771.

[14] R.D. Nowak, "Wavelet-based Rician noise removal for magnetic resonance imaging", IEEE Trans. Image Process. 8 (1999) 1408–1419.

[15] J.L. Starck, E.J. Candes, D.L. Donoho, " The Curvelet transform for image denoising", IEEE Trans. Image Process. 11 (2002) 670–684.

[16] V.G. Ashamol, G. Sreelekha, P.S. Sathidevi, "Diffusion-based image denoising combining curvelet and wavelet", in: Proceedings of 15th InternationalConference on Systems, signals and Image processing, Bratislava, 2008, pp.169–172.

[17] M.N. Do, M. Vetterli, "The contourlet transform: an efficient directional multiresolution image representation", IEEE Trans. Image Process. 14 (2005)2091–2106.

[18] P. Latha, R. Subramanian, "Medical image denoising using X-lets", in: Proceedings of Annual IEEE India Conference, New Delhi, 2006, pp. 1–6.

[19] Mohan, V. Krishnaveni, Yanhui Guo, "A survey on the magnetic resonance image denoising methods", Biomedical Signal Processing and Control 9 (2014) 56–69.