# FILTERED- BACK PROJECTION ALGORITHM ON COMPUTED TOMOGRAPHY (CT) SCAN

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**Abstract -** *This project has done to learn how to perform filtered-back projection algorithm on computed tomography (CT) projection data of a Shepp-Logan head phantom to reconstruct image. Based on a given data file containing fan-beam x-ray data, a program has been created to perform CT Reconstruction on the data. Filtered back projection algorithm has been used to reconstruct CT data and two spatial domain image processing algorithms histogram equalization and gamma correction have been used to enhance the image.*

***Key Words**: Computed tomography (CT), Shepp-Logan head phantom, Fan-beam Projection, X-ray, Histogram equalization, Gamma correction.*

## 1. INTRODUCTION

The main focus of this object is to study the algorithms of fan beam projection of computed tomography and to reconstruct the image from the projection datas using back projection algorithms. This project has been completed using 100 angles of projection (from $0°$ $to$ $360°$), each angle of projection consisting 127 rays. Whole project has been accomplished through three main steps.

First, every single projection data R (pixel value) of 127 rays in the each of 100 angles has been changed to intermediate projection values R'.

Second, Designing a modified Ram-Lak Filter (tapped by a hamming window) and filtering the intermediate data with this filter.

Third, result of the second step is a filtered projection data Q of 127 rays for each of the 100 rotation angles. Now to reconstruct in a XY 2D domain, for every $(x, y)$ combination pixel value $f(x, y)$ has been calculated by back projection algorithm using the Q values. In the following chapters, this report will introduce a brief background of related technologies, detailed steps of project implementation, introduction of project results and conclusion.

## 2. BACKGROUND

X-ray computed tomography (X-ray CT) is a technology that uses computer-processed X- rays to produce tomographic images (virtual 'slices') of specific areas of a scanned object, allowing the user to see inside the object without cutting [?]. Two types of projections are being used:

  (a)  Parallel beam and
  (b)  Fan beam projections.

In parallel projections, a source-detector combination has to linearly scan over the length of a projection, rotate through a certain angular interval, and then scan linearly over the length of the next projection [1]. This is actually slower process since it takes time to collect all the data.
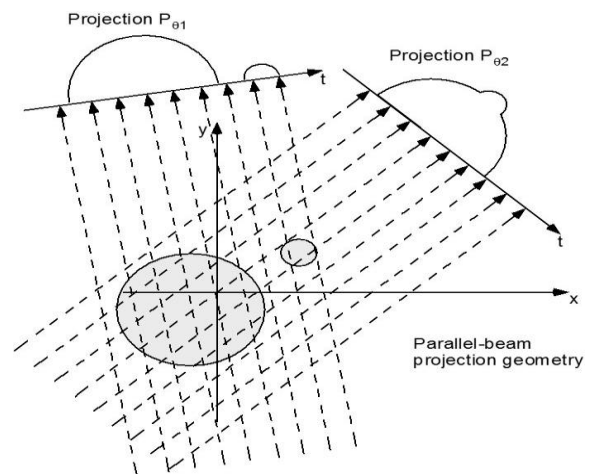


Figure 1: Parallel-beam projection

A much faster way is to use fan projections. One now uses a point source of rays emitting a fan shaped beam. On the other side of the object a bank of detectors is used to make all the measurements in one fan simultaneously. The source and the entire bank of detectors are rotated to generate the desired number of fan projections.

There are two types of fan projections depending upon whether a projection is sampled at equiangular or equispaced intervals [1].
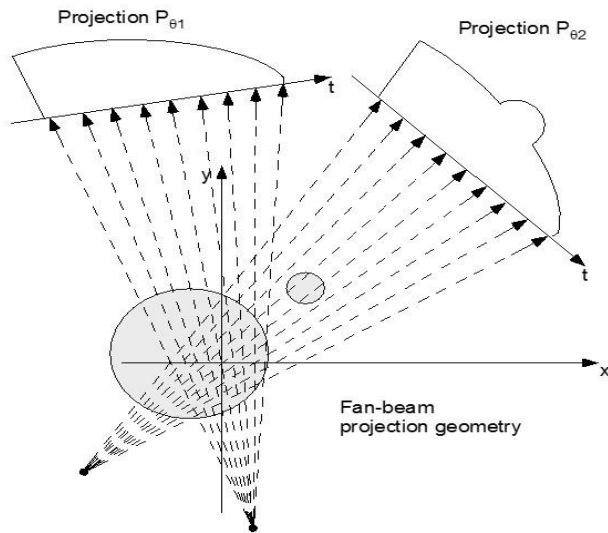


Figure 2: Fan-beam projection

In this project, equiangular fan beam projection data have been used where all the rays for a single rotation angle maintains equal angle difference among them. Reconstructed image of just using projection data results into blur image due to distortion. To avoid this effect projection data need to be filtered before back projection [2]. This is the reason to name this method filtered back projection algorithm

## 3. METHODS

For this project, the input is a $127 * 100$ matrix R. Each column of this matrix contains projection data of $127$ rays for every rotation angle $\beta_i (i = 1,2,3, \ldots \ldots ,100)$. The rays are counted from $n = -63$ to $n = 63$ where $n = 0$ is central ray. So for a particular rotation angle (column), top most element is projection data is for $n = -63$ and the bottom most one is for $n = 63$ numbered ray.

The whole implementation steps of the project can be written as below:

**Step 1:** To calculate the angular distance $\alpha$ between two consecutive rays for any rotation angle. To do that, maximum angle $\gamma_{max}$ has to be calculated, which is the angle between central and $n = 63$ or $-63$ numbered ray, using the formula $\gamma_{max} = \sin^{-1}\left(\frac{1}{D}\right)$.

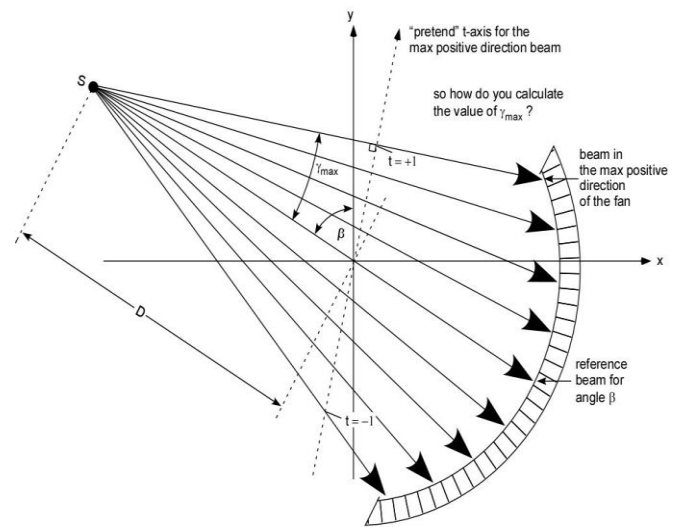Then dividing $\gamma_{max}$ by 63 gives $\alpha$.



Figure 3:  Geometry for calculating $\gamma_{max}$

**Step 2:** To calculate intermediate projection data $R'$ from $R$ by using $R'(n\alpha) = R(n\alpha) * D * \cos(n\alpha)$. The $R'$ matrix size is $127 * 100$.  It has been zero padded to new size $256 * 100$ then has to be shifted to frequency domain by FFT to be convolved with the filter (explained in next step).[1]

**Step 3:** To design a 256 size modified Ram-Lak filter (cite) to filter the R' data and smooth it with a Hamming window. The equation for filter is

$$g(n\alpha) = \begin{cases} \dfrac{8}{\alpha^2} & : n = 0 \\ 0 & : n\ is\ even \\ -\dfrac{1}{2}\left(\dfrac{1}{\pi * sinn\alpha}\right)^2 & : n\ is\ odd \end{cases}$$

**Step 4:** To obtain filtered projection data Q for each ray (corresponds to $n\alpha$) of each rotation angle $\beta$ by multiplying $R'(n\alpha)$ by $g(n\alpha)$ tapped by hamming filter.

$$Q_{\beta i}(n\alpha) = R'_{\beta i}(n\alpha) * g(n\alpha)$$

Then Q datas have been taken to spatial domain by IFFT and then has been cropped to 127*100 to get back to the original size.

$$f(x,y) = R'_{\beta i}(n\alpha) * g(n\alpha)$$

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[1]zero padding has been used to avoid circular convolution which tends to greater statistical error

**Step 5:** Now Q datas are used to reconstruct an image of $256*256$ size of $(x, y)$ space. Both x and y are chosen from range -1 to 1. Then for every $(x, y)$ combination, the pixel $f(x, y)$ has been calculated.

But, before that for each $(x, y)$, two entities has to be calculated for every rotation angle $\beta$. One is angle $\gamma'$ between the central beam and line from the beam source to particular $(x, y)$ position. Other one is distance L from the beam source to that $(x, y)$. Following equations will return these two values.

$$rise = x*\cos(\beta) + y*\sin(\beta)$$

$$run = D + x*\sin(\beta) - y*\cos(beta)$$

$$\gamma' = \tan^{-1}(rise/run)$$

$$L^2 = rise^2 + run^2$$

Now for each $(x, y)$, all the 100 rotation angles have returned 100 $\gamma'$ and $L^2$ . But Q (127*100) matrix has 127 elements for each rotation angle corresponding to their angular distance from reference beam varying from $-63\alpha\,(-\gamma_{max})$ to $63\alpha\,(+\gamma_{max})$. $\gamma'$ can be aligned to either of these rays or not. To overcome this issue, $Q_{\beta i}$ for a particular $i^{th}$ rotation angle has been calculated by interpolation of adjacent 2 values of $Q(n\alpha)$ for that particular rotation angle $\beta_i$.

Then, for every $(x, y)$ position pixel value $f(x, y)$ has been calculated with the following equation.

$$f(x, y) = \Delta\beta * \sum_{i=1}^{100} \frac{Q(\gamma')}{L^2(x, y, \beta_i)}$$

Here $(Q/L^2)$ term has been summed for all 100 rotation angles for individual $f(x, y)$.

**Step 6:** At last Pixel datas $f(x, y)$ from previous step have been gone through two image enhancement techniques histogram equalization and gamma correction.

## 4. RESULTS

This project has used fan beam CT projection datas of Shepp-Logan head phantom. If the reconstructions steps work properly, reconstructed image should show some high pixel values due to tumors (in $y = -0.609$). A $1D\ f(x)$ dataset also has been produced for fixed $y = -0.609$  to verify the result  properly.
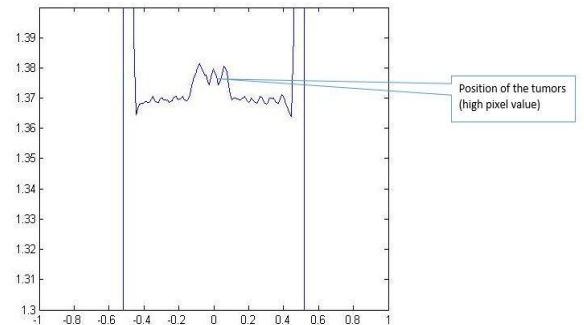


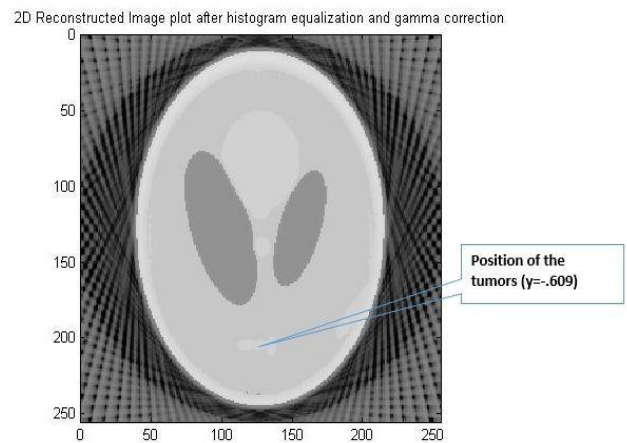Figure 4:  Pixel values plot against x for constant $y = -0.609$



Figure5:  Reconstructed image in $f(x, y)$  space

## 5. CONCLUSION

This project gives a great knowledge about various CT projection systems and reconstruction of projection datas using Back Filtered Projection algorithm. Some key issues have to be maintained in the working progress of this project. First, properly zero padding, second, proper shifting of filters and hamming windows, third, properly interpolating Q values. Some other image enhancement algorithms may return better quality image.

The Final two sections will be the Appendix and the References, which can appear in     either order according to your preference. The Appendix is where your code listings appear, and the References section lists the sources you used and that you cited in other parts of the document.

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