Contact Stress Analysis of Composite Spur Gear using Photo-Stress Method and Finite Element Analysis

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Abstract -*Gears have a wide variety of applications from watches to very large mechanical units like lifting devices and automotives. Gear is one of the most critical component in a mechanical power transmission system and in most industrial rotating machinery. New gear design is needed because of the increasing performance requirement.Gear teeth normally fail when load is increased above certain limit. Therefore it is required to explore alternate material for gear manufacturing. Composite materials provide adequate strength with weight reduction and they have emerged out as a better replacement for metallic gears. The contact stress in the mating gear is the key parameter in gear design. This paper represent contact stress analysis of steel gear and composite gear using Hertz equation and by Finite Element Analysis using Ansys 16.0 Workbench. Also experimental stresses are calculated using Photo-Stress Method.In this work, Aluminium Silicon Carbide is used as a gear material. When compared, the results of both theoretical method and FEA show a good degree of agreement with experimental results.*

***Key Words*:Contact Stress, Finite element Analysis, Hertz Equation, Spur gear**

1.INTRODUCTION

The rapid growth of heavy industries such as vehicle, shipbuilding and aircraft industries require extensive application of gear technology. Spur gear is a cylindrical shaped gear in which the teeth are parallel to the axis. Spur gears are easy to manufacture and it is mostly used to transmit power from one shaft to another shaft up to certain distance & it is also used to vary the speed & Torque. e.g. Watches, gearbox etc. The replacement cost of spur gear is very high and also the system down time is one of the effect in which these gears are part of system. Failure of gear causes breakdown of system which runs with help of gear. E.g. automobile vehicle. So it becomes very important to increase the strength of gear to avoid the failure.

Gears analysis in the past was performed using analytical methods, which required a number of assumptions and simplifications. In general, gear analyses are multidisciplinary, including calculations related to the tooth stresses and to tribological failures such as like wear. Designing highly loaded spur gears for power transmission systems that are both strong and quiet requires analysis methods that can easily be implemented and also provide information on contact and bending stresses, along with transmission errors. The finite element method is capable of providing this information. The finite element method is very often used to analyze the stress state of an elastic body with complicated geometry, such as a gear. The life and performance of gear teeth are directly related to the ability of the teeth to withstand contact stresses. Contact stresses may produce pitting within the contact area and eventually lead to tooth failure. In spite of the importance of contact stresses in gears, comprehensive analyses of these stresses have not been extensively reported in the literature.

Composites provide much improved mechanical properties such as greater strength to weight ratio, increase in hardness, and hence less chances of failure. So this work is concerned with replacing metallic gear with gear of composite material of Aluminium Silicon Carbide so as to improve performance of machine and to have longer working life. P.B. Pawar has developed a metal matrix composite of Aluminum based Silicon Carbide [1]. The composition of Silicon Carbide is varied in Aluminum and mechanical tests were performed. They proposed to use this material for power transmitting element like gears. Author P.B. Pawar has manufactured the spur gear from composite material of Aluminum Silicon Carbide. He has done FEA using Ansys 14.0 and concludes that composite gears offer improved properties over steel alloys and can be used as alternative for replacing metallic gears [2]. Neelima Devi and co authors worked on the mechanical characterization of Aluminum Silicon Carbide [3]. They found that the weight to strength ratio for composite is about three times that of mild steel and it is two times less in weight than aluminum of same dimension. Seok-Chul Hwang presents a contact stress analysis for tooth in contact of gears during rotation [4]. Contact stress analysis for spur and helical gears is carried out between two gear teeth at different contact positions during rotation. The variation of contact stress values during rotation is compared with the values of contact stress at the lowest point of single tooth contact. Ali Raad Hassan has developed a program to plot paired teeth in contact [5]. This program was run each 30 of rotation of pinion to create 10 cases. The program gave graphic results for different FE models and stress analysis was carried out in ANSYS. Sushil Kumar Tiwari found out the contact stress and bending stress for involute spur gear teeth in meshing by finite element method and the results are checked with those obtained by Lewis formula, Hertz equation and AGMA/ANSI equations [6]. They observed that Hertz theory is the primary basis of contact stress calculation and for determining bending stress in a pair of gear, Lewis formula is used. Vivek Karaveer has done the modelling of FEA of spur gear using ANSYS 14.5. He has compared the stress values and deformation for steel and grey cast iron [7].

Mohammad Jebran Khan reports the contact stress analysis of Stainless Steel spur gears by theoretical method using Hertz equations and by Finite Element Analysis using FEA software ANSYS 14.0 Workbench. The spur gear is sketched and modelled in ANSYS Design Modeller and the contact stress analysis is done in Mechanical ANSYS Multiphysics [8]. Prabhakar Purushothaman emphases on the contact stress. The value of contact stress is very important, as the stress value changes with contact area. The higher the contact area the stress generated will be less and for lesser contact area high stress will be generated [9]. Santosh Patil observed that contact behavior in loaded gears along the line of action changes tremendously. the contact pressure for a pair of mating spur gears has been determined by Finite Element Analysis (FEA) using ANSYS software. The contact pressure results are then verified by twin-disc experimental results and the Hertzian contact pressure equation [10]. Santosh Patil studies the effect of coefficient of friction along the line of action of a spur gear.He determines the shape function which define the change of contact stress along the line of action of meshed gear. The result shows an increase in contact stress with the increase in coefficient of friction [11].

2. DESIGN OF GEAR

The material properties of steel and Aluminium Silicon Carbide composite are given in the table 1.

Table -1: Material properties of gear materials

Material Property	Steel	AlSiC
Young's Modulus	210 GPa	150 GPa
Poisson's Ratio	0.3	0.3
Ultimate tensile strength N/mm^2	200	151

The comparative study of steel gear and composite gear is done. So the basic design of spur gear is same for both the gears. The various parameters of gear design are given in the table 2 below.

Table -2: Gear design parameters

Parameter	Gear Pair
No. of teeth	20
Gear Ratio	1
Module	4.5
Pressure angle	20
Pitch diameter	90
Face width	45
Center Distance	90
Torque (Nm)	302
Speed (rpm)	1000

3.THEORETICAL ANALYSIS OF CONTACT STRESS USING HERTZ THEORY

Earle Buckingham (1926) have used theory of Hertz to calculate the contact stress between a rotating pair of teeth while transmitting power by treating the pair of teeth in contact as cylinders of radii equal to the radii of curvature of the mating involutes at the pitch point. According to Hertz theory, when two cylinders are pressed together, the contact stress is given by

$$\sigma_c = \frac{2P}{\pi BL} \tag{1}$$

$$B = \sqrt{\left[\frac{2P\left(\frac{1-\mu_1^2}{E_1}+\frac{1-\mu_2^2}{E_2}\right)}{\pi l\left(\frac{1}{d_1}+\frac{1}{d_2}\right)}\right]} \tag{2}$$

Where,

σ_c = maximum value of contact stress (N/mm2)

P = force pressing the two cylinders together (N)

B = half width of deformation (mm)

L = axial length of cylinders (mm)

d_1, d_2 = diameters of two cylinders (mm)

E_1, E_2 = modulii of elasticity of two cylinder materials (N/mm2)

μ_1, μ_2 = poisson's ratio of the two cylinder materials (unitless)

Substituting the value of half width of deformation *B*, in equation (1) & squaring both sides we get,

$$\sigma_c^2 = \frac{1}{\pi}\left(\frac{P}{L}\right)\left[\frac{\left(\frac{1}{r_1}+\frac{1}{r_2}\right)}{\left(\frac{1-\mu_1^2}{E_1}+\frac{1-\mu_2^2}{E_2}\right)}\right] \quad (3)$$

If we treat the material of both the cylinders as same, then the modulus of elasticity and poisson's ratio will be equal. Therefore substituting $E_1 = E_2 = E$ and $\mu_1 = \mu_1 = \mu$ in equation (3) we get

$$\sigma_c^2 = \frac{1}{2\pi}\left(\frac{P}{L}\right)\left[\frac{\left(\frac{1}{r_1}+\frac{1}{r_2}\right)}{\left(\frac{1-\mu}{E}\right)}\right] \quad (4)$$

Now applying this equation to a pair of spur gear teeth in contact, we need to replace the radii *r1* &*r2* by the radii of curvature at the pitch point

$$r_1 = \frac{d_{pp}\sin\phi}{2} \quad \text{and} \quad r_2 = \frac{d_{pg}\sin\phi}{2}$$

Now, since the pinion and gear have equal geometry in all respects as given in table 2, we have,

$$d_{pp} = d_{pg} = d_p$$

$$r_1 = r_2 = r = \frac{d_p \sin\phi}{2} \quad (5)$$

From (4) and (5) we get

$$\sigma_c^2 = \frac{1}{\pi(1-\mu)}\left(\frac{PE}{Lr}\right) \quad (6)$$

For same material of pinion and gear as stainless steel, then for stainless steel, from table 1 poisson's ratio μ =0.3.

$$r = r_p \sin\phi \quad \text{and} \quad P = \frac{P_t}{\cos\phi}$$

The length *L* is same as the face width *b* of spur gears, therefore replacing *L* by *b* in equation. Substituting this value of in equation (6) and solving we have

$$\sigma_c = 0.6747\left[\frac{P_t E}{b r_p \sin\phi\cos\phi}\right]^{\frac{1}{2}} \quad (7)$$

The tangential load acting on the tooth can be obtained by

$$P_t = \frac{2T}{d_p} = \frac{2\times 302}{90} = 594.54N$$

Putting the value of young's modulus E, face width b, pitch circle radius r_p and pressure angle in equation (7) , we get the value of contact stress:

For Steel

$$\sigma_c = 0.6747\left[\frac{594.5\times 210\times 10^3}{45\times 45\sin 20\cos 20}\right]^{\frac{1}{2}} = 54.62\frac{N}{mm^2}$$

For Composite material

$$\sigma_c = 0.6747\left[\frac{594.5\times 150\times 10^3}{45\times 45\sin 20\cos 20}\right]^{\frac{1}{2}} = 41.23\frac{N}{mm^2}$$

4.STRESS ANALYSIS USING PHOTO STRESS METHOD

PhotoStress is a widely used full-field technique for accurately measuring surface strains to determine the stresses in a part or structure during static or dynamic testing. With the PhotoStress method, a special strain-sensitive plastic coating is first bonded to the test part. Then, as test or service loads are applied to the part, the coating is illuminated by polarized light from a reflection polariscope. When viewed through the polariscope, the coating displays the strains in a colorful, informative pattern which immediately reveals the overall strain distribution and pinpoints highly strain areas. With an optical transducer (compensator) attached to the polariscope, quantitative stress analysis can be quickly and easily performed. Permanent records of the overall strain distribution can be made by photography or by video recording.

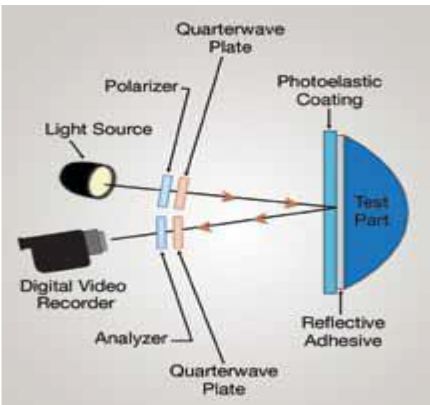


Fig-1: Schematic representation of a reflection polariscope

Fig-2: Reflection Polariscope Apparatus

Fig-3: Loading arrangement of gear

Figure 1 shows the schematic representation of a reflection polariscope. Figure 2 shows the actual reflection polariscope used to perform the experimentation. Figure 3 shows the loading arrangement made to provide the input torque to the pinion.

The basic relationship for strain measurement in a photoelastic coating can be expressed as follows:

$$\varepsilon = \frac{N_n \lambda}{2kt} \tag{8}$$

Where:

ε= principal strains in coating

N_n = normal-incidence fringe order

λ = wavelength of yellow light (22.7μin, or 575 nm)

t = thickness of PhotoStress coating = 3mm

k = strain-optic coefficient of coating =0.15

Assuming the strains in the coating precisely replicate those in the test-part surface, and assuming the part is stressed below its proportional limit, Hookes law can be applied as follows to determine the difference of principal stresses:

$$\sigma = \frac{E}{1+\mu} \times \varepsilon \tag{9}$$

Where:

σ = principal stresses in test part

E = elastic modulus of test material

μ = Poisson's ratio of test material

The preceding relationships implicitly assume that the strains in the test part are unaffected by the presence of the bonded photoelastic coating, and that the strains in the coating are uniform through the coating thickness and equal to those in the surface of the test part. These assumptions are quite well satisfied for typical metal castings, forgings, and robust structural members, since the coating is much lower in elastic modulus, and is usually thin compared to the section depth of the test part.

Putting the value of fringe order we get from reflection polariscope for a given loading in above equation 8 and 9, we can get the stress in the component at that loading.

Table 3: Observation table for pair of steel gear

Sr. No.	Load (Kg)	Torque (Nm)	Fringe Order N_n	Stress σ (MPa)
1	10	100	0.41	42.307
2	20	200	0.48	49.544
3	30	300	0.57	58.832

Table 3: Observation table for pair of composite gear

Sr. No.	Load (Kg)	Torque (Nm)	Fringe Order N_n	Stress σ (MPa)
1	10	100	0.44	32.435
2	20	200	0.51	37.592
3	30	300	0.58	42.762

5. FINITE ELEMENT ANALYSIS

Finite Element Method is the easy technique as compared to the theoretical methods to calculate the stress developed in teeth of gears. Therefore FEM is widely used for the stress analysis of mating gears. FE analysis is done in ANSYS Workbench 16.0 to determine the maximum contact stresses for steel and composite material. Also the deformation is found out for both the gears. CAD model of gear is created in CREO 2.0. It is imported as a IGES file in ANSYS 16.0.

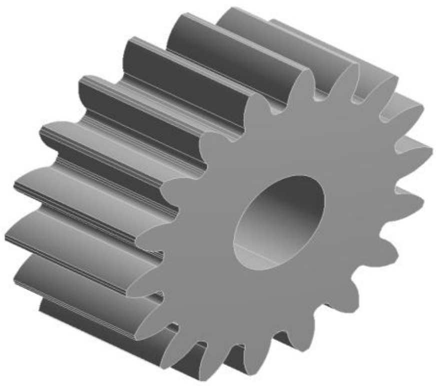


Fig -4: CAD model of gear

5.1 Meshing

Meshing is done using Hexagonal mesh with number of elements of 499900 and number of nodes of 2125654. The element size is 0.8 mm. This mesh is used as it is fine and gives least number of elements with good results. So the calculation time is reduced.

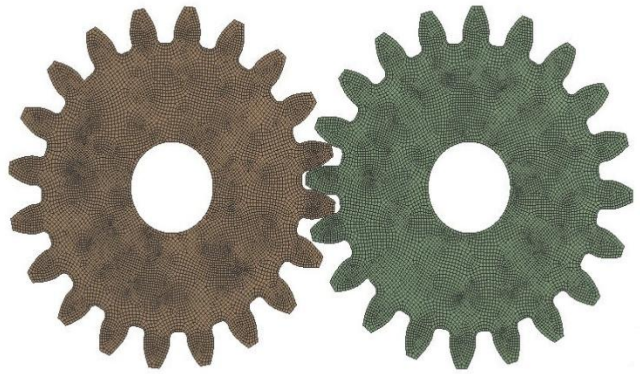


Fig -5: Mesh Model of gear

5.2 Boundary Conditions

Fixed support is applied on inner rim of the gear. Frictionless support is applied on the inner rim of pinion to allow its tangential rotation. Moment of 302Nm is applied on the surface of second gear.

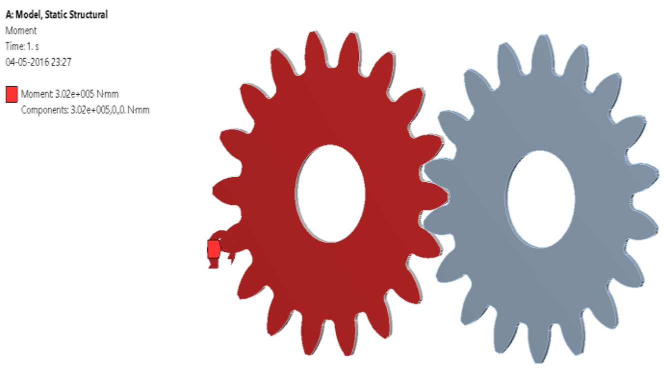


Fig -6: Boundary Conditions of Ansys

6. RESULT AND DISCUSSION

Contact stress for steel and aluminium silicon carbide is calculated in ANSYS 16.0. Figure 7 shows the contact stress for steel which gives a stress of 52.14 MPa. Figure 8 shows contact stress for composite gear which gives a stress value of 39.18 MPa. Chart 1 presents the comparison of stress of steel and composite gear in bar chart form for maximum loading in all three methods.

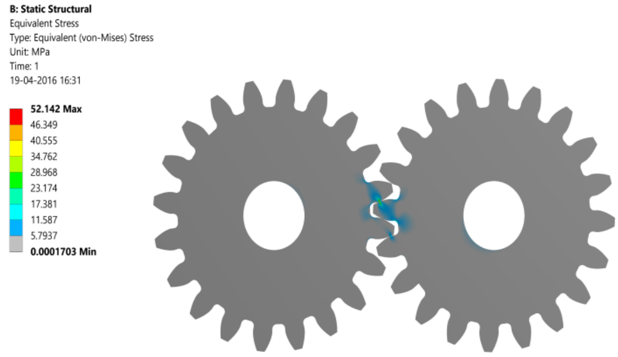


Fig -7: Stress Diagram of Steel

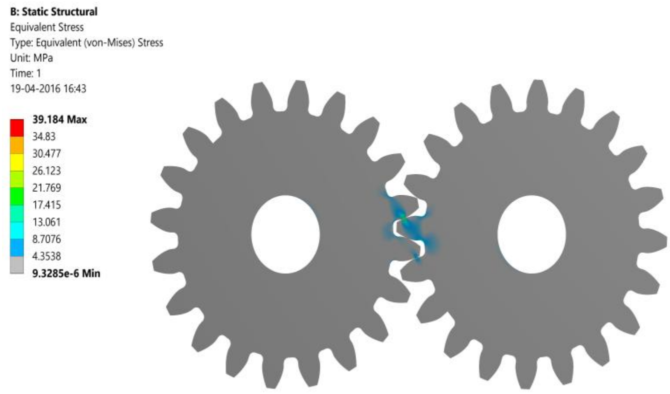
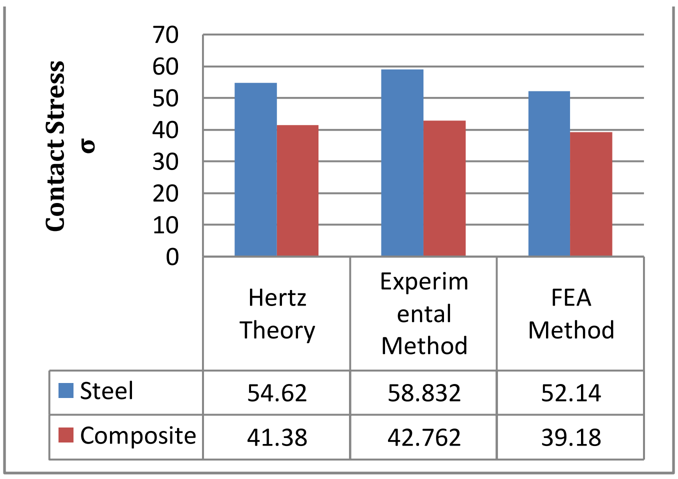


Fig -8: Stress Diagram of Aluminium Silicon Carbide



	Hertz Theory	Experimental Method	FEA Method
Steel	54.62	58.832	52.14
Composite	41.38	42.762	39.18

Chart -1: Stress Comparison Analytical, FEA and Experimental Method

Percentage difference of stress for steel and composite material using analytical method, FEA and Experimental method is calculated. It is shown in table 3.

Table-5: Percentage difference between analytical and FEA

	Steel	Composite Material	% Reduction in Stress
Hertz Equation	54.62	41.38	24.24
FEA Method	52.14	39.18	24.87
Experimental Method	58.832	42.762	27.31
% Difference	12.83	9.14	

7. CONCLUSIONS

Here the theoretical maximum contact stress is calculated by Hertz equation. Photo stress method is used form experimental validation. Also the FE analysis of spur gear is done to determine the maximum contact stress by ANSYS 16.0. It was found that the results from both Hertz equation and Finite Element Analysis are comparable with the experimental method. The results are well withinthe difference of 13%. Also it is observed that stress is reduced by nearly 25% due to the use of composite material.

ACKNOWLEDGEMENT

The authors are thankful to Dr. S. P. Danao, Principal AISSMS COE, Professor A. V. Waghmare, Head, Department of Mechanical Engineering, AISSMS COE for their constant encouragement and cooperation throughout the work.

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