LQR Control of Piezoelectric Actuators

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Abstract—*Piezoelectric actuators (PEAs) uses the inverse piezoelectric effect of piezoelectric materials to generate forces and displacements. The PEA can be modeled as a linear dynamic system with matched uncertainties. Pole placement method is one of the classic control theories that is used in system control for desired performance. This method helps to set the desired pole location and to move the pole location of the system to that desired pole location to get the desired system response. Linear Quadratic Regulator (LQR) is the optimal theory of pole placement method. To find the optimal gains in LQR the optimal performance index should be defined .*

Key words**-Piezoelectric devices, LQR control, uncertain systems, end-effector, Matlab.**

1.INTRODUCTION

Piezoelectric actuators (PEAs) have been used in micro and nano positioning systems due to their fine displacement resolution and large actuation force [1]. In these applications, accurate models of PEAs are required to understand their dynamic behaviors and controller design. A common category of PEA models takes the form of a cascade of three sub-models, each of which representing the effect of hysteresis, creep, and vibration dynamics, respectively [2]. Most of the PEAs have a non-negative input voltage range and their corresponding hysteresis behaviors subject to such one-sided input range are referred to as one-sided hysteresis which contains an initial ascending curve in addition to the hysteresis loops. A number of models for the PEA have been reported, and they can be generally classified into two categories: phenomenon-based models and physics-based models. The phenomenon-based models of PEA are developed based on the experimental results alone. The hysteresis and the vibration dynamics are combined to form a dynamic or rate-dependent hysteresis model for PEAs. In the physics-based models of PEA the linear and nonlinear effects are decoupled by means of individual sub-models of PEAs. In [5] the PEA was modeled as a cascade of a nonlinear sub-model for the rate-independent hysteresis and a linear sub-model for the vibration dynamics.

Based on the models developed for PEAs, various control schemes have been developed and reported to improve ASSthe PEA performance. A significant number of such control schemes are open-loop inversion based or feed forward [1], in which the control action is generated based on the inverse of the PEA model. The feed forward controllers are developed to compensate for the rate-independent hysteresistime applications. Such feed forward controllers works in the cases with low operating frequencies, where hysteresis is the dominant effect. There are two problems associated with the feed forward schemes, which include an accurate model for PEA and the computational effort to invert the model. So that the controllers are developed based on the linear nominal model of the PEA dynamics. The nonlinearity and uncertainty due to hysteresis and external-loading changes are treated as disturbances to be suppressed. For performance improvement at both low- and high- frequencies, a high-gain feedback is desirable. But this may not be feasible due to the system stability. With the increase in the operating frequency, there will be a fast phase loss in the frequency response of the closed-loop system due to the high-frequency PEA dynamics. This tends to destabilize the system. As a result, high gains are limited for use at high frequencies. One method for improvement is the use of a notch filter to lower the first resonant peak of the system, thus increasing the gain margin [8]. Other method for improvement is the use of disturbance observers [3] to estimate and then provide the PEA with a portion of the control input required for disturbance compensation, which allows for the use of low-gain feedback.

Robust controllers [3] can be designed to minimize the effects of the disturbances based on a cost function. If the disturbances can be treated as an unknown input applied to the PEA through the same channel as the known input, referred to as matched uncertainty or unknown input, the effect of the disturbances on the PEA performance can be theoretically completely rejected by the use of sliding-mode-based –controllers. The theory of optimal control is concerned with operating a dynamic system at minimum cost. The LQR algorithm reduces the amount of work done by the control systems engineer to optimize the controller.

The objective of this paper are to develop a LQR control for PEA. The organization of the rest of this paper can be summarized as follows. The modeling of PEA are explained in Section 2. The LQR control for PEA-driven system is developed in Section 3. Simulation results is developed in Section 4.

2. PEA MATHEMATICAL MODELING

The modeling and control of PEAs has proven to be a challenging task. Fig.1 shows the schematic of a PEA with the end-effector connected to the base through flexure hinges and driven by a piezoelectric element under an input voltage of $u(t)$, $f_e(t)$ is the external load applied to the end-effector and y(t) the displacement of the end-effector or the system output. The PEA is represented as a physics-based model. The linear and nonlinear effects of the PEA are decoupled by means of individual sub-models that are connected in cascade.

The block diagram of PEA model is shown in Fig. 2. In this, the blocks of H, V and F_c represents the PEA hysteresis, vibration dynamics and creep respectively; $f(t)$ and $y_0(t)$ represents represent the internal actuation force and the output displacement of the end-effector without creep, respectively [2].

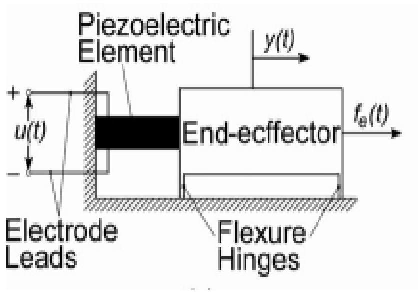

Fig-1: Schematic of a PEA

The block H represents the nonlinear hysteretic relationship between the input voltage, $u(t)$ and the internal actuating force, $f(t)$. These hysteresis is the dominant form of PEA nonlinearity [4] and can be represented by means of rate-independent hysteresis models, such as the classical Preisach hysteresis model [1],or the differential-equation-based model [5].

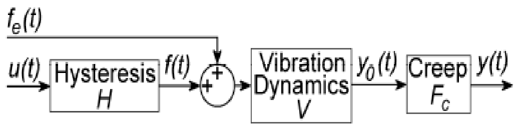

Fig-2: Physics-based model of PEA

For SM-based controllers, the effect of hysteresis is treated as a matched unknown input to the block *V*, hence the accurate representation of hysteresis is not needed. The block *V* represents the vibration dynamics relating the internal actuating force, *f(t)*and the external force, $f_e(t)$, to the end-effector displacement without considering the creep. The vibration dynamics of a PEA can be approximated by a linear combination of several second-order systems or one second-order system if the mass of the end-effector driven by the piezoelectric element is much larger than that of the piezoelectric element itself. The block $F_c$in Fig.2 represents the creep, which can be either linear [1]or nonlinear [7]. Here, a linear sub-model is assumed to be used for$F_c$and then the blocks of *Fc* and *V* are swapped without changing the output displacement, *y(t).* If the second-order system is used for the block *V* and the approximation error, along with the effects of $f_e(t)$, *H* , and *Fc* , are lumped together as the matched unknown input to the block *V* , one has

$$\ddot{y}(t) + 2\zeta\omega_n\dot{y}(t) + \omega_n^2 y(t) = \omega_n^2 K_1[f(t) + f_e(t) + \varepsilon_0(t)] \quad (1)$$

where ξ,$\omega_n$and $K_1$are the damping ratio, the natural frequency, and the steady state gain of the second-order system, respectively. The input to *V* is represented by K*u(t)*+K$\varepsilon(t)$, where *K*is a known nominal gain and $\varepsilon(t)$ is an unknown input added to $u(t)$. The output induced by $\varepsilon(t)$ accounts for effects such as hysteresis, creep, the external loads, and the error induced by approximating *V* with a second-order system.

If the input voltage V is applied to the piezoelectric material, the output displacement X will be generated. The relationship between V and X is given by,

$$x = AX + BV \quad (2)$$

$$Y = CX \quad (3)$$

where A,B and C are the system constants. To determine these constants system identification methods can be used.

$$f(t) = k_2 u(t) + \varepsilon_1(t) \quad (4)$$

The unknowns are lumped together to be considered as the matched unknown input to the second order system.

$$\varepsilon(t) = k_2^{-1}[f_e(t) + \varepsilon_0(t) + \varepsilon_1(t) \quad (5)$$

On the following assumption $x_1 = y$ $x_2 = \dot{y}$ and

$k = k_1 k_2$

$$x + 2\zeta\omega_n x_2 + \omega_n^2 x_1 = \omega_n^2 K_1[f(t) + f_e(t) + \varepsilon(t)]$$ (6)

The state space can be represented as,

$$x_1 = x_2$$

$$x_2 = -\omega_n^2 x_1 - 2\xi\omega_n x_2 + k\omega_n^2[u + \varepsilon]$$ (7)

The state space representation for the PEA is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\xi\omega_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ K\omega_n^2 \end{bmatrix} (u + \varepsilon)$$

$$\dot{X} = AX + Bu + B\varepsilon$$

$$y = [1 \quad 0][x_1 \quad x_2]^T = CX$$

This equation is referred to as the nominal model of the PEA if $\varepsilon(t)$=0. The states of the PEA model are $x_1$and x_2. These states represents the displacement and the velocity of the end-effector. The model parameters were identified and given by ξ = 0.82, ω_n= 5450 rad/s, and K = 0.142μm.

3. LQR CONTROL

The case where the system dynamics are described by a set of linear differential equations and the cost is described by a quadratic function is called the LQ problem. One of the main results in the theory is that the solution is provided by the linear-quadratic regulator (LQR). The cost function can be defined as a sum of the deviations of key measurements, desired altitude or process temperature, from their desired values. The LQR algorithm reduces the amount of work done by the control systems engineer to optimize the controller. However, the engineer have to specify the cost function parameters, and compare the results with the specified design goals. The LQR algorithm is also an automated way of finding an appropriate state-feedback controller. The value of R can be assumed.The command [K,P,E]=lqr(A,B,Q,R,N) solves the Algebraic Riccati Equation.

$$\dot{x} = Ax + Bu.$$ (10)

$$\dot{x} = (A − BK)x.$$ (11)

The closed loop state space representation of LQR with gain K will be,

$$A_{new} = A - B * k$$

$$B_{new} = B$$

$$C_{new} = C$$

$$D_{new} = D \qquad (12)$$

$$K = lqr(A, B, Q, R) \qquad (13)$$

$$Q = W * C' * C \qquad (14)$$

4. SIMULATIONS

Tracking control of PEAs is a challenging task. A LQR controller is simulated for PEA tracking.

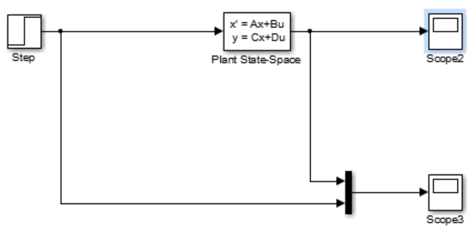

Fig-3: Simulation without controller

The step input given is 5*10^{-6}m. The result for this simulation is obtained in Fig.4.

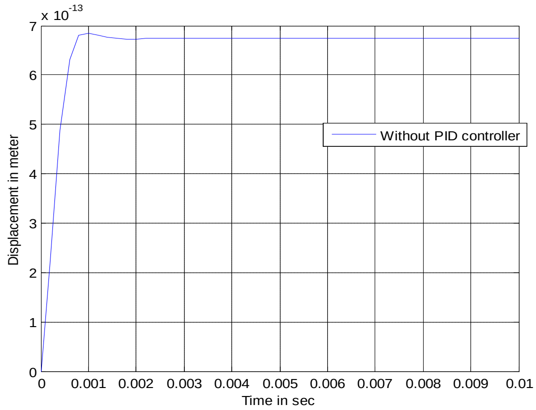

Fig-4: Result for tracking control using LQR

The MATLAB coding can be done. The values of K

areobtained.K = 1.0e-0.5* 0.6735 and K1 =1.0e- 0.5*0.0001. The simulation by using the K value is as shown.

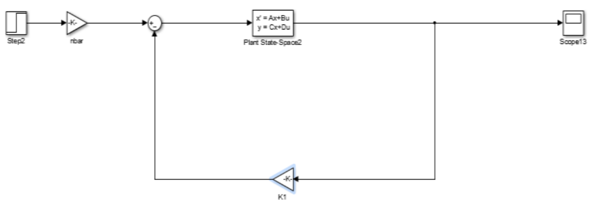

Fig-5: Simulation using LQR controller

The result for this simulation is obtained in Fig.6. The precompensation filter is used to track the desired path.

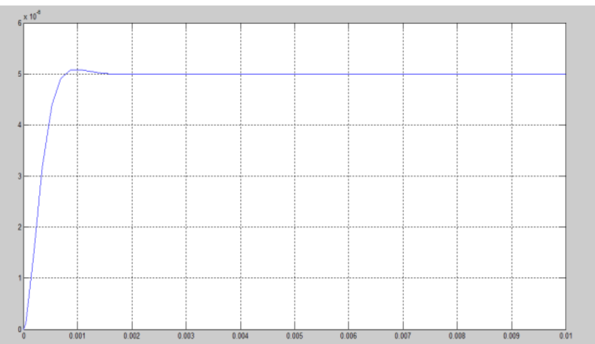

Fig-6: Result for LQR control

5. CONCLUSION

Tracking control of PEAs has been proven to be a challenging task, due to the involvement of the PEA nonlinear properties such as hysteresis, creep and dynamics. A number of control schemes based on the state feedback have been developed for improved performance. This paper presents the modeling and development of LQR control for PEA tracking. The existing control methods shows promising for use in the PEA tracking control due to their capability of rejecting matched nonlinearities.

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