

# Speed Controller Design of PMSM using Balanced Model Reduction Technique and CRA Method

K.Navaya<sup>1</sup>, Dr. N.Prema kumar<sup>2</sup>

<sup>1</sup>PG student, Dept of EEE, Andhra University (A), Vishakhapatnam, India

<sup>2</sup> Professor, Dept of EEE, Andhra University (A), Vishakhapatnam, India

\*\*\*

**Abstract** Model order reduction gives systematic approximation of complex models. This paper deals with speed control of permanent magnet synchronous motor (PMSM) using (CRA) characteristic ratio assignment method and the balanced reduction (BR) theory to obtain reduced-order models from dynamic system equivalents of electric system. BR allows obtaining reduced-order models via a process in which the original asymptotic stable system is internally balanced. The balanced system is truncated according to its dominant dynamics. The simulation and response[11] are obtained by using MATLAB software.

**Key Words:** Balanced reduction (BR), Permanent Magnet Synchronous Motor (PMSM), Characteristic ratio assignment method (CRA)

## 1. INTRODUCTION

Permanent magnet synchronous motors (PMSM) are typically used for high-performance and high-efficiency motor drives. High-performance motor control is characterized by smooth rotation over the entire speed range of the motor, full torque control at zero speed, and fast acceleration and deceleration. To achieve such control, vector control techniques are used for PM synchronous motors. Compared to a DC motor, the PMSM misses a commutator therefore it is more reliable than a DC motor. The PMSM also has advantages when compared to an AC induction motor. PMSM machines have the unique advantages of unsurpassed efficiency and power density characteristics, which are primarily responsible for their wide appeal [1] [2].

On the other hand, PMSM machines are synchronous, which certainly requires accompanying power electronics, but they also provide the basis for achieving high-quality actuator control. The speed control of PMSM motor is done by design of controller by CRA method, adjustment of speed of the motor and the damping ratio can be determined by only one parameter [6]. Therefore, this technique is user friendly for tuning controller under the requirement of the system where coming to Model order reduction concerns systematic approximation of complex models. The topic has received a great deal of attention in the control community at least since the early 1980's

The High-order systems (HOS) are always costly and tedious. Model order reduction (MOR) [7] is used for Simplifying complicated problems in HOS.

In this paper the selection of order reduction technique is based on balanced truncation method. The procedure of implication is based on using mathematical approaches of balanced truncation[3]. It is based on introducing a special joint measure of controllability and observability for every vector in the state space of an LTI system [5]. Then, the reduced model is obtained by removing those components of the state vector which have the lowest importance factor in terms of this measure

## 2. MATHEMATICAL MODEL OF A PMSM DRIVE

The most convenient way to determine the performance characteristics of machines is by means of their electrical equivalent circuits. The steady state model of the PMSM is used for studying steady state performance of the system only [1] this implies that all the electrical transients are neglected during load changes and frequency variations. Fig1 gives the steady state equivalent circuit of a synchronous machine. These equivalent circuits can become very elaborate when saturation, armature reaction, harmonic reactance, and other nonlinear effects are introduced

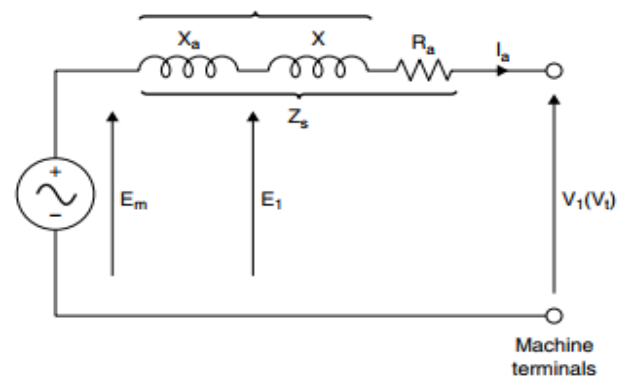


Fig 1: Steady-state equivalent circuit of PM Synchronous Machine

(Here  $X$  = leakage reactance,  $X_a$  = armature reaction reactance,  $X_s = X_a + X$  = synchronous reactance,  $R_a$  = armature resistance,  $Z_s$  = synchronous impedance,  $V_1 (V_t)$  = terminal voltages,  $E_m$  = magnetizing voltage)

The modeling of PMSM is done through transfer function approach which is given below

$$V_q = R_s i_q + p\lambda_q + W_e \lambda_d \tag{1}$$

$$V_d = R_s i_d + p\lambda_d + W_e \lambda_q \tag{2}$$

$$T_e = \left(\frac{3}{2}\right) P(\lambda_{af} i_q + (L_d - L_q) i_d i_q) \tag{3}$$

$$T_e = T_1 + BW_r + JpW_r \tag{4}$$

Where

$$W_e = PW_r \tag{5}$$

$$\lambda_d = L_d i_d + \lambda_{af} \tag{6}$$

$$\lambda_q = L_q i_q \tag{7}$$

Assuming all the equations from (1) to (3) are non-linear and forced to zero according to vector control of PMSM. Then equations (1) to (3) can be solved as given below

$$V_q = R_s i_q + L_q P i_q + W_e \lambda_{af} \tag{8}$$

$$V_q = -W_e L_q i_q \tag{9}$$

$$T_e = \left(\frac{3}{2}\right) P(\lambda_{af} i_q + K_t i_q) \tag{10}$$

The transfer function of PMSM is given below

$$\frac{W_r(s)}{V_q(s)} = \frac{PK_A S + K_t}{2S(R + L_q S)(JS + B) + K_t K_A P \lambda_{af}}$$

Where the overall transfer function of the practical PMSM motor is given as

$$G(s) = \frac{537.3s^4 + 1239s^3 + 928.9s^2 + 223.4s + 2.142}{137.2s^4 + 503.3s^3 + 1236s^2 + 1676s + 1035s + 225.4s + 2.152}$$

### 3. Balanced Truncation Method

As we know that the higher the order of the model, will be more complex [8-9]. Therefore, there is need to reduce higher order model to lower order model. This method was first proposed by Gustafson where reduced order models are obtained by neglecting higher order terms from numerator polynomials and denominator polynomials As we know that

the higher the order of the model, will be more complex. Therefore, there is need to reduce higher order model to lower order model. The proposed method of model reduction is balanced reduction method Consider the stable liner time invariant LTI high-dimensional System

Let

The original  $n^{th}$  order linear time invariant system is

$$\dot{x} = f(x, u) \tag{11a}$$

$$y = g(x, u) \tag{11b}$$

Where from equation (11) it is assumed to be asymptotically stable and characterized by the system matrices A, B, C and D where in general D is assumed to be zero

$$\dot{x} = Ax + Bu \tag{12a}$$

$$y = Cx + Du \tag{12b}$$

Two major characteristic of dynamic system are its controllability ( $W_c$ ) i.e., it represents a quantitative measure of the coupling of inputs to the state behavior, and its observability ( $W_o$ ) represents strength of the coupling between outputs and the state behavior[10]

$$W_c = \int_0^\infty e^{At} B B^T e^{ATt} dt \tag{13a}$$

$$W_o = \int_0^\infty e^{ATt} C^T C e^{At} dt \tag{13b}$$

Equation (13) satisfies the Lyapunov equation

$$AW_c + W_c A^T = -BB^T \tag{14a}$$

$$A^T W_o + W_o A = -C^T C \tag{14b}$$

Using a new co-ordinates system via transformation matrix to make the internal responses by  $e^{At} B$  and  $e^{ATt} C^T$  equal in magnitude;

$$W_{cb} + W_{ob} = \Sigma \tag{15}$$

Where  $W_{cb}$  and  $W_{ob}$  corresponds to controllability balanced co-ordinates and observability co-ordinates systems respectively where  $\Sigma$  is a diagonal matrix consists of Hankel singular value  $\sigma_i, i=1, \dots, n$ , also known as second order modes. Where the Hankel singular value are given by the square roots of the eigenvalues of the product of the two gramians  $W_c$  and  $W_o$  i.e,

$$\sigma_i = \sqrt{\lambda_i(W_c W_o)} = \sqrt{\lambda_i(W_o W_c)} \tag{16}$$

Similarly Transformation  $T_b$  can be found from the eigenvalue problem

$$W_c W_o = T_b \Lambda_b T_b^{-1} \tag{17}$$

Such that the balanced domine, i.e.,  $W_{cb}$  and  $W_{ob}$  become equal and diagonal as in eq (15) the new co-ordinate system given by eq (17) allows obtaining the internally balanced system  $(A_b, B_b, C_b)$  as

$$\begin{aligned} \dot{x}_b &= T_b^{-1} A T_b x_b + T_b^{-1} B u \\ A_b x_b + B_b u \end{aligned} \tag{18a}$$

$$\begin{aligned} y &= C T_b x_b \\ C_b x_b \end{aligned} \tag{18b}$$

Balanced system in eq (18) gives the reordering and truncation modes (deleting states associated with the smallest singular values) can be applied to obtain the reduced  $r^{th}$  order model  $(A_r, B_r, C_r)$  and with  $r < n$  and it is shown that eq (18) results in to two asymptotically stable and internally balanced subsystem and that system is chosen as the reduced order system

$$\begin{aligned} \dot{x}_r &= A_r x_r + B_r u \\ y &= C_r x_r + D_r u \end{aligned}$$

And the reduced order of the system is given as

$$Gr(s) = \frac{0.03375s^2 - 0.1609s + 4.413}{s^2 + 1.523s + 4.433}$$

**4. CHARACTERISTIC RATIO ASSIGNMENT METHOD:**

The desired control response would be attainable by an all-pole commensurate fractional order transfer function. This closed loop transfer function is considered as follows [6]

$$F(S) = \frac{\delta_0}{\delta_n s^{n\nu} + \delta_{(n-1)} s^{(n-1)\nu} + \dots + \delta_1 s^\nu + \delta_0} \tag{19}$$

The characteristic ratio  $\alpha_i$  for the system has the following relation

$$\alpha_i = \frac{\delta_i^2}{\delta_{i-1} \delta_{i+1}} \tag{20}$$

Parameters of  $\delta_i$  the system (19) are expressed in function of the characteristic ratio and the generalized time constant,

$$\delta_1 = \tau^\nu \delta_0$$

$$\delta_i = \frac{\delta_0 \tau^{i\nu}}{\alpha_{i-1} \alpha_{i-2} \alpha_{i-3} \dots \alpha_1^{i-1}} \text{ For } i = 2 \dots n \tag{21}$$

It is also possible to make the previous relation into a recursive form

$$\delta_i = \frac{\delta_{i-1} \tau^\nu}{\alpha_{i-1} \alpha_i}$$

In this section, the design of a fractional PI controller with time specification is developed. The structure of the PI can be written as

$$C(S) = k_p + \frac{k_i}{s^\lambda} \tag{22}$$

Let consider the following open loop transfer function

$$G(S) = \frac{b_0}{a_1 s^{2\nu} + a_0} \tag{23}$$

The obtained closed loop transfer function by applying the fractional PI controller is given by

$$F(S) = \frac{b_0 k_p s^\nu + b_0 k_i}{a_1 s^{2\nu} + (a_0 + b_0 k_p) s^\nu + b_0 k_i} \tag{24}$$

The characteristic equation is expressed by

$$E_c = a_1 S^{2\nu} + (a_0 + b_0 k_p) s^\nu + b_0 k_i \tag{25}$$

This equation is identified to the characteristic polynomial  $\delta(S)$

$$\delta(S) = \delta_2 S^{2\nu} + \delta_1 S^\nu + \delta_0 \tag{26}$$

Hence, the controller parameters can be determined using relations (25) and (26):

$$\delta_0 = \frac{\delta_2 \alpha_1}{\tau_1^{2\nu}} \tag{27}$$

$$\delta_0 = \frac{a_2 \alpha_1}{\tau_1^{2\nu}} \tag{28}$$

Controller design is given as

$$k_p = \frac{\delta_1 - a_0}{b_0} \tag{29}$$

$$k_i = \frac{\delta_0}{b_0} \tag{30}$$

Where the output results with controller are shown in the fig:4

### 5. SIMULATION RESULTS

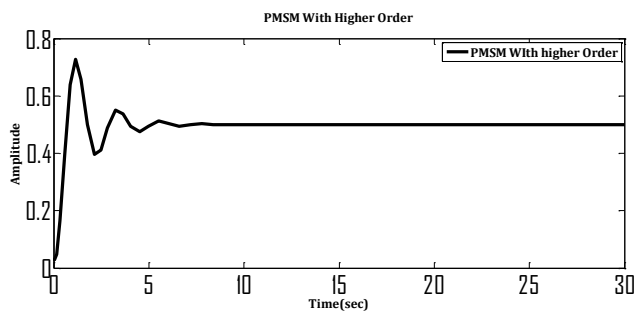


Fig: 2 Step response of PMSM with Higher Order TF

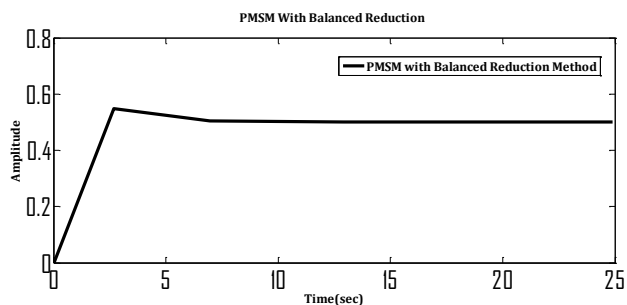


Fig: 3 Step response of PMSM with Balanced Reduction Method

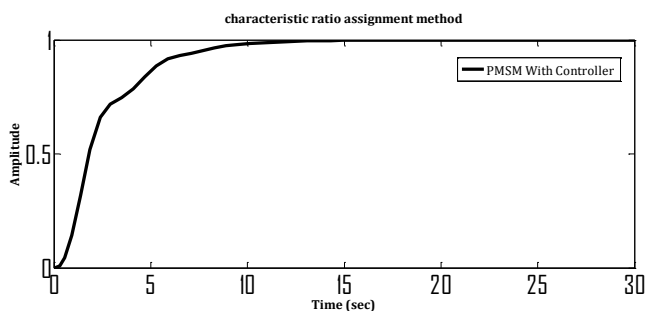


Fig: 4 Response of PMSM with Controller Using CRA tuning Method

From the above responses of PMSM fed with inverter in Fig.2, and reduced order system using Balanced reduction method in Fig.3 and PMSM with controller using CRA method is in Fig.4 give the time response where all the stability parameters can be easily found and the peak time and rise time of the PMSM with inverter and reduced order response are calculated and they are given with 0.708, 0.663 and 1.661, 1.652 respectively, PMSM with controller is having zero maximum peak over shoot where the stability parameters and comparison is given in the Table.1 specified and the combined response of the system is given in Fig.8 to ensure the optimum result by comparison

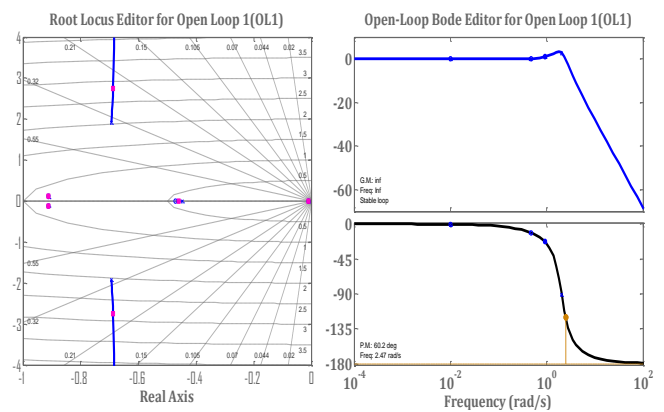


Fig: 5 Root Locus and frequency response of Higher Order system

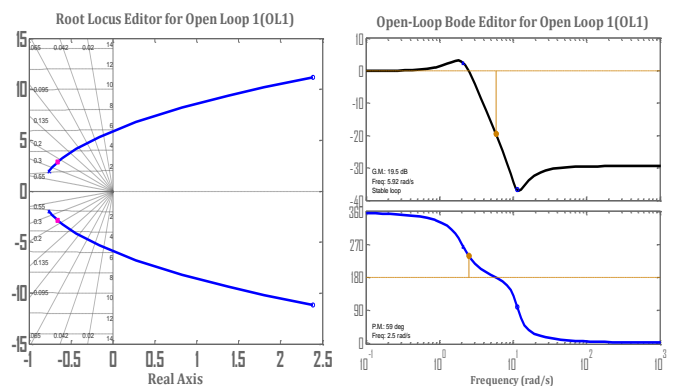


Fig: 6 Root Locus and Frequency Response of the plant with balanced reduction Method

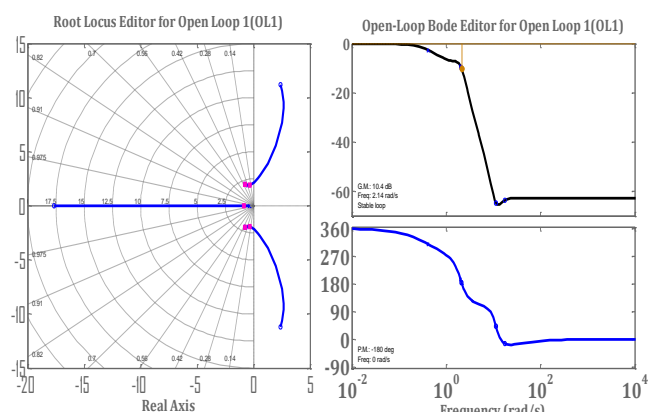


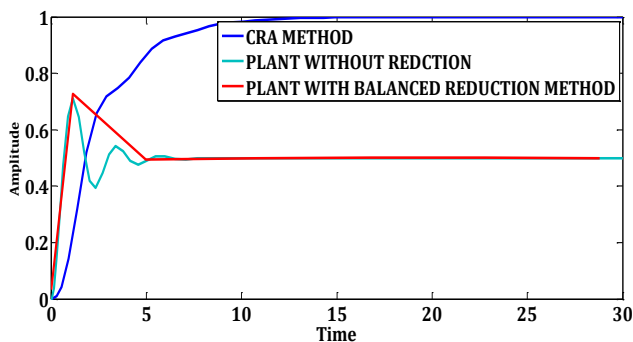
Fig: 7 Root Locus and Frequency Response of the plant with controller using CRA method

Where the Root locus plots give us the poles and zeros that are located and from Fig.5 and Fig.6 comparing and considering the dominant poles that are obtained after reducing are considered and the reduced second order considered for designing the controller where in Fig.7 the gain is increased and steady state error is decreased the frequency response also gives phase margin, gain margin all stability parameters are considered in Table 1 and all the

comparisons are done in the same plot is shown in Fig.8 and the results are obtained through the MATLAB software [11]

**Table -1: Response specifications (time and frequency domains) of PMSM with different configurations**

Stability Parameters	PMSM TF	PMSM WITH ORDER REDUCTION	PMSM WITH CRA CONTROLLER
RISE TIME (tr)	0.708	0.663	4.741
PEAK TIME (tp)	1.661	1.652	18.82
SETTLING TIME (ts)	5.238	5.203	9.480
MAXIMUM PEAK OVER SHOOT (Mp%)	27.2	28.6	0
PEAK	1.266	1.286	0.999
STEADY STATE ERROR (ess)	0.994	1	0
PHASE MARGIN (deg)	60.2	58.8	-180
GAIN MARGIN(dB)	inf	19.5	10.3
PHASE CROSS OVER FREQUENCY	60.17	58.76	-180
GAIN CROSS OVER FREQUENCY	INF	9.46	3.29



**Fig: 8** Combined overall response

## 6. CONCLUSIONS

This paper deals with speed control of PMSM where because of its wide range of applications. As it is of higher order the order reduction techniques are used we use balanced truncation (BR) method to achieve optimum order reduction of the plant where it is convenient to design a controller, the desired controller we use is CRA to get better performance of speed control of PMSM

## REFERENCES

- [1] Grzegorz SIEKLUCKI, Akademia Górniczo-Hutniczaw Krakowie "Analysis of the Transfer-Function Models of Electric Drives with Controlled Voltage Source"
- [2] Transfer function determination for a PM synchronous AC motor
- [3] M. Dalir and M. Bashour "Applications of Fractional Calculus". Applied Mathematical Sciences, Vol. 4, pp. 1021 - 1032.2010
- [4] Emna Ouhibi, Maher Ben Hariz, and Faouzi Bouani "Design of fractional order PI controller using the CRA method"
- [5] Henrik Sandber" Parameterized Model Order Reduction Using Extended Balanced Truncation"
- [6] Jutarut Chaorai-ngern, Arjin Numsomran, Taweepol Suesut, Thanit Trisuwannawat and Vittaya Tipsuwanporn." PID Controller Design using Characteristic Ratio Assignment Method for Coupled-Tank Process"
- [7] Zur Erlangung des Grades eines Doktors der Naturwissenschaften (Dr. rer. nat.) am Fachbereich Mathematik & Informatik der Freien Universita't Berlin "Comparison of Balance Truncation and Singular Perturbation Approximation with Application to Optimal Control"
- [8] Dr.S.Janardhanan "Model Order Reduction and Controller Design Techniques"
- [9] Abner Ramirez, Senior Member, IEEE, Ali Mehrizi-Sani, Senior Member, IEEE "Application of Balanced Realizations for Model-Order Reduction of Dynamic Power System Equivalents"
- [10] Sudipta Ghosh, and Nilanjan Senroy, Member, IEEE "A Comparative Study of Two Model Order Reduction Approaches for Application in Power Systems"
- [11] David Houcque Northwestern university "Introduction to MATLAB for engineering students"

## Biographies



K. Navya, M. Tech. (Control Systems Engineering), Dept of Electrical Engineeirn, Andhra University (A), Vishakhapatnam-530003. India



Dr. N. Prema Kumar, Professor of Electrical Engineering,, ANDHRA UNIVERSITY, Visakhapatnam- 530003.