Integrated methodology for the seismic design of reinforced embankments with berms

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Abstract - *The purpose of this study is to present an integrated methodology for the design of reinforced soil slopes with composite geometry (berms) that are subjected to seismic loading. This solution is based on the kinematic theorem of limit analysis and on the quasi-static approach and concerns cohesionless soils that are expected to deform plastically following the Coulomb yield criterion. For the implementation of this methodology, a software application has been designed and presented. Via this application the horizontal peak ground acceleration is defined based on the seismotectonic characteristics of each area and seismic design of reinforced slopes with berms is performed based on the plane failure mechanism. Additionally, the slope yield acceleration is calculated and the expected permanent displacement as a criterion of slope vulnerability in the case of seismic risk increase. The results of the solution are compared with correspondent ones that derive from conventional methods and elasto-plastic finite element stress analysis. Finally, continuous reinforced slopes and slopes with berms with the same height and mechanical characteristics are designed and compared and the advantages of the slopes with berms are highlighted.*

Key Words: Geosynthetics, Reinforced slopes, Limit Analysis, Seismic Loading, Composite Geometry, Failure Mechanisms.

1. INTRODUCTION

Soil is used widely as a foundation and structural material mainly in road constructions, railway projects, dams, embankments, tunnels etc. It is a relatively inexpensive and abundant construction material, capable of providing high strength in compression but virtually no strength in tension. Like other construction materials with limited strength, soil can be reinforced with materials such as strips, grids, sheets, rods and fibres in order to form a composite material that has improved mechanical characteristics [1]. Due to their composite behaviour and energy absorption capacity, soil reinforcement systems exhibit high earthquake resistance [2] and are used for high embankments, erosion control, protection against landslides, rockfalls and other applications. Key to the operation of a reinforced earth system is the compatibility of each material, relative to the mechanical and geometrical characteristics, so that the loading can be transferred between the components [3]. The influence of reinforcement materials on slope stability can be seen in Figure 1 where the maximum angle of repose of a scaled sand slope without reinforcement tested in a geotechnical centrifuge at 100 g is 32^0 while with an additional reinforcement an angle of 63^0 was be reached [4].

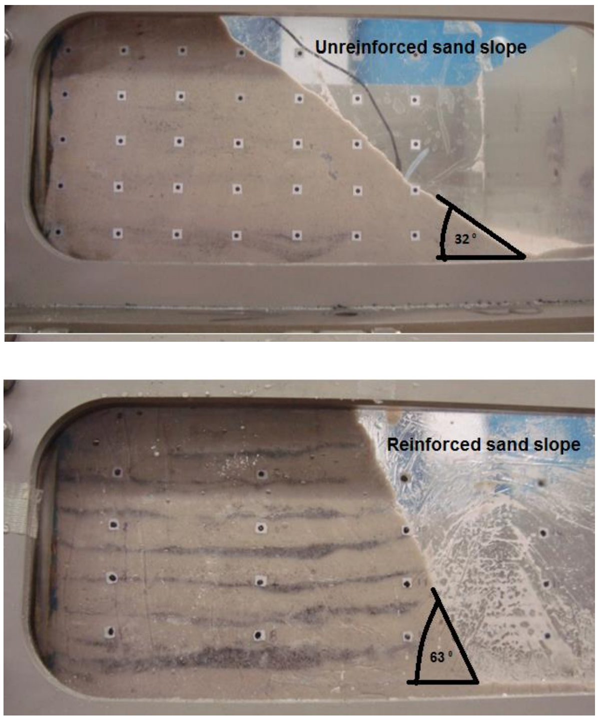

Figure 1. Deposition of sand slopes without and with reinforcement layers.

2. INVESTIGATION METHODS OF REINFORCED SLOPES

Most geotechnical constructions are investigated by applying experimental, numerical and analytical methods with main goal to define their behaviour and failure mechanisms due to static and seismic loading [5].

Experimental investigation requires detailed physical modelling of the constructions and large or small scale

models can be built and tested. The main forces that are applied and define the behaviour of soil structures are those of self weight and therefore the simple gravity filed is inadequate when small scale models are examined [6].
Numerical modelling in geotechnics requires detailed simulation of the mechanical and geometrical characteristics of the models. Finite element stress analysis can be applied and for the case of reinforced slopes special attention should be given on the simulation of the interface between soil and reinforcement [7].

Analytical investigation of reinforced slopes is also applied for the design of reinforced slopes that are subjected to static and seismic loading and/or in order to examine their vulnerability. The methods used are based on the limit equilibrium method [8] and on the limit analysis method [9]. The limit equilibrium method is traditionally applied to obtain approximate solutions in soil stability problems and entails assumed failure surfaces of various simple shapes such as plane, circular and log-spiral. With this assumption, each of the stability problems can be reduced to one of finding the most critical position for the failure surface of the shape chosen. In this method, an overall equation of equilibrium, in terms of stress resultants can be written for a given problem. This makes it possible to solve various problems by simple statics.

In contrast to limit equilibrium method, the limit analysis method considers the stress-strain relationship of the soil in an idealized manner. This idealization (expressed by the flow rule) establishes the limit theorems on which limit analysis is based. Within this framework, the approach is rigorous and the techniques are in some cases much simpler than those of limit equilibrium. The plastic limit theorems of Drucker et al can be employed to obtain upper and lower bounds of the collapse load for stability problems. If the same collapse mechanisms are applied, the results of the limit equilibrium method and the limit analysis method are identical.

Bishop and Janbu analysis methods are also applied to obtain solutions in soil stability problems. The stability of slip surfaces is analyzed using slice limit equilibrium methods and the safety factors of circular or non-circular failure surfaces in soil slopes are evaluated.

Comparing the finite element analysis method and an analytical solution, the first provides a comprehensive approach where stress-strain analysis can be performed, soil-reinforcement interface can be taken into account and the developed stresses along the surface of the reinforcement can be defined. This can only be achieved by performing finite element stress analysis and not by applying analytical solutions however, an analytical solution can provide an accurate, closed form solution with small computational cost and therefore can be used as a fast tool for the design of soil structures.

3. DESIGN OF HIGH REINFORCED SLOPES WITH BERMS

Reinforced slope stability is related to the geometrical and mechanical characteristics of the construction. Height and slope inclination along with static and seismic loading conditions determine the amount of the reinforcement required to prevent failure. Slopes with small height and gentle inclination demand relatively low reinforcement. On the other hand, high and steep slopes are vulnerable to earthquake loading and therefore require long reinforcement with large tensile strength. Moreover, the erosion due to the water flow is another possible problem that affects high and steep slopes. The surface water covers larger distance along high and steep slopes with high velocity, leading to erosion. For these reasons, there is need for another approach for the design and construction of high slopes in order to limit these problems.

In the current study, the analytical expressions for the design of high reinforced slopes with berms are presented, based on the kinematic theorem of limit analysis and on the quasi-static approach. The plane failure mode is examined and a software application in Delphi environment based on these expressions has been created and presented. Moreover, representative examples are noted in order to demonstrate the software features as well as the influence of the seismic loading on the amount of the reinforcement required to prevent failure.

The results obtained, are imported into a representative 2D limit equilibrium slope stability programme and Bishop Analysis is performed. The safety factors calculated by this analysis show that the amount of the reinforcement calculated by the analytical solution is also adequate when Bishop Analysis is performed.

Additionally, the results are imported into a 2D elasto-plastic finite element stress analysis program and the strength reduction factors and failure mechanisms of the models are defined. The strength reduction factors calculated by this analysis show that the amount of the reinforcement calculated by the analytical solution is also adequate when finite element stress analysis is performed. Finally continuous reinforced slopes and slopes with berms are designed and compared and the advantages of the latter are shown.

In the following, the theorems and methods applied are presented as well as the proposed method for the design of high reinforced slopes with berms. In particular, the kinematic theorem of limit analysis and the quasi-static approach are briefly described; the earthquake-induced permanent ground displacement prediction proposed by Ambrasseys is noted [10] as well as the Ausilio et al. [11] equations for the seismic design of reinforced slopes applied for the purposes of the current study.

3.1 Kinematic theorem of limit analysis

The kinematic theorem of limit analysis is based on the upper – bound theory of plasticity and states that the slope will collapse if the rate of work done by external loads and body forces exceeds the energy dissipation rate in any kinematically admissible failure mechanism [12].

$$\int_V \sigma_{ij}^* \varepsilon_{ij}^* dV \geq \int_S T_i \, v_i dS + \int_V X_i v_i^* dV, \quad \text{i, j=1,2,3...} \quad (1)$$

Where ε^*_{ij} is the strain rate in a kinematically admissible velocity field, σ^*_{ij} is the stress tensor associated with ε^*_{ij}, velocity $v^*_i=v_i$ on boundary S (given kinematic boundary condition), X_i is the vector of body forces (unit weight and the distributed quasi-static inertial force), and S and V are the loaded boundary and the volume, respectively.
In this study, pore water pressure and potential liquefaction are not considered. The rate of external work is due to soil weight and inertia force induced by the seismic loading and the only contribution to the energy dissipation is assumed to be provided by the reinforcement. In addition, it is assumed that the energy dissipation is performed only by the tensile strength of the reinforcement, while resistance to shear, bending and compression are ignored.

3.2 Quasi- static approach and seismic coefficient.

According to the quasi-static approach, a static force with horizontal direction represents the seismic influence on the failure soil mass. This force is estimated by the product of seismic intensity coefficient and weight of the potential sliding soil mass. This approach is a widely accepted method, despite the fact that it neglects the acceleration history.
The evaluation of the seismic coefficient can be accomplished with various empirical predictive relations based on the seismotectonic environment of each region.
Ambraseys (1995) predictive relations for peak horizontal ground accelerations generated by earthquakes in the European area and are used for the purposes of the current study.

$$\log(a_h) = -1.09 + 0.238M_s - 0.0005r - \log(r) + 0.28P \quad (2)$$

if no account is taken of the focal depth, where, $r=(d^2+6^2)^{0.5}$, d is the source distance in km, M is the surface wave magnitude, P is 0 for 50 percentile values and 1 for 84 percentile.

If the effect of the focal depth h (km) is allowed for, the equation becomes:

$$\log(a_h) = -0.87 + 0.217M_s - 0.00117r - \log(r) + 0.26P \quad (3)$$

where $r=(d^2+6^2)^{0.5}$

3.3 Permanent displacement

Applications via the pseudo-static approach indicate that the amount of the required reinforcement increases for high seismic coefficient affecting significantly the construction costs. A slope can undergone limited permanent displacement prior failure and several relationships have been proposed in order to predict the permanent displacement due to seismic loading. In the current study, equations formed by Ambraseys and Menu [13] are applied for the estimation of the expected permanent displacement for a given seismic coefficient and in particular:

$$\log U = 0.9 + \log\left[\left(1-\frac{k_{cr}}{k_h}\right)^{2.53}\left(\frac{k_{cr}}{k_h}\right)^{-1.09}\right] + 0.3t \quad (4)$$

Where U the horizontal permanent displacement in cm, k_{cr} the critical yield acceleration for a slope, k_h the seismic coefficient and t the confidence level resulting from a normal distribution.

3.4 Seismic design of reinforced slopes.

According to Ausilio et al the rate of external work done by soil weight and inertial force is:

$$\dot{W} = (G_1 + G_2 + ... + G_n)V\sin(\Omega-\phi) + k_h(G_1 + G_2 + ... + G_n)V\cos(\Omega-\phi) \quad (5)$$

Where G_1, G_2...G_n, indicate the weight of the soil wedge for every one step with different expressions for local and global failure mode and k_h the horizontal seismic coefficient.
The energy dissipation is

$$\dot{D} = V\cos(\Omega-\phi)\sum_{i=1}^{n} T_i \quad (5)$$

Where T_i is the force of the i^{th} layer per unit width
Moreover, Ling et al [14] suggested that the tensile strength T_i can be calculated approximately by the following equation:

$$T_i = K\gamma z_i d_i \quad (6)$$

where K represents the total reinforcement in a normalized form with the following expression:

$$K = \frac{\sum_{i=1}^{n} T_i}{(1/2)\gamma H^2} \quad (7)$$

And d_i is the tributary area of layer i.
Equation 6 owing to Equation 7 becomes:

$$\dot{D} = \frac{1}{2}V\cos(\Omega-\varphi)K\gamma H^2 \quad (8)$$

In the next section the new method proposed for the seismic design of reinforced slopes with berms and vulnerability evaluation are presented and discussed, based on the above equations.

3.5 Seismic design of reinforced slopes with berms

According to the method presented in the current study, a high and steep reinforced slope can be divided into more slopes (berms) with smaller height and scaled inclination.

Different height, inclination, soil mechanical characteristics and bench width can be chosen and the tensile strength and length of the required reinforcement can be calculated for each berm separately.

For example, a slope with 30 m height and average inclination 3:2 can be divided into 5 slopes with 6 m height each and scaled inclination as shown in Figure 2.

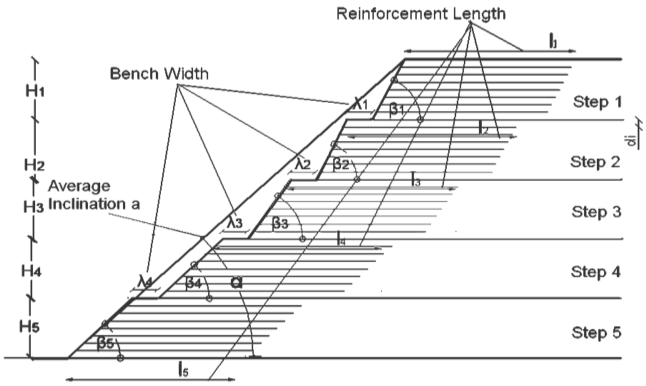

Figure 2. Reinforced slope with five berms.

In order to determine the amount of the required reinforcement of each berm, various potential failure mechanisms, based on the plane failure mode, are examined and the most critical ones are used for the final design.
In the plane failure mechanism, it is assumed that the reinforced soil mass translates as a rigid body with velocity V (Figure 3) and height H of the slope and angle Ω that the failure plane forms with the horizontal, specify the mechanism. Main goal is to determine the critical value of Ω (for a slope with given height) and therefore the critical failure mechanism. Once the critical failure mechanism is defined, the amount of the required reinforcement can be calculated.

For the current method, the plane failure mode is applied twice, once in order to ensure that the tensile strength and length of the reinforcement are adequate against local stability (Figures 3 and 4) and then in order to ensure global stability (Figure 5). For the local stability failure mode each step is examined separately and it is assumed that the soil weight of the upper steps is taken into account as an overburden, as shown in Figures 3 and 4. For example, for the local stability failure mode for the 3rd step, G_1,G_2,G_3 are taken into account, while for the 2nd step, G_1 and G_2 are taken into account.

For the global stability failure mode, the soil weight of every one step is taken into account as shown in Figure 5.

It is necessary that both failure modes are applied, since the failure mode that concerns local stability provides critical results for the upper steps while the failure mode for global stability for the lower steps.

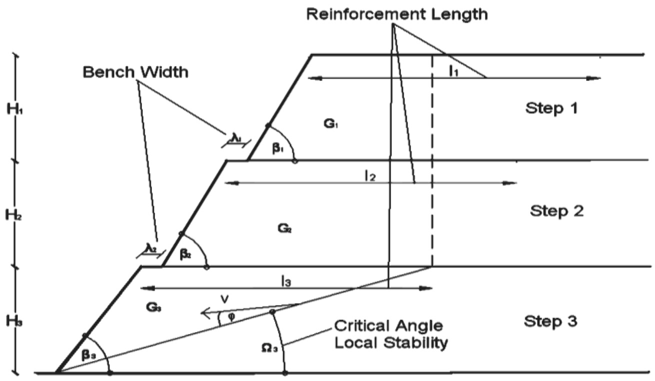

Figure 3. Local stability failure mode, 3rd step.

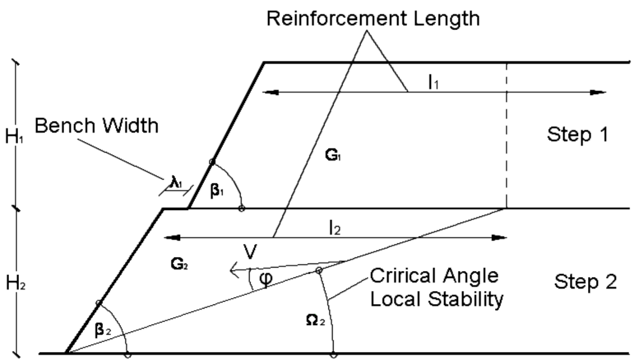

Figure 4. Local stability failure mode, 2nd step.

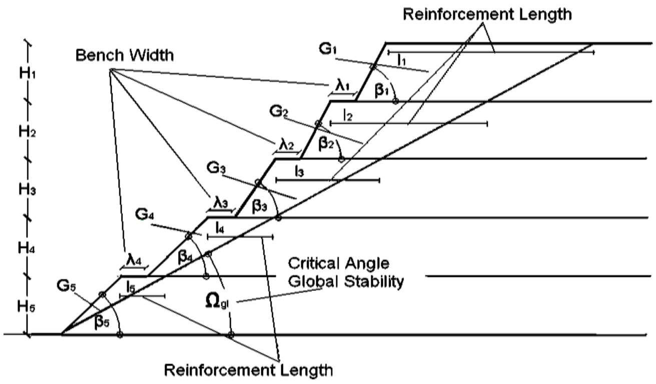

Figure 5. Global stability failure mode

The rate of external work done by soil weight and inertial force is:

$$\dot{W} = (G_1+G_2+....+G_n)V\sin(\Omega-\phi)+k_h(G_1+G_2+....+G_n)V\cos(\Omega-\phi) \quad (4)$$

Where G_1, G_2...G_n indicate the weight of the soil wedge for every one step with different expressions for local and global failure mode and k_h the horizontal seismic coefficient.
The energy dissipation equals to

$$\dot{D} = V\cos(\Omega-\phi)\sum_{i=1}^{n} T_i \quad (5)$$

Where T_i is the force of the i^{th} layer per unit width

Moreover, Ling et al (1997) suggested that the tensile strength T_i can be calculated approximately by the following equation:

$$T_i = K\gamma z_i d_i \tag{6}$$

where K represents the total reinforcement in a normalized form with the following expression:

$$K = \frac{\sum_{i=1}^{n} T_i}{(1/2)\gamma H^2} \tag{7}$$

And d_i is the tributary area of layer i.
Equation 6 owing to Equation 7 becomes:

$$\dot{D} = \frac{1}{2} V \cos(\Omega - \varphi) K \gamma H^2 \tag{8}$$

Equating the rate of external work done by soil weight and inertial force to the energy dissipation, the total reinforcement in a normalized form K can be calculated:
a) For Local Stability for every berm i:

$$K_i = \frac{2G_i \tan(\Omega_i - \varphi_i) + 2k_{hi} G_i}{\gamma_i H_i^2} \tag{9}$$

Which attains a maximum value when $dK_i/d(\Omega)=0$
where G_i has the following expression for the reinforced :

$$G_i = G_{i1} + G_{i2} + ..G_{ij} + ... + G_{ii} \tag{10}$$

where j the number of the upper steps that influence the specific local mechanism. In addition j<i should be taken into account.
G_{ii} is the soil weight of the specific step examined with the following expression:

$$G_{ii} = \frac{0.5\gamma_i H_i^2 \sin(\beta_i - \Omega_i)}{\sin\Omega_i \sin\beta_i} \tag{11}$$

And G_{ij} the soil weight of the upper steps with the following expression:

$$G_{ij} = (2(l_i - \lambda_{i-1} - \lambda_{i-2} - ... - \lambda_j - \frac{H_{i-1}}{\tan\beta_{i-1}} - \frac{H_{i-2}}{\tan\beta_{i-2}} - ... - \frac{H_{j+1}}{\tan\beta_{j+1}}) - \frac{H_j}{\tan\beta_j}) \frac{H_j}{\tan\beta_j}\gamma_j \tag{12}$$

if i-1>j+1, while for i-1<j+1,

$$G_{ij} = (2(l_i - \lambda_j - \frac{H_j}{\tan\beta_j}) - \frac{H_j}{\tan\beta_j}) \frac{H_j}{\tan\beta_j}\gamma_j \tag{13}$$

For example for a 4-step slope the total soil weight that is taken into account for local stability failure mode for the 4th step (lower step), is given by the following expression:

$$G_4 = G_{41} + G_{42} + G_{43} + G_{44} \tag{14}$$

where:

$$G_{41} = (2(l_4 - \lambda_3 - \lambda_2 - \lambda_1 - \frac{H_3}{\tan\beta_3} - \frac{H_2}{\tan\beta_2}) - \frac{H_1}{\tan\beta_1}) \frac{H_1}{\tan\beta_1}\gamma_1 \tag{15}$$

$$G_{42} = (2(l_4 - \lambda_3 - \lambda_2 - \frac{H_3}{\tan\beta_3}) - \frac{H_2}{\tan\beta_2}) \frac{H_2}{\tan\beta_2}\gamma_2 \tag{16}$$

$$G_{43} = (2(l_4 - \lambda_3) - \frac{H_3}{\tan\beta_3}) \frac{H_3}{\tan\beta_3}\gamma_3 \tag{17}$$

$$G_{44} = \frac{0.5\gamma_4 H_4^2 \sin(\beta_4 - \Omega_4)}{\sin\Omega_4 \sin\beta_4} \tag{18}$$

While for the same 4-step slope the soil weight that is taken into account for local stability failure mode for the 3rd step is given by the following expression:

$$G_3 = G_{31} + G_{32} + G_{33} \tag{19}$$

Where:

$$G_{31} = (2(l_3 - \lambda_2 - \lambda_1 - \frac{H_2}{\tan\beta_2}) - \frac{H_1}{\tan\beta_1}) \frac{H_1}{\tan\beta_1}\gamma_1 \tag{20}$$

$$G_{32} = (2(l_3 - \lambda_2) - \frac{H_2}{\tan\beta_2}) \frac{H_2}{\tan\beta_2}\gamma_2 \tag{21}$$

$$G_{33} = \frac{0.5\gamma_2 H_3^2 \sin(\beta_3 - \Omega_3)}{\sin\Omega_3 \sin\beta_3} \tag{22}$$

By substituting the maximum values of K_i to Equation 6, the maximum required tensile strength Ti can be calculated for every step i.
Moreover, the distance between the failure surface and the edge of the slope is:

$$l_i = \frac{H_i \sin(\beta_i - \Omega_i)}{\sin(\beta_i)\sin(\Omega_i)} \tag{23}$$

b) Global Stability for the whole slope:

$$K_{gl} = \frac{2(G_1+G_2+...+G_n)\tan(\Omega_{gl} - \varphi_{gl}) + 2k_h(G_1+G_2+...+G_n)}{\gamma(H_1+H_2+...+H_n)^2} \tag{24}$$

Which attains a maximum value when $dK_{gl}/d(\Omega)=0$
where φ_{gl} the average friction angle of the slope, K_{gl} the average expected horizontal acceleration, n the number of the steps of the slope and $G_1, G_2, \ldots G_n$ have the following expressions:

$$G_1 = (\frac{H_1 + 2H_2 + ... + 2H_n}{\tan\Omega_{gl}} - \frac{H_1}{\tan\beta_1} - \frac{2H_2}{\tan\beta_2} - ... \frac{2H_n}{\tan\beta_n} - 2\lambda_1 - 2\lambda_2 - ... - 2\lambda_{n-1})\gamma_1 \frac{H_1}{2} \tag{25}$$

$$G_2 = (\frac{H_2 + 2H_3 + ... + 2H_n}{\tan\Omega_{gl}} - \frac{H_2}{\tan\beta_2} - \frac{2H_3}{\tan\beta_3} - ... \frac{2H_n}{\tan\beta_n} - 2\lambda_2 - 2\lambda_3 - ... - 2\lambda_{n-1})\gamma_2 \frac{H_2}{2} \tag{26}$$

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$$G_i = (\frac{H_i + 2H_{i+1} + 2H_{i+2} + ... + 2H_n}{\tan\Omega_{gl}} - \frac{H_i}{\tan\beta_i} - \frac{2H_{i+1}}{\tan\beta_{i+1}} - \frac{2H_{i+2}}{\tan\beta_{i+2}} - ... \frac{2H_n}{\tan\beta_n} - 2\lambda_i - 2\lambda_{i+1} - ... - 2\lambda_{n-1})\gamma_i \frac{H_i}{2} \tag{27}$$

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$$G_n = (\frac{H_n}{\tan\Omega_{gl}} - \frac{H_n}{\tan\beta_n})\gamma_n \frac{H_n}{2} \tag{28}$$

The required tensile strength of the reinforcement for each layer derives by substituting again the maximum value of K_{gl} to Equation 6.

Moreover, the distance between the failure surface and the edge of the slope is:

$$l_i = \frac{H_i + H_{i+1} + \ldots + H_n}{\tan(\Omega_i)} - \frac{H_i}{\tan(\beta_i)} - \frac{H_{i+1}}{\tan(\beta_{i+1})} - \ldots - \frac{H_n}{\tan(\beta_n)} - \lambda_i - \lambda_{i+1} \ldots - \lambda_{n-1} \quad (29)$$

The results that derive from local and global failure mode are compared and the maximum values for every one step are used for the final design. As mentioned before, local stability calculations give longer reinforcements with larger tensile strength for the upper steps and global stability calculations for the lower steps. By applying both analyses, the multi step reinforced soil slope can be designed in a more comprehensive and completed way.

Similarly, the critical acceleration factor can be obtained for:

a) Local Stability for every one step separately:

$$k_{yi} = \frac{K_i \gamma_i H_i^2 - 2(G_1 + G_2 + \ldots + G_i)\tan(\Omega_i - \varphi_i)}{2(G_1 + G_2 + \ldots + G_i)} \quad (30)$$

b) Global Stability for the whole slope:

$$k_{ygl} = \frac{K_{gl} \gamma_i (H_i + H_{i+1} + \ldots + H_n)^2 - 2(G_1 + G_2 + \ldots + G_i)\tan(\Omega_{gl} - \varphi_i)}{2(G_1 + G_2 + \ldots + G_i)} \quad (31)$$

where k_{yi} and k_{ygl}, the yield acceleration factors for local and global stability cases, which attain minimum value for $dk_{yi}/d(\Omega)=0$ and $dk_{ygl}/d(\Omega)=0$ respectively.

In both cases K attains a maximum value when $dK_i/d(\Omega)=0$ and the distance between the failure surface and the end of the slope is calculated by Equations 22 and 27 for local and global failure mode respectively.

4. SOFTWARE IMPLEMENTATION

For the implementation of the methodology the software application has been designed in Delphi environment with the following features:

1) Calculation of the Horizontal Peak Ground Acceleration (HPGA) based on the Ambraseys (1995) predictive relations for peak horizontal ground accelerations (Eq. 2&3). An independent value can also be chosen.

2) Seismic analysis of reinforced slopes with berms, according to the plane failure mechanism for local and global failure modes. The software user imports the soil mechanical characteristics (φ, γ), height H, inclination β of each step, bench width λ and the number of the reinforcement layers n. It is assumed that reinforcement layers are of equal length for each berm and of the same spacing for the whole slope (d_i). An appropriate value of the HPGA is also imported, based on step 1.

3) Seismic analysis results display: the angle that specifies the critical failure mechanism (Ω_i and Ω_{gl}), the maximum total reinforcement in a normalized form (K_i and K_{gl}), the maximum tensile strength for each layer (T_i and T_{gl}), and the necessary length of the reinforcement (l_i and l_{gl}) are defined. In addition, the required tensile strength and length of the reinforcement for local and global stability analysis are compared and the maximum values are used for the final design.

4) Calculation of the critical yield acceleration factors kyi and k_{ygl}, for a reinforced slope with berms, in the case of the seismic risk upgrade and/or in order to study an already constructed slope. The software user imports the soil mechanical characteristics (φ, γ), height H, inclination β of each step, bench width λ and number of the reinforcement layers n as well as tensile strength and length of the reinforcement. During this analysis, no value of the HPGA is imported and the critical yield acceleration factors k_{yi} and kygl that lead slope to failure are calculated for each berm and for the whole slope respectively. The results of this analysis can be used in conjunction with step 1, in order to define the expected permanent displacement that is described in the following step 5.

5) Definition of the expected permanent displacement as a criterion for the reinforced slopes with berms vulnerability. The expected Horizontal Peak Ground Acceleration (HPGA) based on step 1 is imported as well as the critical yield acceleration factors k_{yi} and k_{ygl}, for a trust level 't' and the value of the expected permanent displacement is defined in cm.

6) The software developed is also capable of performing static analysis, for a desirable Safety Factor. The method used is similar to the seismic design one and has been described in Kapogianni et al (2008).

In Figure 6 the flow chart of the software application is presented. As can be noted, there is an interaction between various analysis phases, which has also been described in the above steps. In Figure 7 a typical seismic analysis of a reinforced slope with five berms can be seen.

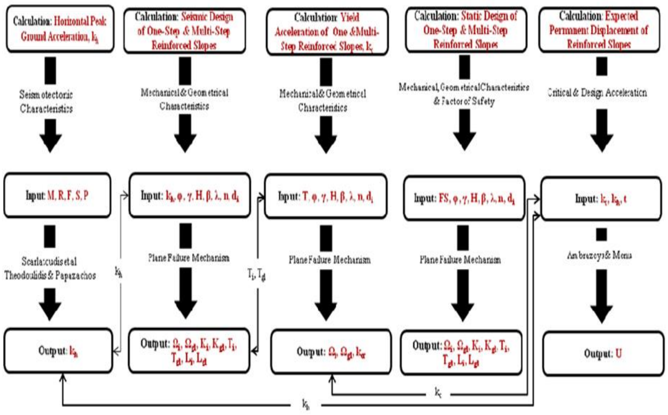

Figure 6. Application flow chart.

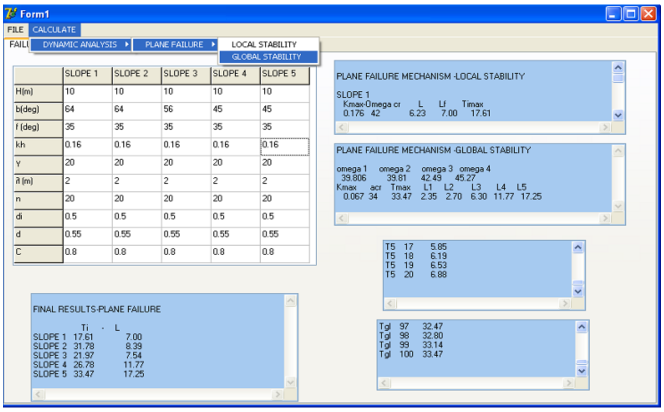

Figure 7. Typical Seismic Analysis.

5. EXAMPLES OF MULTI-STEP AND ONE-STEP REINFORCED SLOPES:

In this section, calculations are carried out in order to demonstrate how the program works and how geometry, seismic loading and soil properties affect the required tensile strength and length of the reinforcement.

In the first example it is assumed that soil is cohesion less with unit weight γ=18kN/m^3 and angle of soil shearing resistance φ=35^0. The slope consists of 5 steps with inclination: β_1=2:1, β_2=2:1, β_3=3:2, β_4=1:1 and β_5=1:1(vertical: horizontal).The height of each step is 10 m and the reinforcement consists of 20 equally spaced layers for each slope (d_i=0.5 m). The plane failure mechanism is applied for local and global stability and the critical mechanism is defined, initially for every one step (local stability) and then for the whole slope (global stability).
In general, for steeper slopes and at lower seismic coefficient, the critical mechanism is located at $\Omega>\varphi$ and for slopes with gentle inclination and higher seismic coefficient at $\Omega<\varphi$. For example, for a one-step slope with φ=35^0, β=45^0 and k_h=0.16, the critical mechanism is at Ω=34^0. On the other hand, for a one-step slope with φ=35^0, β=65^0 and k_h=0.16, the critical mechanism is at Ω=42^0 and for kh=0.36, Ω is smaller than φ (Ω=33^0).

The seismic influence on the amount of necessary reinforcement of every one step is illustrated. Moreover the effect of the bench width λ between the 5 steps is shown. Finally, a one step and a multi step slope are compared.
Figures 8 and 9 show the required tensile strength at different values of the seismic coefficient k_h for every one step. As can be expected, the required tensile strength increases with increasing seismic coefficient k_h. In addition, for the 5th step, the most tensile strength is required, due to the influence of the weight of the upper steps. In Figure 9, where bench width λ between the 5 steps is larger, it can be seen that the necessary tensile strength is reduced and especially for the lower layers of the slope and at higher seismic coefficient. Figures 10 and 11 illustrate how λ affects the necessary tensile strength for every one step. Steps 2, 3, 4 and 5, require less tensile strength as distance λ increases, while for the 1st step, as expected the same tensile strength is required.
Furthermore, Figures 12 and 13, illustrate the comparison of a one-step reinforced slope with a multi-step reinforced slope for different values of the seismic coefficient. It is assumed that the one-step has average inclination α (Figure 1), associated with the geometrical characteristics of the multi-step slope through the following equation:

$$\tan(a)=\frac{(H_1+H_2+\ldots+H_5)}{\frac{H_1}{\tan(\beta_1)}+\frac{H_2}{\tan(\beta_2)}+\ldots+\frac{H_5}{\tan(\beta_5)}+\lambda_1+\lambda_2+\lambda_3+\lambda_4} \quad (27)$$

Specifically, for the case of the multi-step mentioned before (β_1=2:1, β_2=2:1, β_3=3:2, β_4=1:1 and β_5=1:1), the corresponding average inclination for λ = 0 is slightly less than 3:2 and the height of the one-step slope is H_{total}=50m. The soil mechanical characteristics are the same. As shown in Figures 12 and 13, the required tensile strength of the multi-step slope is significantly lower than the required tensile strength of the one-step slope. The inclination is taken as 56^o (3:2) for the single step slope, whilst the average inclination α for λ=2 and 4 is 48^o and 46^o respectively, hence accounting for the reduction in reinforcement.

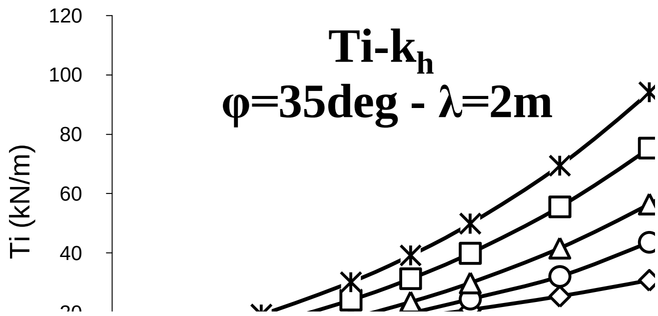

Figure 8. Seismic influence on the necessary tensile strength for λ=2m.

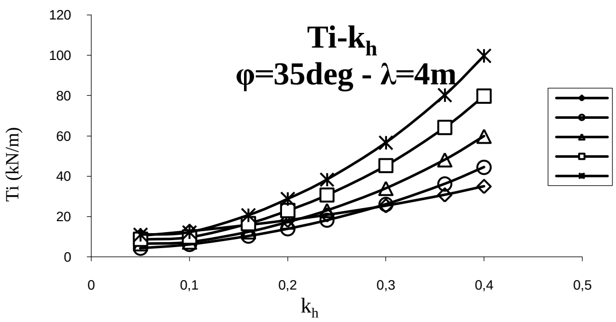

Figure 9. Seismic influence on the necessary tensile strength for λ=4m.

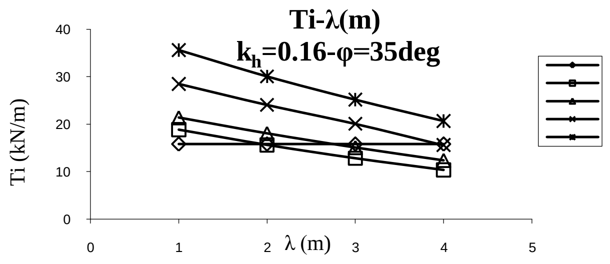

Figure 10. Effect of distance λ between the 5 steps, for kh=0.16.

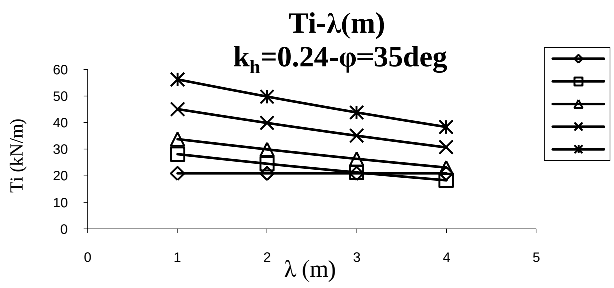

Figure 11. Effect of distance λ between the 5 steps, for kh=0.24.

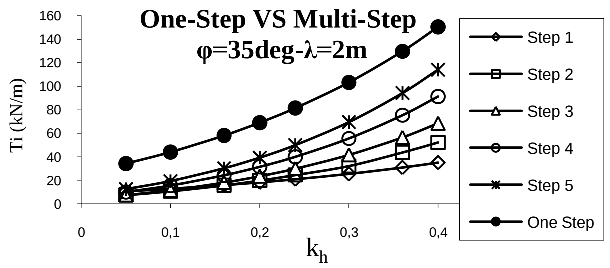

Figure 12. One-step (α= 3:2) versus multi-step (λ=2m), at different seismic coefficients.

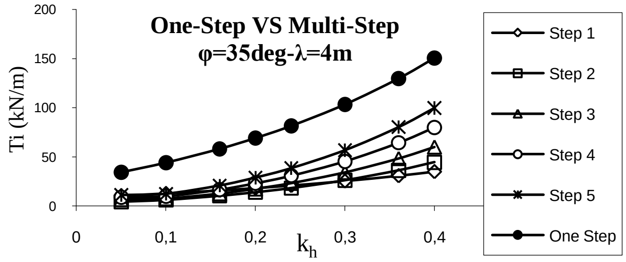

Figure 13. One-step (α= 3:2) versus multi-step (λ=4m), at different seismic coefficients.

6. COMPARISON WITH CONVENTIONAL NUMERICAL METHODS

The results from the calculations carried out in the previous section, are imported in a into a representative 2D limit equilibrium slope stability program (Slide) and the safety factors of the models are calculated in order to examine whether the reinforcement calculated by the analytical solution gives satisfactory values of SF. Moreover, the strength reduction factor (SRF) is defined, with the help of a 2D elasto-plastic FESA program (Phase).

Models with SF higher than 1, indicate that the reinforcement is adequate against failure and models with SF lower than 1, imply that the limit equilibrium analysis is more critical that the analytical solution. In addition, due to the transient nature of ground motion, models with SF lower than 1 experience only a finite displacement rather than a complete failure. The multi-step model has the same soil geometrical and mechanical characteristics with the previous example ($H_{1\text{-}5}$=10m, γ=20kN/m^3, φ=35°, d_i=0.5m β_1=2:1, β_2=2:1, β_3=3:2, β_4=1:1, β_5=1:1). Results for different values of bench width (λ=1, 2 and 3) are presented in Table

1. The one-step model with 50 m height and average inclination α=3:2 is also analyzed (Table 1).

Specifically, Table 1 shows that for the case of the one-step, the reinforcement calculated by the analytical solution is adequate also for limit equilibrium analysis. For lower seismic coefficient the SF of the multi-step models also take values higher than 1, but as seismic coefficient increases, the SF decrease. Moreover, the required total reinforcement in a normalized form K is lower for multi step slopes. This proves that the construction of high slopes with steps can also be an economical and practicable solution.

Table 2, demonstrates the SRF of the one step and multi step models. Models with SRF higher than 1, indicate that the reinforcement is adequate against failure and models with SRF lower than 1, imply that the finite element stress analysis is more critical that the analytical solution. In addition, due to the transient nature of ground motion, models with SRF lower than 1 experience only a finite displacement rather than a complete failure.

Table 1. Bishop analysis with data from the analytical solution

Seismic Coefficient	One Step		Multi Step					
			λ=1m		λ=2m		λ=3m	
	K Limit Analysis	F.S. Bishop Analysis	K Limit Analysis	F.S. Bishop Analysis	K Limit Analysis	F.S. Bishop Analysis	K Limit Analysis	F.S. Bishop Analysis
0.05	0.16	1.1	0.10	1.1	0.08	1.0	0.08	1.1
0.10	0.20	1.1	0.12	1.1	0.10	1.1	0.10	1.1
0.15	0.24	1.1	0.16	1.1	0.14	1.1	0.12	1.1
0.20	0.32	1.1	0.22	1.1	0.18	1.1	0.16	1.1
0.25	0.40	1.1	0.28	1.1	0.24	1.0	0.22	1.0
0.30	0.48	1.1	0.36	1.1	0.32	1.0	0.28	1.0
0.35	0.56	1.0	0.44	1.0	0.40	1.0	0.40	1.0

Table 2. Finite element stress analysis with data from the analytical solution

Seismic Coefficient	One Step		Multi Step					
			λ=1m		λ=2m		λ=3m	
	K Limit Analysis	SRF FESA	K Limit Analysis	SRF FESA	K Limit Analysis	SRF FESA	K Limit Analysis	SRF FESA
0.05	0.16	1.1	0.10	1.1	0.08	1.1	0.08	1.1
0.10	0.20	1.1	0.12	1.2	0.10	1.1	0.10	1.0
0.15	0.24	1.1	0.16	1.1	0.14	1.0	0.12	1.1
0.20	0.32	1.0	0.22	1.1	0.18	1.1	0.16	1.0
0.25	0.40	1.0	0.28	1.1	0.24	1.0	0.22	1.0
0.30	0.48	1.0	0.36	1.0	0.32	1.0	0.28	1.0

7. CONCLUSIONS

The expressions derived in this study based on the kinematic theorem of limit analysis can be conveniently used for the design of high reinforced slopes with steps. The amount of the reinforcement required to prevent failure due to static and seismic loading can be calculated for local and global failure modes and the most critical one can be used for the final design. In general, the critical mechanism can be defined for $\Omega>\varphi$ for steeper slopes or/and at lower seismic coefficient and for $\Omega<\varphi$ for slopes with gentle inclination or/and at higher seismic coefficient. The calculations carried out indicate that reinforcement increases with an increased seismic force and reduces as bench width λ increases and that the inclination of each step affects the amount of the reinforcement. Additionally, the required tensile strength of the multi-step compared to that of the one-step is significantly reduced, while the potential erosion due to the water flow is limited. Moreover the tensile strength of the reinforcement calculated for both local and global failure mode can give satisfactory safety factors when limit equilibrium analysis is performed and satisfactory strength reduction factors when finite element stress analysis is performed. Finally, the critical acceleration factor k_y can be obtained and static analysis with the desirable factor of safety can be performed.

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