Numerical Investigation of Heat Performance Enhancement for a Double-Pipe Heat Exchanger at Annulus with Continuous Helical Baffles using Al_2O_3 as Nanofluid

Amit Kumar Singh[1], H.C.Thakur[2]

[1,2]*Department of Mechanical Engineering, Gautam Buddha University, Greater Noida,, India*

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Abstract - *Main goal of our project is to maximize the heat transfer rate and minimizing the pressure drop as much as possible to increase the performance enhancement factor for heat exchanger. As we know that by increasing Reynolds number Nusselt number increases thus increasing the heat transfer rate but in doing so pressure drop is also increased that is not beneficial for the operating of heat exchanger so we used passive technique that is the use of nanofluid in place of water as cold fluid in the annulus side to increase heat transfer rate. In this project we keep the mass flow at 0.1, 0.15, 0.2, 0.25, 0.3 Kg/s.* AL_2O_3 *is used as nanofluid with volumetric concentration of 1%, 2%, 3%, 4% respectively and found that heat transfer rate is increased while there is a decrement in pressure drop. And we have also plotted the graph for heat transfer, pressure drop and performance enhancement factor*

Key Words: Continuous helical baffles, Water to water heat exchanger, Heat transfer enhancement, Double pipe, Friction, Numerical

1. INTRODUCTION

Double-pipe exchanger is made of two pairs of pipes that are concentric to each other. There can be same or two different fluids that are transferring heat from inner to outer pipes or vice versa. The fluids usually flow in counter clockwise directions. They are mostly used in applications involving relatively low flow rates and high temperatures or pressures. Other advantages include low installation cost, ease of maintenance, and flexibility.

One of the starting studies performed on double pipe heat exchangers was done by Mozley [1] who numerically and experimentally developed a case to study various dynamic characteristics of a special DPHEs. Those were derived from simple mathematical models. From this research frequency responses was studied it matched experimental results. Research has been done to increase the heat transfer rate of heat exchangers which helps in saving energy, money and time. It also helps in increasing thermal rating and extending the overall working life of the device. This is done by using various passive geometrical features to increase the heat transfer capacity. These innovations reduces the thermal resistance but increasing the pressure drop , as reported by Jian et al. [2] Bhadouriya et al. [3].

In this study, Rennie and Raghavan [4] studied DPHE. The study was conducted using both parallel and counter flow arrangements. During research it was concluded that performance evaluation criterion was identical for both configurations, also there was a huge difference in the heat transfer to the counter flow configuration due to its high temperature difference as compared to its counterpart Wilson plots were used to determine the heat transfer rates of inner tube and the annulus.

In this research Bhadouriya et al. [5] studied DPHE both experimentally and numerically. The objective was to research on the heat transfer and pressure drop. Boundary condition of annulus wall was taken as uniform temperature wall for outer flow. The important point of the study was to research the effect of twist ratio of the inner tube on the flow characteristics. Air and water were taken as the working fluid in the outer and inner tube of the heat exchanger, respectively.

Following the pervious researches, in another experimental investigation, Yadav [6] investigated the effects of half-length twisted tapes on heat transfer and pressure drop of a double pipe U-bend heat exchanger. The twisted tapes were in the inner tube of a DPHE which caused a 40-percent increase in heat transfer in comparison with smooth tube

Prasad et al. [7] studied the effect of trapezoidal-cut twisted tapes in a DPHE. The nanofluid used in this investigation was water-based Al_2O_3 which was in turbulent flow regime. The results showed that the heat transfer rate increase in the annulus was higher than that of the inner tube which this was mainly due to the secondary flows in the annulus. They also observed that the increase in nanofluid concentration and also the twist ratio leads to a higher heat transfer and pressure drop. Heat transfer performance of a double pipe heat exchanger at annulus side with continuous helical baffles was studied by Maakoul et al.[8] numerically The research was conducted for different mass flow rate (0.1 -0.3 kg/s) and baffle spacing (0.025–0.1 m). The results showed an increase in heat transfer and pressure drop as compared to the simple double-pipe exchanger.

Nomenclature

	Latin symbols	u	average velocity, m/s
Across	cross-flow area at the annulus side, m2	x; y; z	Cartesian coordinate
B	baffle spacing, mm		
cp	specific heat capacity, J/(kg K)		Greek symbols
c_i	coefficients in k e model	Γ	generalized diffusion coefficient
D_s	internal annulus diameter, m	ε	turbulent kinetic energy dissipation rate, m2/s3
D_o	external tube diameter, m	k	thermal conductivity, W/(m K)
D_h	hydraulic diameter, m	μ	dynamic viscosity, kg/(m s)
h	average heat transfer coefficient, W/(m2 K)	ν	kinematic viscosity, m2/s
k	turbulent fluctuation kinetic energy, m2/s2	q	density, kg/m3
L	tube total effective length, m	σ_k	Prandtl number for k
f	friction factor	σ_ε	Prandtl number for ε
m	mass flow rate, kg/s	ϕ	nanoparticle volume fraction
Pr	Prandtl number		
		Subscripts	
Δp	pressure drop, Pa		
		in	inlet
Q	heat transfer rate, W		
		out	outlet
Re	Reynolds number		
		a	annulus side
Nu	Nusselt number		
		t	tube side
Tin	hot water inlet temperature, oC		
		turb	turbulent
tin	cold water inlet temperature, oC		

2. NUMERICAL MODEL

The main goal of this paper is to study heat transfer performance for double-pipe heat exchanger at annulus side with continuous baffle of different spacing. This leads to change in the heat transfer, pressure drop, velocity in the annulus flow domain and thus the net heat transfer rate and pressure drop varies from initial values. This structure is made of steel to facilitate better heat transfer and resistance to corrosion. The D structure is shown in fig 1the baffle spacing is varied between 100mm to 0.25 mm and the heat performance of the structure is analyzed using water and nanofluid. Even though it seems that the structure is simple but when analyzing the fluid flow it becomes complex due to introduction of baffles on annulus side.

In this paper first the base paper [8] result is validated for double pipe heat exchanger when water is taken as fluid on annulus side and inner tube side. The flow in both tubes is taken as counter flow. The best part of the research is that due to introduction of baffle the overall length of the exchanger remains the same but the flow path of the fluid is increased. The only variation in the geometry is that the baffle spacing varies from 100mm to 25mm. More geometry details are listed in Table 1. Also during further research water in the annulus side is replaced with Al_2O_3 nanofluid.

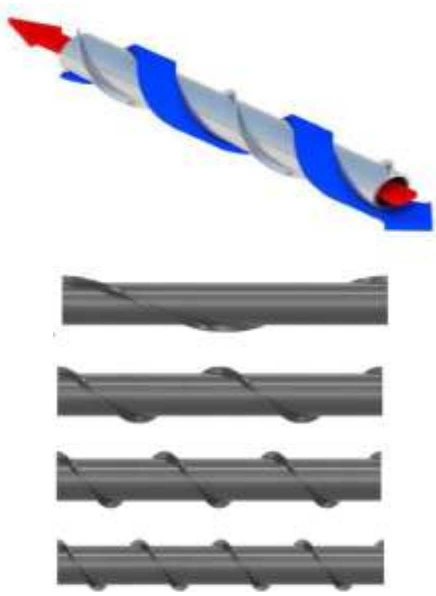


Figure 1 geometry (3D & 2D)

Table 1 Structural parameters

Material:	Stainless steel
Inner tube internal diameter	10 mm
Inner tube thickness	1 mm
Outer tube external diameter	16 mm
Outer tube thickness	1 mm
Annulus side baffles spacing	100–25 mm
Heat exchanger length	100 mm

2.1 Governing Equations

Continuity equation:

$$\left(\frac{\partial u_i}{\partial x_i}\right) = 0 \qquad (1)$$

Momentum equation

$$\left(\frac{\partial u_i u_j}{\partial x_i}\right) = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}((v + v_{turb})\ (\frac{\partial u_i}{\partial x_j} + \frac{\partial uj}{\partial xi})] \qquad (2)$$

Energy equation

$$\frac{\partial u_i T}{\partial x_i} = \rho\ \frac{\partial}{\partial x_i}((\frac{v}{Pr} + \frac{v_t}{Pr_{turb}})\frac{\partial T}{\partial x_i}) \qquad (3)$$

To solve the problem that arises by turbulent flow which was caused by the introduction of baffle spacing on annulus side Realizable k e turbulence model was selected in energy. The equations for realizable k e model are given below:

Turbulent kinetic energy k equation:

$$\frac{\partial u_i k}{\partial x_i} = \frac{\partial}{\partial x_i}((v + \frac{v_t}{\sigma k})\frac{\partial k}{\partial x_i}) + \Gamma - \varepsilon \qquad (4)$$

Turbulent energy dissipation equation

$$\frac{\partial u_i}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\left(v + \frac{v_t}{\sigma\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_i}\right) + C_1\Gamma\varepsilon - C_2\frac{\varepsilon^2}{k+\sqrt{v\varepsilon}} \quad (5)$$

where C represents the generation of turbulence kinetic energy k due to the mean velocity gradients The wall is set to scalable wall function

2.2 Domains definitions, mesh sensitivity and boundary conditions

At geometry creation we have to make a straight double pipe heat exchanger with continuous baffle on annulus side. First we have to draw a circle of Dia 10mm and then another concentric circle of Dia 12mm at the center the select both the sketch and click generate surface. Similarly from the center point draw another sketch of two circle of Dia 16mm and 17mm. again select the second sketch and click on generate surface. After this draw a new sketch that are two straight line between inner tube outer circumference and outer tube inner circumference also the distance of both straight line from axis is 0.5mm on both side. Again select this sketch and click on generate surface. Now select the first two sketches and select extrude. In this select direction as both forward and backward with extrude length of 50mm. Now separately select the third sketch and select revolve in the pitch section select the pitch according to the baffle spacing. Then select direction as forward and backward. Also select the axis that is passing through the center of the circle around which the helical baffle will be rotated.

Meshing is done in the mesh window. In this portion a mix of unstructured tetrahedral and wedge (Prism) grid is used. Inflation option is used near the wall of tubes

Table 2 Mesh quality

Nodes	Elements	Average skewness	Average orthogonal quality
286288	392755	0.270605	0.844705

The no slip condition for all the solid walls

The software FLUENT is used to calculate the fluid flow and heat transfer in the computational domains. The governing equations are iteratively solved by the finite-volume formulation with the SIMPLE algorithm. The second-order upwind scheme is adopted for the momentum, energy, turbulence and its dissipation rate. The pressure term is treated with the standard scheme.

Outer wall is considered as insulated wall. In this condition heat is transferred from hot fluid to cold fluid through a solid steel wall that is the inner wall of heat exchanger and cold fluid gets warmer and hot fluid gets colder. For inlet of hot fluid was set as velocity inlet. Here the velocity of inlet is kept constant at 0.1kg/s mass flow rate throughout the entire analysis. The inlet temperature of the hot fluid was taken as 312K. Similarly for inlet of cold fluid was set as velocity inlet. Here the velocity of inlet was varied by changing the mass flow rate. The mass flow rate of the working fluid was varied between 0.1, 0.15, 0.2, 0.25, 0.3 kg/s. The inlet temperature of cold fluid was taken as 288K. For the outlet of cold fluid and hot fluid was takes as Pressure outlet boundary condition.

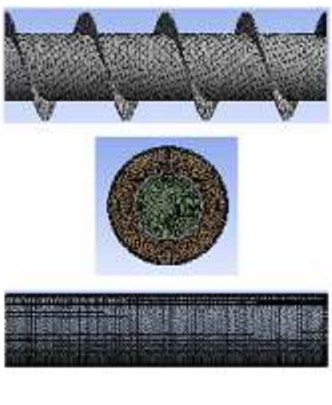


Figure 2 meshing of geometry

At the outlet the gauge pressure was taken as zero atmospheric pressure. The wall of the outer tube is taken as insulted wall condition, this done because there is no heat exchange taking place through that wall The fluid property of water was assumed to be continuous throughout the analysis with respect to temperature.

3. DATA REDUCTION

3.1 Annulus side velocity

$$u_a = \bar{m}_a/(\rho_a A_{cross}) \quad (6)$$

3.2 Characteristics cross section area for helically baffle annulus side

$$A_{cross} = 0.5B(D_s - D_o) \quad (7)$$

With the mean velocity value, the Reynolds number for the annulus side is determined by:

$$Re = (\rho_a u_a D_o)/\mu \quad (8)$$

3.3 Heat transfer rate:

Heat transfer rate of the annulus side fluid (cool water):

$$Q_a = m_a Cp_a(T_{out\ a} - T_{in\ a}) \quad (9)$$

Heat transfer for hot fluid tube

$$Q_t = m_t Cp_t(T_{in\ t} - T_{out\ t}) \quad (10)$$

3.4 Nusselt number

$$Nu = (h_a D_h)/k \quad (11)$$

When using nanofluid in place of water

3.5 Thermal conductivity:

For calculating thermal conductivity of water based Nanofluid Maxwell [9] formula is used. Also Brownian effect is neglected. The effective thermal conductivity of nanofluid is given by:

$$K_{nf} = K_f \frac{|(K_{np} + K_f) - 2\phi(K_f - 2K_{np})|}{|(K_{np} + K_f) + \phi(K_f - 2K_{np})|} \quad (12)$$

3.6 Specific heat and density:

Calculation of specific heat and density is straight forward. They can be based on the physical guideline of the blend administer these outcomes are in great understanding with experiment information. The specific heat is formula [9]:

$$(Cp)_{nf} = (1-\phi)(Cp)_f + \phi(Cp)_{np} \quad (13)$$

3.7 Density is calculated by using formula given by[9]

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_{np} \quad (14)$$

3.9 Viscosity

$$\mu_{nf} = \mu_f(1+2.5\phi) \quad (15)$$

4. RESULTS AND DISCUSSION

The research work was done in two parts. First water was taken as fluid in both the inner and outer pipe and the results obtained from it are compared with results from the base paper. Second the fluid in the annulus side is replaced with Al_2O_3. Also nanofluid concentration is varied from 1% to 4% and the results obtained are shown below. In order to verify the results graph with various parameters like heat transfer coefficient, pressure drop, thermal enhancement factor with respect to mass flow rate is plotted and is compared to the graph obtained from the base paper. This showed satisfactory result

4.1 Fluid flow characteristics

In Fig. 3 velocity streamline simulation has been shown. It can be seen that the max velocity is at the inlet of the fluid. It can also be seen that due to introduction of continuous helical baffle on annulus side, helical flow is initiated. Due to helical path the same length of double pipe exchanger provides a greater flow path for the fluid hence increasing the heat transfer effect. This leads more compact exchanger design. Also due to helical path the turbulence of the fluid is increased which is beneficial in energy transfer. From analyzing the fig 3 it can be seen that the velocity is directly proportional to mass flow rate but inversely proportional to the baffle spacing. The total flow path length can be determined by following formulae

$$\frac{L}{B} * \sqrt{(\pi D)^2 + B^2} \tag{16}$$

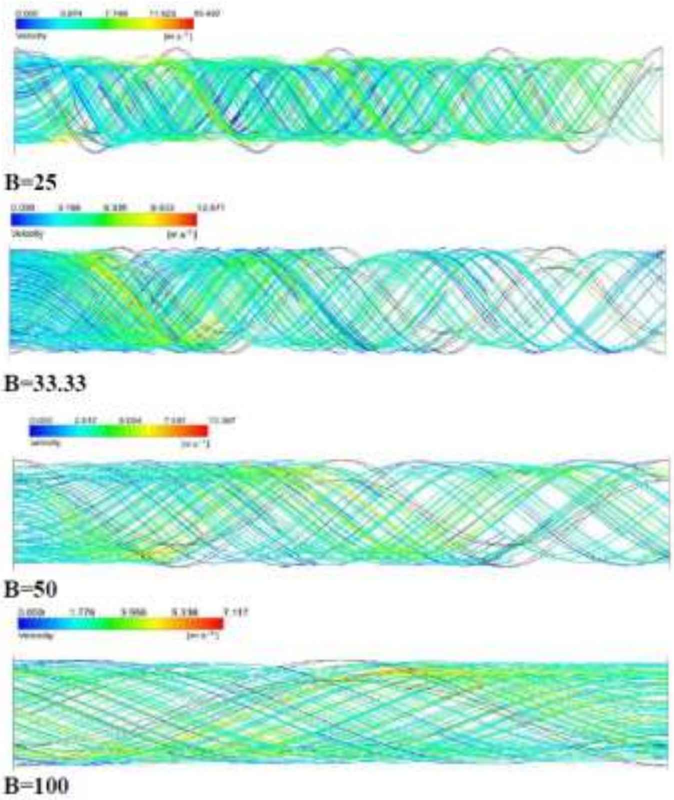


Figure 3 Annulus side velocity streamlines at m= 0:3 kg/s for different baffle spacing when using water as fluid

The velocity is at max at the inlet of the annulus this can be explained by the fact that there is a sudden decrease in the cross section area. Hence the velocity can only be increased to maintain the same mass flow rate at the annulus side. This can be used to explain the sudden rise in velocity when increasing the mass flow rate at the annulus side.

4.2 Heat transfer performance

From Fig. 4(A) results for the base paper validation is shown. From graph it can be seen that results almost matches the graph drawn in the base paper [8]. In this the heat transfer coefficient at the annulus side is shown foe different baffle spacing as well as for simple double pipe heat exchanger. Results obtained showed the benefits of having continuous baffle spacing on annulus side as compared to simple double pipe heat exchanger. The average increment of the overall heat transfer coefficient on annulus side for 100, 50, 33.33, 25mm is 5%, 17%, 30% and 45% respectively. The results obtained approximately have same as in the base paper.

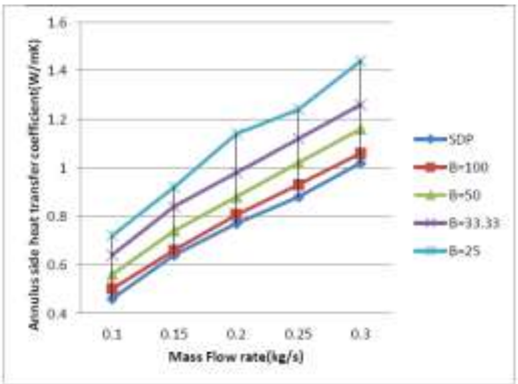


Figure 4(A)

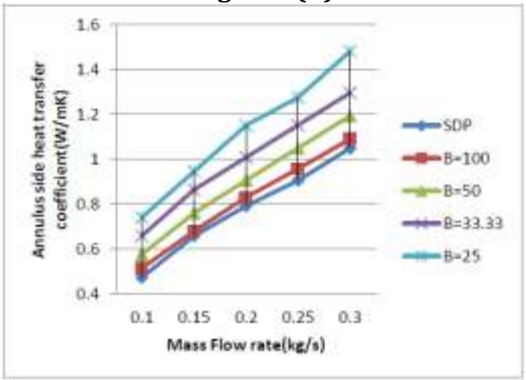


Figure 4(B)

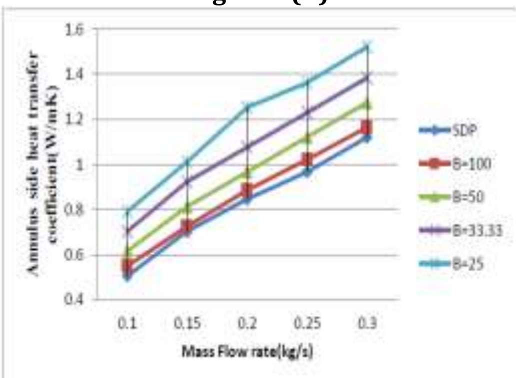


Figure 4(C)

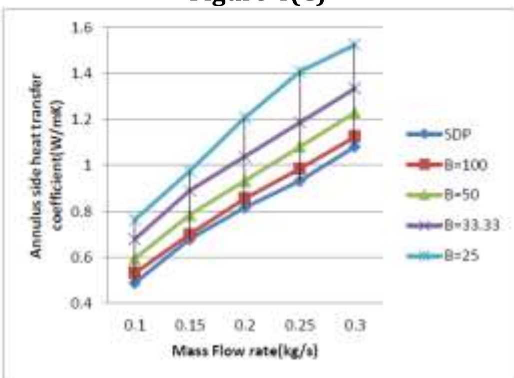


Figure 4(D)

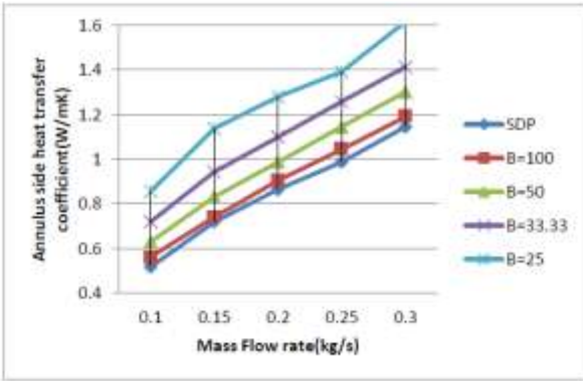


Figure 4(E)

Figure 4(A)(B)(C)(D)(E). Annulus side heat transfer coefficient (*10^4) w.r.t. mass flow rate for different baffle spacing

Fig.4 (B), (C), (D), (E) demonstrate the heat transfer coefficient results for the annulus side with same condition of baffle spacing and mass flow rate as used in base paper [8]. But the difference is that in this case Al_2O_3 nanofluid of concentration from 1% to 4% respectively is used. The results from the graph shows that there is an average increase in 2.88%, 5.90%, 9.83% and 12.21% for volumetric concentration of 1%, 2%, 3% and 4% of nanoparticle respectively. The increase in heat transfer can be explained by the fact that with the addition of nanoparticle increases the overall thermal conductivity of base fluid and enhancing the heat transfer results. Also from the graph it can be interpreted that the heat transfer coefficient rate is directly proportional to mass flow rate and inversely proportional to baffle spacing as smaller the baffle spacing greater is the flow path and hence greater is the heat transfer.

4.3 Nusselt number

From Fig. 5(A) results for the base paper [8] validation is shown. From graph it can be seen that results almost matches the graph drawn in the base paper. In this the Nusselt number in the form of $NuPr^{-1/3}$ at the annulus side is shown foe different baffle spacing as well as for simple double pipe heat exchanger. Results obtained showed the benefits of having continuous baffle spacing on annulus side as compared to simple double pipe heat exchanger.

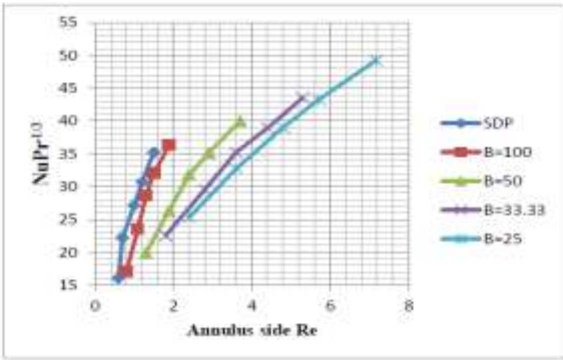
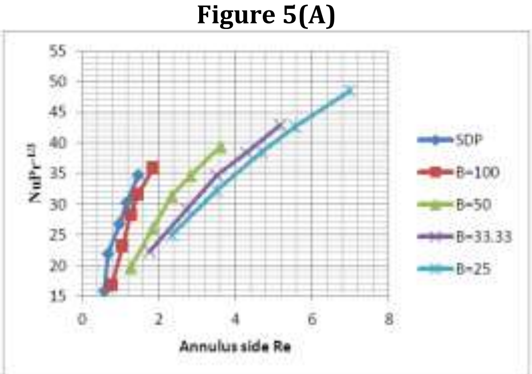


Figure 5(A)



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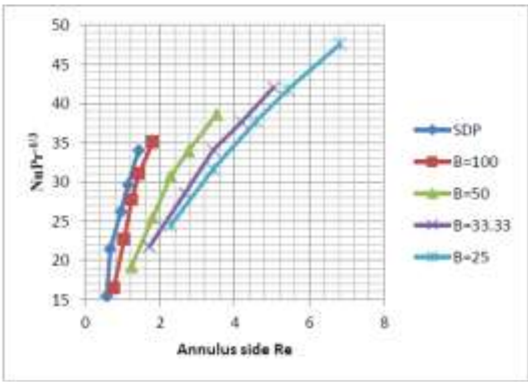


Figure 5(C)

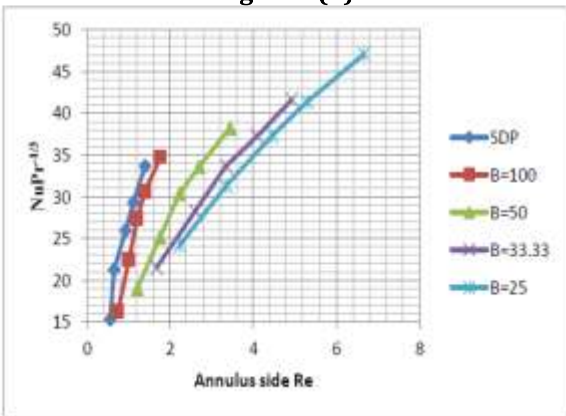


Figure 5(D)

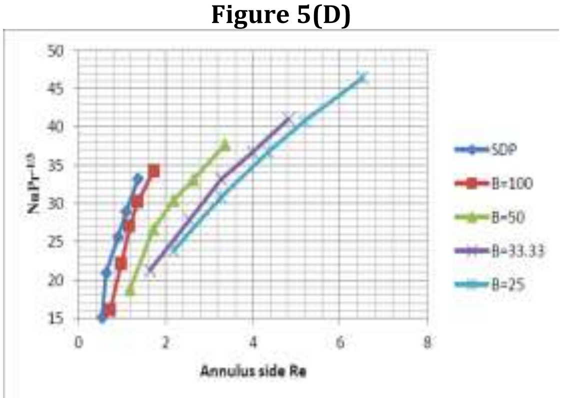


Figure 5(E)

Figure 5(A)(B)(C)(D)(E). Annulus side Nusselt number w.r.t. Reynolds number (*10^4) for different baffle spacing

From this it can be seen that with increasing mass flow rate and decreasing baffle spacing heat transfer of the heat exchanger increases.

$$NuPr^{-1/3} = 0.04572Re^{0.6098}(1+B)^{7.565} \quad (17)$$

From equation 17 it can be seen that the value of $NuPr^{-1/3}$ is inversely proportional to dynamic viscosity which increases with increasing the nanoparticle concentration hence with increasing nanofluid concentration the value for $NuPr^{-1/3}$ begins to decrease. The results from the fig 5(B), (C), (D), (E) shows that there is an average decrease of 1.49%, 3.22%, 4.31% and 5.64% for volumetric concentration of 1%, 2%, 3% and 4% of nanoparticle respectively.

4.4 Pressure drop

The pumping power that is supplied to the heat exchanger is an important factor in determining the net overall efficiency of that particular heat exchanger. Since simple double pipe heat exchangers can only be used in the field with low pumping power hence it is most important to reduce the net pressure drop for the heat exchanger so that it can be most efficient. When the continuous baffle was provided at the annulus side the pressure drop was increased due to following reasons

- Fluid flow path was increased
- Due to sudden reduction in cross sectional area to maintain the mass flow rate, fluid velocity has to increase.

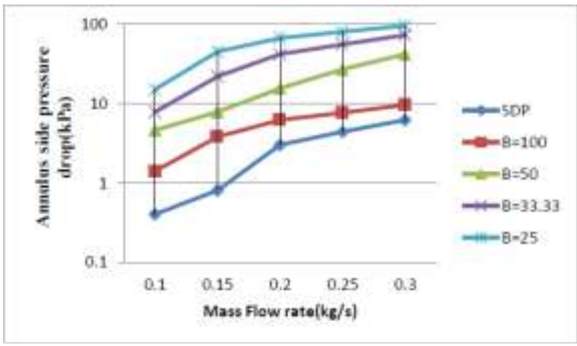


Figure 6(A)

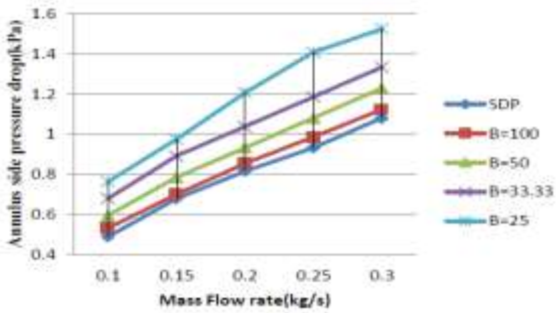
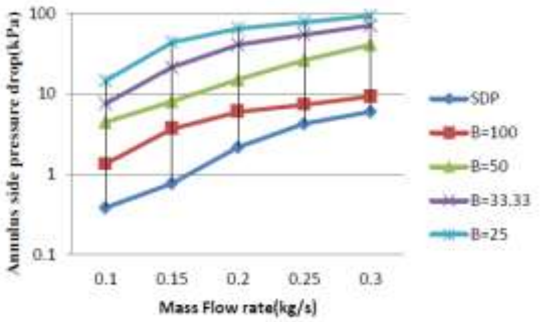


Figure 6(B)



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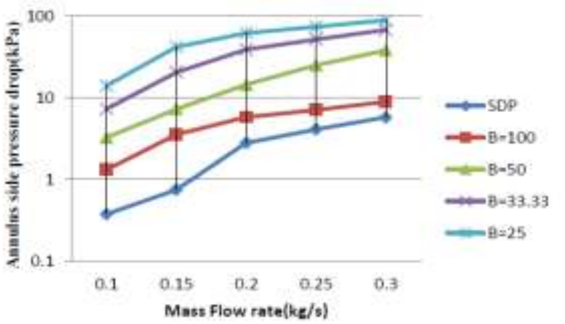

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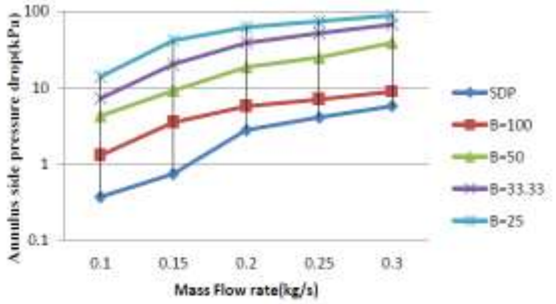


Figure 6(E)

Figure 6(A)(B)(C)(D)(E). Annulus side pressure drop w.r.t. mass flow rate for different baffle spacing

In fig 6(A) results from base paper [8] is validated. It can be seen from the graph that pressure drop directly depends on the mass flow rate. Also as baffle spacing is decreased the flow path length increase to maintain the overall fluid flow till the end, pressure has to be increased. Hence pressure drop is inversely proportional to the baffle spacing.

$$\Delta p_a = 4f\left(\frac{L}{D_h}\right)\rho_a\frac{{u_a}^2}{2}\left(1+\frac{L}{B}\right) \quad (18)$$

Fig 6 (B), (C), (D), (E) demonstrates the pressure drop in the outer tube flow region. In these cases Al_2O_3 nanofluid of concentration from 1% to 4% respectively is used. The results from the graph shows that there is an average decrease of 2.29%, 4.58%, 6.69% and 8.60% for volumetric concentration of 1%, 2%, 3% and 4% of nanoparticle respectively. The decrease in pressure drop can be explained by the fact that it is directly proportional to friction factor and inversely proportional to density of fluid. With the addition of nanoparticle both friction factor and density increase but the increase of density is greater as compared to friction factor. Hence the values of pressure drop on annulus side decreases with increment in the nanoparticle concentration.

4.5 Thermal performance enhancement factor

In earlier times researchers mainly focused on increasing heat transfer irrespective of pumping power loss. This approach is good when the size and pumping power requirement of heat exchanger are considered secondary when comparing it to heat transfer.

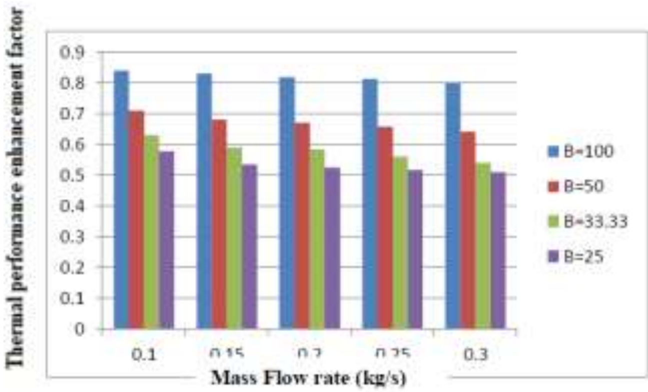


Figure 7(A)

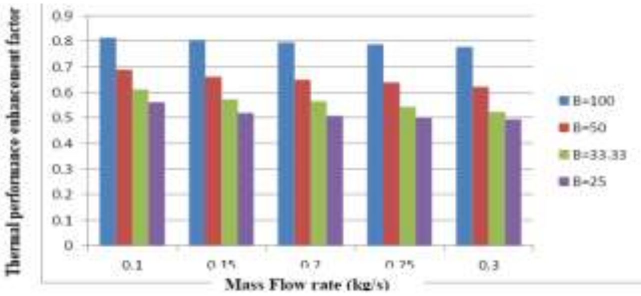
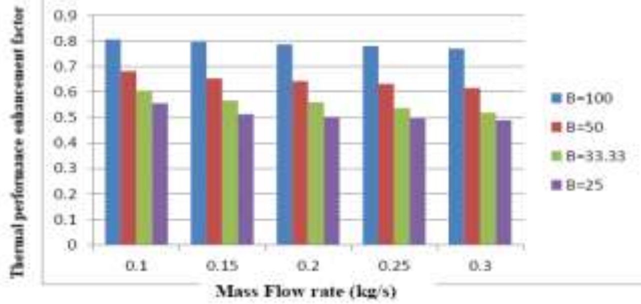


Figure 7(B)



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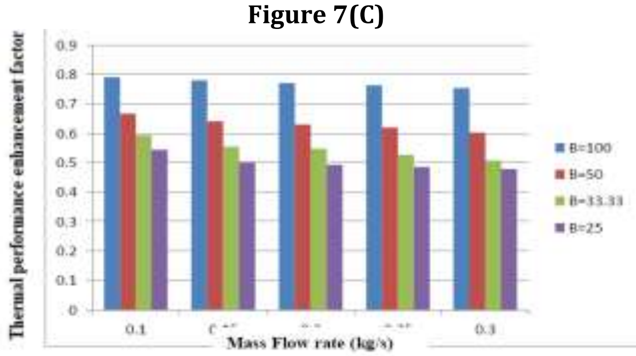

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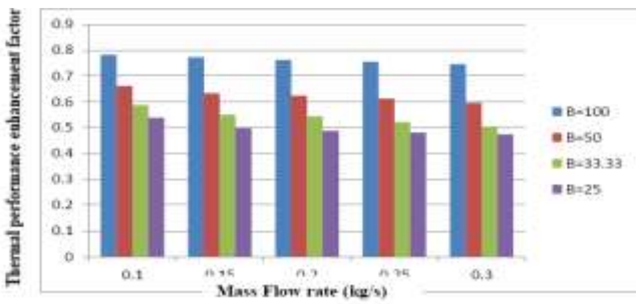

Figure 7(E)

Figure 7(A)(B)(C)(D)(E). Thermal performance enhancement factor w.r.t. mass flow rate for various baffle spacing

But in this paper research is done on those types of heat exchangers that are used in small machinery with low pumping power. Hence we cannot rely on only increasing heat transfer and neglecting pressure drop. So after analyzing the problem a new term was introduced which was called as thermal performance enhancement factor, represented by symbol η. It is the comparison between Nusselt number and pressure drop of annulus side with baffles to Nusselt number of ordinary double pipe heat exchanger without any modifications. This quantity is given by:

$$(\eta) = \left(\frac{Nu}{Nu_o}\right)\left(\frac{\Delta p}{\Delta p_o}\right)^{-1/3} \quad (19)$$

In Fig. 7(A) results from base paper [8] is validated. It can be seen from the graph that thermal performance enhancement factor is directly proportional to baffle spacing. Also it decreases with the increasing mass flow rate.

This result can be explained by the fact that when the baffle spacing is decreased the pressure drop is tremendously increased as compared to increment in Nusselt number this leads to decrement in the thermal performance enhancement factor, this result is similar to base paper.

From fig 7 (B), (C), (D), (E) When the nanofluid is used the average percentage of change in thermal performance enhancement factor are 0.97%, 0.96%, 0.94% and 0.93% for volumetric concentration of 1%, 2%, 3% and 4% of nanoparticle respectively.

5 CONCLUSIONS

A numerical study has been carried out for tube and tube heat exchanger with continuous helical baffle on annulus side subjected to different boundary conditions. Heat transfer coefficient, product of Nusselt number and Prandtl number, pressure drop, thermal enhancement factor with respect to mass flow rate for different baffle spacing is plotted. Following observations has been noticed-

- Increasing the mass flow rate increases the heat transfer coefficient on annulus side.
- With decreasing the baffle spacing heat transfer coefficient is increased.
- Increasing the mass flow rate increases the pressure drop on annulus side.
- With decreasing the baffle spacing pressure drop is exponentially increased
- The product of Nusselt number and Prandtl number increases with decreasing baffle spacing and increasing mass flow rate.
- Overall the thermal enhancement factor for the mass flow rate of 0.3 kg/s and baffle spacing of 33.33mm is the most balanced of all the results.

When Al_2O_3 nanofluid is used on annulus side with different boundary conditions following observations has been noticed-

- As the concentration of nanofluid is increased the thermal conductivity of the nanofluid is enhanced hence the heat transfer coefficient is also increased. The effect of mass flow rate and baffle spacing remains the same as when water is used in annulus tube.
- Pressure drop is directly proportional to friction factor but inversely proportional to density of nanofluid hence when comparing at the same mass flow rate as used in the base paper [8] the pressure drop decreases with increasing volumetric concentration of the nanofluid because the increase of density is greater as compared to friction factor.

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