VIBRATION ANALYSIS OF SIMPLY SUPPORTED BEAM WITH VARYING CRACK DEPTH AND LOCATION BY USING ANSYS

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Abstract - *The existence of damage in a structure results in changes of global dynamic characteristics. Therefore, relatively simple vibration measurements of a structural response and extraction of information on natural frequencies, damping or mode shapes, make damage detection feasible. Presence of crack in a structure cause vibration in the system, thus an accurate and comprehensive investigation about vibration of cracked dynamic structures seems to be necessary. This paper deals with the damage assessment technique for the non-destructive detection and size estimation of cracks in a simply supported beam using finite element method of analysis by ANSYS programme. The purpose of the study is to present the results of experimental and numerical analyses of damage detection.*

Key Words: **Natural Frequency, Mode shape, Crack, ANSYS**

1. INTRODUCTION

During the service life of civil structure, damage identification plays a vital role by providing timely damage assessment, which improves safety and maintains high performance and reliability for civil structures. Many researchers develop various non-destructive methodologies for early detection of crack location & depth; size and pattern of damage in a structure have been in use worldwide. The recent trends in crack detection in beam like structures are generally done by using fuzzy logic, neural network, artificial intelligence...etc. These methods are based on the fact that modal parameters (notably frequencies and mode shapes, and modal damping) are functions of the physical properties of the structure (mass, damping, and stiffness). Any change in the physical properties, such as reduction in stiffness resulting from cracking or loosening of a connection, will cause detectable change in the modal properties. The vibration data such as frequency and mode shape are very important parameters for detecting the damage in structures and a number of research works was carried out on detection of damage using frequency or mode shape.

2. LITERATURE REVIEW

The research in the past few decades on cracked structures and rotors is well documented in a review paper by H.R.Oz [1] considered the transverse vibrations of an Euler-Bernoullibeam carrying a concentrated mass and assumed six boundary conditions and eleven different mass location for calculating the natural frequencies by FEM analysis. Cubic interpolation function for the vertical displacement is taken. The approximate results are compared with exact and other approximate solutions and concluded that the FEM results are very close to the exact frequency value also increasing the no of element increases the accuracy of the natural frequency. P.N Saavendra& L.A. Cuitino [2] evaluated the additional flexibility that the crack can be generated in its vicinity by using the strain energy density function given by linear fracture mechanic theory. Using the linear fracture mechanics theory a new cracked element stiffness matrix is developed which can be used in a standard finite element programme in order to analyse the static and dynamic behavior of different structure. E. Viola et. al. [3] investigated the effect of crack on the stiffness matrix and consistent mass matrix for the crack beam element by the help of FEM analysis.by examining the dynamic behavior of the structure due to the local flexibility introduced by the cracks, the crack position and magnitude are identified. J. T Kim & Stubbs [4] presented a methodology to non-destructively locate and estimate size of crack in structure or which only few natural frequencies are available. The proposed methodology was presented in two parts. The first part outlined the location and size of crack directly from change in frequencies of the structure. And the second part demonstrated the feasibility and practicality of crack detection by accurately locating and sizing cracks in test beam. The authors concluded that it is possible to localize a crack and estimate the crack size in a beam type structure with knowledge of natural frequencies of only a few natural frequencies measured before and after damage. N. Dharmaraju et al. [5] considered

Euler-Bernoulli beam element in the finite element analysis. In this the transverse surface crack is considered to remain open. A local compliance matrix of four degrees of freedom is considered for the modelling of a crack. This compliance matrix contains diagonal and off-diagonal terms. A harmonic force of given amplitude and frequency is used to excite dynamically the beam. The present identification algorithms have been illustrated through numerical examples. Hai-Ping Lin [6] has studied an analytical transfer matrix method, is used to solve the direct and inverse problems of simply supported beams with an open crack. The crack is modelled as a rotational spring with sectional flexibility. The natural frequencies of a cracked system can easily be obtained through many of the structural testing methods. When any two natural frequencies of a cracked simply supported beam are obtained from measurements, the location and the sectional flexibility of the crack can then be determined from the identification equation and the characteristic equation. A.S. Bouboulas, N.K. Anifantis [7] used the FEM method to evaluate different crack scenarios without changing the discretization. a cracked element is developed incorporating a non-propagating edge crack into a structural beam member. The additional flexibility that the crack generates is evaluated using strain energy density factors given by linear fracture mechanics. Based on this flexibility the stiffness matrix of the cracked beam element is deduced. The governing equations of the element were treated as in the usual way of finite element procedures. In that formulation, the crack depth and location were considered as independent global design parameters. Thus, that element was appropriate for static and dynamic analysis of skeletal structures with crack-like damages. The convergence, accuracy and computational efficiency of the presented element are validated through comparisons with experimental and numerical results available from the literature. Christides and Barr [8] developed a one-dimensional cracked beam theory at same level of approximation as Bernoulli-Euler beam theory. Dimaragonas [9] presented a review on the topic of vibration of cracked structures. His review contains vibration of cracked rotors, bars, beams, plates, pipes, blades and shells. Shen and Chu [10] and Chati, Rand and Mukherjee [11] extended the cracked beam theory to account for opening and closing of the crack, the so called "breathing crack" model. A.S. Sekhar [12] has examined different multiple-cracks, their Effects, identification method in vibration structures such as beams, rotors, pipes, etc. It brings out the state of research on multiple cracks effects and their recognition.

3. BASIC EQUATIONS

Consider the beam cross-section with various forces acting on it. Net force on the element

$$Q - \left[Q + \frac{\partial Q}{\partial x}dx\right] = dmacceleration \ldots\ldots\ldots\ldots (1)$$

$$-\frac{\partial Q}{\partial x}dx = (\rho A dx)\frac{d^2y}{dt^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$\frac{\partial Q}{\partial x} + \rho A\frac{\partial^2 y}{\partial t^2} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Considering the moments about A, we get

$$M - \left[M + \frac{\partial M}{\partial x}dx\right] + \left[Q + \frac{\partial Q}{\partial x}dx\right]dx = 0 \ldots\ldots\ldots\ldots (11)$$

$$-\frac{\partial M}{\partial x} + Q + \frac{\partial Q}{\partial x}dx = 0 \ldots\ldots\ldots (4)$$

Neglecting higher order derivatives

$$\frac{\partial Q}{\partial x}dx = 0 \qquad \ldots\ldots\ldots (5)$$

Hence $Q = \frac{\partial M}{\partial x}$

$$\frac{\partial Q}{\partial x} = \frac{\partial^2 M}{\partial x^2} \qquad \ldots\ldots\ldots (6)$$

From equation (1) & (2)

$$\frac{\partial^2 M}{\partial x^2} = \rho A\frac{\partial^2 y}{\partial t^2} \qquad \ldots\ldots\ldots (7)$$

From Beam theory

$$M = -EI\frac{\partial^2 y}{\partial x^2} \ldots\ldots\ldots (8)$$

$$\frac{\partial^2 M}{\partial x^2} = -EI\frac{\partial^2 y}{\partial x^2} \ldots\ldots (9)$$

Comparing equation (3) & (4)

$$EI\frac{\partial^4 y}{\partial x^4} + \rho A\frac{\partial^2 y}{\partial t^2} = 0 \ldots\ldots\ldots (10)$$

$$\frac{\partial^4 y}{\partial x^4} + \left(\frac{\rho A}{EI}\right)\frac{\partial^2 y}{\partial t^2} = 0 \ldots\ldots\ldots (11)$$

Equation (5) is governing partial differential equation for transverse vibration of any uniform beam.

Normal mode solution of above equation is,

$Y(x,t) = X(x).e^{-i\omega_1}$ ....(12)

This makes equation (5) as,

$\frac{d^4X}{dx^4} - \frac{\rho A}{EI}\omega^2 X = 0$ ....(13)

The solution [5] of equation (7) is,

$X(x) = C_1\cos(x\lambda) + C_2\sin(x\lambda) + C_3\cosh(x\lambda) + C_4\sinh(x\lambda)$
.......(14)

Where $\lambda = \left(\frac{\rho A}{EI}\omega^2\right)^{\frac{1}{4}}$

For simply supported beam, the displacement and bending moments are zero of both the ends. Thus the boundary conditions are,

$y(0,t) = 0 \ldots at..x = 0$ .... (bc.1)

$y(1,t) = 0 \ldots at..x = i$ ....(bc.2)

$\frac{d^2X}{dx^2} = 0 \ldots at..x = 0$ ....(bc.3)

$\frac{d^2X}{dx^2} = 0 \ldots at..x = 1$ ....(bc.4)

Application of boundary condition (1) gives,

$0 = C_1 + C_2$ ....(15)

Application of boundary condition (3) gives,

$0 = -C_1 + C_2$ ....(16)

Thus, $C_1 = C_2 = 0$

Now equation (7) becomes

$X(x) = C_4\sin(x\lambda) - C_4\sinh(x\lambda)$ ....(17)

Application of boundary condition (2) gives

$0 = C_2\sin(l\lambda) + C_4\sinh(l\lambda)$........(18)

Application of boundary condition (4) gives

$\frac{d^2X}{dx^2} = -\lambda C_2\sin(l\lambda) + \lambda C_4\sinh(l\lambda)$ ....(19)

$0 = -\lambda^2 C_2\sin(l\lambda) + \lambda^2 C_4\sinh(l\lambda)$ ....(20)

Solution of the above equation is possible if,

$\sin(l\lambda) = 0$; $\sin(0) = 0$;$\sin(\pi) = 0$;$\sin(2\pi) = 0$ .....

$\sin(n\pi) = 0$

Thus $l\lambda = n\pi$ and hence $\lambda = \frac{n\pi}{l}$

Now from equation (8)

$\lambda = \left(\frac{\rho A}{EI}\omega^2\right)^{\frac{1}{4}} \omega_\pi = (n\pi)^2\sqrt{\frac{EI}{\rho A l^4}}$ .... (21)

4. FEA SIMULATION USING ANSYS

Finite element model of a simply supported beam with or without crack is developed in ANSYS environment. The dimensions of the beam are shown in Fig. Structural steel beams have been considered for making Specimens. The modulus of elasticity and densities of beams have been measured to be 69GPa and 2643 Kg/m³ respectively. Theoretical analysis is done by Euler's beam theory.

Table -1:

Density (kg/m³)	2643
Poisson's Ratio	0.28
Elasticity Modulus(Gpa)	69
Length (mm)	600
Thickness (mm)	25
Depth (mm)	25

(Properties of Beam)

4.1MODEL OF UNCRACKED BEAM BY ANSYS

Finite element software, ANSYS version 17.2 is used For free vibration analysis of the crack free and cracked beams. In order to perform numerical analysis, modal and structural analysis of the beam is performed following the steps.

In ANSYS workspace after selecting static structural preference the constant material properties like (density, young's modulus, poisson's ratio) are defined as per thesis requirement. Then the geometry of the beam(length ,breath and height) was created as decided for the thesis and the dimensioning was properly done .The entire beam was converted to elemental mess in the " generate mess step" .the mess type was chosen to be course from the drop down menu. The boundary conditions (support conditions) for the beam – analysis were introduced .Finally the solutions were obtained in "solver mode".

Beam length, thickness and depth are along X axis, Y axis and Z axis respectively in ANSYS coordinate system. A 3400-node three dimension structural solid element under SOLID 540 was selected to model the beam because it is suitable for all structural analysis and it is mid node element which gives the more accurate result. Fig shows finite element model of a cracked beam. The modal analysis of cracked and crack free beams are performed. First 8 modes have been selected as for both crack and crack free beam as first, third and fifth natural frequencies correspond to first three natural frequencies in the transverse direction (Y direction) of vibration. The corresponding mode shapes for both cracked and cracked free beam are also captured. In order to determine the mode shape and modal displacement, node points at the bottom surface of the beam are considered. Distance of each sample point along the lengths is assumed to be 75 mm. First point is taken at fixed end. Since length of the beam is 600 mm, the total number of sample data point is 115. The mode shape of the beam is obtained by plotting transverse displacement (Y direction) of the beam at each sample data point against its position along X direction. The first three mode shape of cracked beam with natural frequency and nodal displacements of the sample have been performed for various crack depth at different location and The output values of these simulations are used as training data for the ANN analysis.

5. RESULTS AND DISCUSSIONS

Initially, a simply supported un cracked beam (600×25×25) mm was analysed by the ANSYS to find out the natural frequency and mode shape. To cross check the ANSYS result the natural frequency of that beam again calculated by the theoretical formula i.e.-

$$\omega_n = (n\pi)^2\sqrt{\frac{EI}{\rho A l^4}}$$

where $n_1\pi$= 3.141 ,$n_2\pi$ =6.282, $n_3\pi$ =9.423
The results of ANSYS for un cracked beam have been checked with the theoretical value which was obtained as follows.

Natural Frequencies of free vibration of uncracked beam

Table: 2

Mode no	**1**	**2**	**3**
Theoretical results(Hz)	160.808	643.5	1447.9
ANSYS results(Hz)	159.66	622.5	1294.2

From the above table it is observed that the experimental results for natural frequency of un-cracked beam are close to the results obtained by theoretical formula. As the difference between the results i.e. theoretical and ANSYS is very less so this value is acceptable.

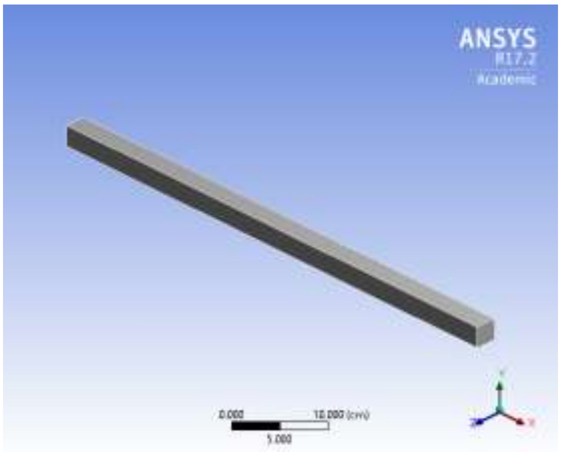

Fig:1 Un-cracked simply supported beam

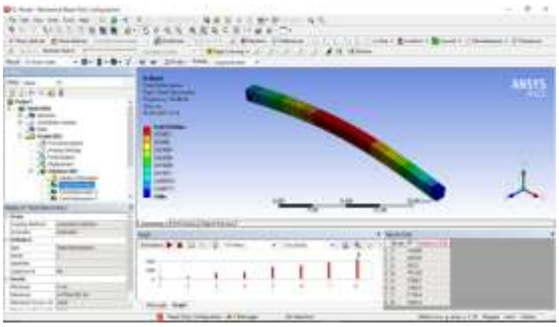

Fig:2 First mode shape of un-cracked simply supported beam

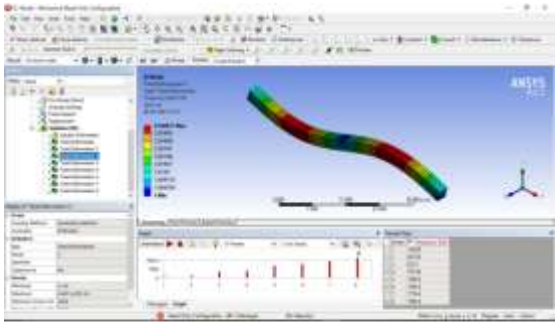

Fig:3 Second mode shape of un cracked simply supported beam

Fig:4 Third mode shape of un-cracked beam

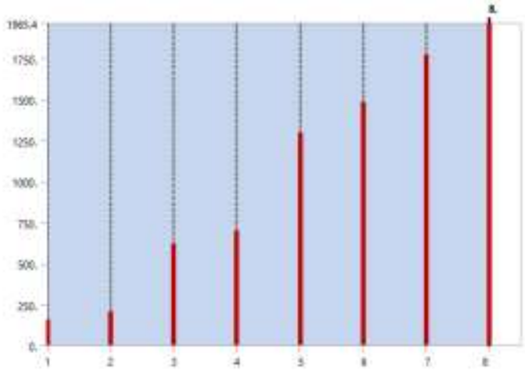

Fig:5 Frequency at each calculated mode.

5.1ANSYS MODEL FOR CRACKED BEAM

Cracks in the beam create changes in geometrical properties so it becomes complex to study the effect of cracks in the beam. The crack modelling has been very important aspect. The analysis has been done using finite element method. FEM software package ANSYS 17.2 has been used. Cracked beam has been modelled and free vibration analysis has been performed considering geometric and material non linearity. The crack is considered to be an open edge Crack with 1mm width on the top surface of the beam has been modelled. It is assumed that crack have uniform depth across the width of the beam.

For modelling of cracked beam the process is same as un cracked beam except while doing geometry an open transverse single crack of 1mm width is taken at the top surface of the beam. Here the cracks are developed at locations of 0.50L of the beam(where L is the length of the beam) with varying crack depth of4mm, 8mm, 12mm, 16mm, 20mm & 24mm of the beam. A total of 6 models are analysed and some of the models are shown in fig. The modal values i.e. natural frequency for 1st mode, 2nd mode , 3rd mode and its corresponding mode shape values each model are found out.

The ANSYS results for cracked beam for crack location at centre (300 mm) with varying depth have been shown in Table.

Table-3:

SL NO.	DEPTH OF CRACK in mm	NATURAL FREQUENCIES in Hz		
		N1	N2	N3
1	4	157.67	622.33	1476.1
2	8	152.82	622.33	1465.3
3	12	142.87	622.05	1446.5
4	16	122.89	621.36	1417.5
5	20	86.381	619.86	1372.9
6	24	22.761	615.89	1267.7

Table-4:

SL NO.	**DEPTH OF CRACK** IN MM	MAXIMUM DEFLECTION in MM		
		D1	D2	D3
1	4	0.04510	0.04454	0.047951
2	8	0.04577	0.04445	0.047595
3	12	0.04703	0.04429	0.047892
4	16	0.04922	0.04475	0.049181
5	20	0.05227	0.04566	0.051245
6	24	0.05487	0.04709	0.05599

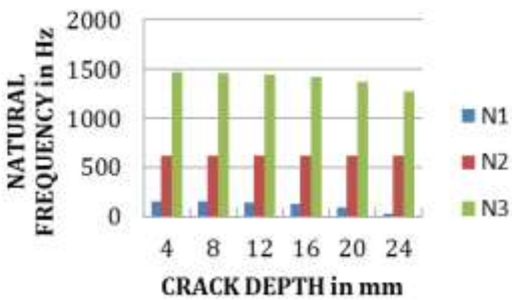

Fig:6 Natural frequency vs. crack depth

6. CONCLUSION

The vibration analysis of a structure holds a lot of significance in its designing and performance over a period of time. Detailed analytical investigations of the effect of crack on the first three modes of vibrating simply supported beam have been presented in this paper. The vibration behavior of the beam is shown to be very sensitive to the crack location, crack depth and mode number. It has been observed that natural frequency change subsequently due to presence of crack depending upon location and size of crack. In case of cracks the frequencies of vibration decreases with increase of relative crack depth and crack location. The results obtained are accurate and expected to be useful to other researchers for comparison. The study in this work is necessary for a correct and thorough understanding of the vibration analysis of a simply supported single crack beam.

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