Harmonic Analysis of Cantilever Beam with and without Cracks

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Abstract - *Crack is one of the serious breakdowns that decrease the serviceable life and lead to collapse of the structures and structural element. Today, failure of structural element and structures mostly takes place due to component debility. Cracks present on the surface of the structural element like beam, accounts changes in stiffness, significantly depending upon the cracks depth and location. Unique features of vibration can be utilized to perceive crack in beams. The presence of cracks which impact the demonstration of structures as well as the vibration parameters like modal natural frequencies, mode shapes. By modal analysis the vibration parameters such as Eigen frequencies and mode shapes are obtained. In the present work, harmonic analysis of cantilever steel and concrete beam cracked, un-cracked and multiple cracks at different depths and locations are analyzed using SAP 2000 software, the repercussion of crack parameters (crack position and crack depth) on the vibration parameters of a cracked cantilever beam are inspected by unique systems using finite element analysis (FEA), using SAP2000 software.*

***Key Words*: Crack deepness, Natural Frequency, Mode Shape, Harmonic analysis, SAP, Cantilever beam etc...**

1. INTRODUCTION

Every structure and its structural elements experiences cracks or damages during its entire life span. Cracks or damages occur either on the surface or inside the structural element. Cracks are visibly repulsive and bothersome for inhabitants, and if the cracks are neglected they can impact the rectitude, security, durability and firmness of the building. Cracks present on the surface of the structural element like beam, accounts changes in stiffness, significantly depending upon the cracks depth and location. These changes, results in a consequential effect on vibration performance of a whole structure. To make sure the secure performance of the structures, it is very essential to know whether their structural elements are rid of cracks, and even if crack is present evaluate their boundary. The approaches largely used for identification of cracks are direct processes such as ultrasound, X-rays, etc. Anyhow, these approaches have confirmed to be inactive and inappropriate in inevitable circumstances, because they need costly and in-detailed investigations. In-order to overcome these kinds of inconveniences, in recent years, investigators has concentrated on higher qualified methods in detecting cracks that is using 'vibration installed damage discover method'.

The vibration based technique acts on this determined influence of mode shapes and frequency. The mode shapes and natural frequencies can be determined either by experimentation or analysis that it would be able to know the global actions of the assembly. The application of vibration based technique has been discovered by a lot of investigators above different procedures because they are non-destructive, economical, quick and for the discovery of one or more cracks in varying constructions and also for detecting the cracks that exists inside the structure which are not visible to naked eye. This method also evaluating the durability of the structures by utilizing assessment of the constructional natural frequencies is explained. It has been displayed in what way evaluations made at one point in the building can be implemented to identify, locate and determine the damage. The system given utilizes finite-element analysis (FEA), because this technique will be allowed to use on any construction.

1.1 Objectives

The main objectives of the present study are as follows:

- To construct the cantilever steel and concrete beam models with appropriate dimensions and to know its response using SAP2000 software.
- To observe the behaviour of fissured, without fissure and multiple fissured beam.
- Harmonic analysis is performed to obtain and to study the response which includes Eigen frequencies, Eigen vectors and displacement of wholesome and fissured beam, for one-of-a-kind fissure places and distance downwards and for multiple fissures of each Cantilever steel and concrete beam.
- And evaluation made between the variations of first three modes of herbal frequencies with fissure intensity whilst fissure area changes.

1.2 Methodology

In this paper discussion made on simple harmonic analysis of a healthy Cantilever steel and concrete beam and a beam with cracks at different locations and depths and for multiple cracks using SAP2000 software. Analysis is carried out for the models are as follows;

MODEL 1: Cantilever steel beam of box section is modeled without and with cracks at different depths (10, 20, 30 mm) and locations (0.5, 1.5, 2.5 m) and multiple cracks.
MODEL 2: Cantilever concrete beam is modeled without and with cracks at different locations (0.2m, 0.5m, 0.8m) and distance downwards (10, 20, 30mm)and multiple cracks.

Table-1: Properties of the beam

Steel Beam

Length of the beam	3 m
Width of the beam	0.092 m
Height of the beam	0.172 m
Thickness of the beam	0.0054 m
Modulus of elasticity	210 Gpa
Poisons ratio	0.3
Density	7850 kg/m³

Concrete Beam

Length of the beam	1 m
Width of the beam	0.05 m
Depth of the beam	0.1 m
Modulus of elasticity	22.36*10⁶ kN/m²
Poisons ratio	0.2
Density	25kN/m³

2. Modal Analysis

The essential accessory in shudder evaluation is modal evaluation. To figure out Eigen frequencies and Eigen vectors of systems and its structural element is the main purpose of the modal evaluation. Eigen frequencies and Eigen vectors of both fissured and without fissure beams through the use of SAP software program has been obtained.

For harmonic analysis following steps considered are as follows:

- Developing the model (cantilever beam) and defining material properties and section properties.
- Meshing and assigning the boundary condition.
- Harmonic aid is utilized to measure outcome for displacement, natural frequency.
- Set harmonic analysis and analyze the model.
- Evaluate the outcome in the fashion of pictorial representation.

2.1 Equation of motion

$M\ddot{X} + C\dot{X} + KX = F(t)$ - For Damped system............. (1)

$M\ddot{X} + KX = F(t)$ - For Un-damped system............ (2)

$M\ddot{X} + KX = 0$ – Free vibration...................................... (3)

<u>Harmonic Analysis</u>

If load is harmonic the response is also harmonic

$F(t) = F_0 e^{iwt}$ .... (4)

Where,

$e^{iwt} = \cos\omega t + i\sin\omega t$

ω – Excitation frequency

t – Time period; **K** – Stiffness matrix; **M** – Mass matrix;

C = **αM** +**βK** – Damping matrix

Excitation Force $F(t) = F_0 e^{iwt}$

Corresponding response is expressed as

$X(t) = X_0 e^{iwt}$ .... (5)

$\dot{X}(t) = X_0 iwe^{iwt}$ .... (6)

$\ddot{X}(t) = X_0 i^2 w^2 e^{iwt}$

$= -X_0 w^2 e^{iwt}$ .... (7)

Substituting equations (5), (6), (7) in (1)

$$M\ddot{X} + C\dot{X} + KX = F(t)$$

$$\{M \times (-X_0 w^2 e^{iwt}) + C \times (X_0 iwe^{iwt}) + K \times (X_0 e^{iwt})\} = F_0 e^{iwt}$$

$$X_0 \times (-M \times w^2 + C \times iw + K) = F_0$$

$$X_0 = \frac{F_0}{(-M \times w^2 + C \times iw + K)}$$

$$(X_0 = X_0(\omega) \text{ For } F_0 = 1)$$

$$X_0(\omega) = \frac{1}{(-M \times w^2 + C \times iw + K)}$$

The progressed strength and weight matrix due to destructions are demonstrated as

$$(K) = (K_0) \pm (\Delta K); \qquad (M) = (M_0) \pm (\Delta M)$$

Where,

(ΔK) = Alternate in strength due to harm

(ΔM) = Alternate weight due to harm

(K_0) = Unique strength matrix of size *n* x *n*

(M_0) = Unique regular weight matrix of size *n* x *n*

As a result damping matrix can be acquired as $(C_0) = \alpha(M_0) + \beta(K_0)$

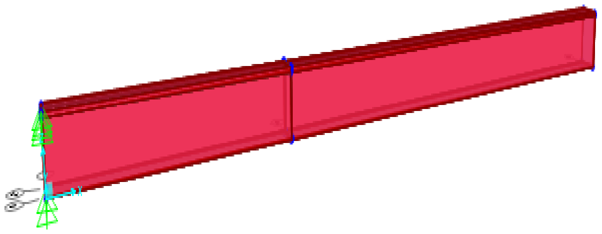

Fig -1: 2D view of Cantilever Healthy Steel Beam

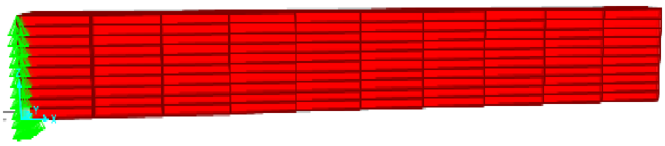

Fig -2: 2D view of Cantilever Healthy Concrete Beam

3. Results and Discussion

In this part cantilever steel and concrete beam models with, without cracks and multiple cracks are developed as shown in fig- 1 and 2. Eigen frequencies, Eigen vectors and displacement of fissured and without fissured beams are obtained by performing modal analysis using SAP2000 software. The crack locations considered for the beam models as 0.5, 1.5, 2.5m for steel and 0.2, 0.5, 0.8m for concrete, for these crack locations crack deepness considered are 10, 20, 30mm.

The comparison for displacement v/s frequency of healthy and cracked beams at different locations graphs has been plotted below.

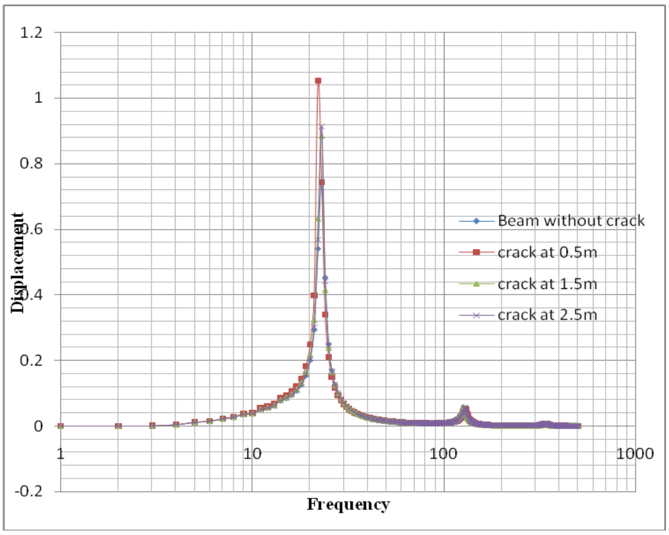

Chart -1: The comparison graph of Frequency v/s Displacement of Healthy steel beam and beam with crack at different locations (0.5, 1.5, 2.5m)

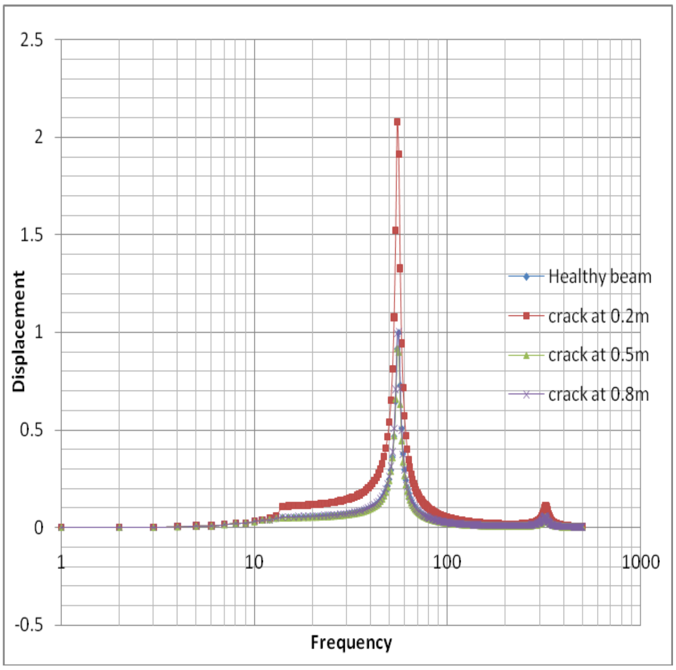

Chart -2: The comparison graph of Frequency v/s Displacement of Healthy concrete beam and beam with crack at unlike locations (0.2, 0.5, 0.8m).

3.1 Eigen frequency of the fissured beam has been reduced compared to a beam without fissure and the Eigen frequency of the beam with fissure near to the fixed support (0.5m) has been in large part decreased in comparison to a beams with fissure away from the support (1.5m, 2.5m) as shown below.

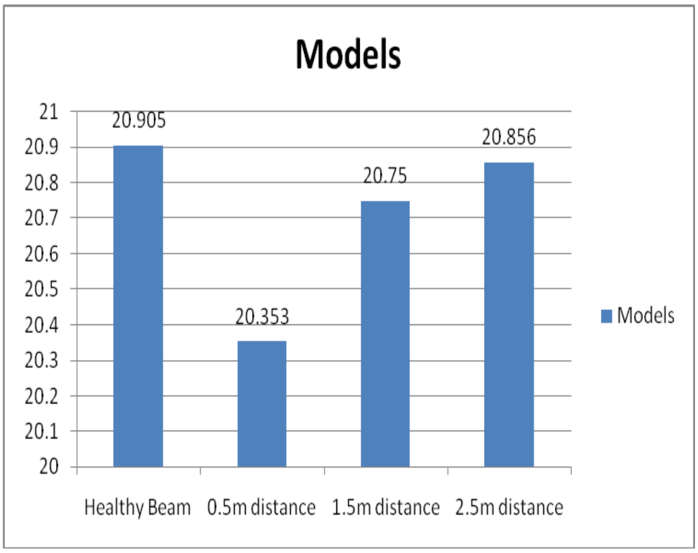

Chart -3: Comparative graph for Variation of Natural Frequency of un-cracked and cracked steel beam at non-identical crack locations

3.2 Eigen frequency of the fissured beam has been reduced compared to a beam without fissure and Eigen frequency of beam with fissure near to the fixed support (0.2m) has been in large part decreased in comparison to a beams with fissure away from the support (0.5m, 0.8m).

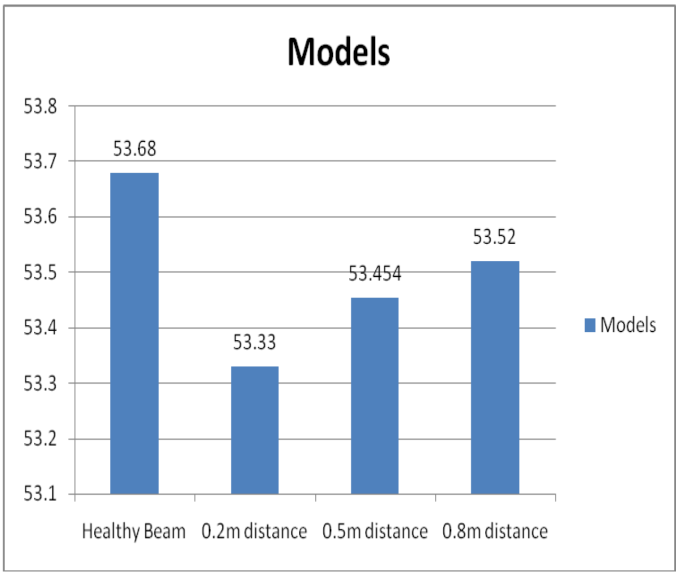

Chart -4: Comparative graph for Variation of Natural Frequency of un-cracked and cracked concrete beam at non-identical crack locations.

4. Conclusions

Based on the results obtained the variations of Eigen frequency and Eigen vectors because of presence of crack on cantilever beam structural element, following conclusions are done:

- Variations in Eigen frequencies and Eigen vectors are because of variation in crack specifications along with crack vicinity and crack deepness.
- Crack existing near to the fixed end support of the cantilever beam decreases the Eigen frequency than compared to crack existing away from the fixed support. In this study from chart 3 - 4 we can observe that reduction in natural frequency is more at crack location 0.2m for concrete and 0.5m for steel compared to 0.5m, 0.8m(concrete)and 1.5m, 2.5m(steel).
- Displacement will be more for cracked beam compared to healthy beam because of reduction in stiffness.
  In the present study from chart 1 and 2 we can observe crack (0.5m steel and 0.2m concrete) located near to the fixed end support has high displacement than crack located away from the fixed support (1.5m, 2.5m steel and 0.5m, 0.8m concrete) and healthy beam.

4.1 Scope of future work:

- Different shapes of beams (I-Beam, round, pipe).
- Structures like communication towers, turbine shafts and structural elements like simply supported beam, fixed beam with point load can be analyzed.
- Damage identification in structures and structural elements for different forms of materials.
- Different forms of cracks and viewpoint.

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