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Semi Prime Ideals in Ordered Join Hyperlattices

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Abstract: In this article we discuss about the semi prime ideals in ordered join hyperlattices.

Introduction: In this paper, we consider order relation ≤ as x ≤ y if and only if y = x ∧ y for all x, y Є L, and we introduce semi prime ideals in ordered join hyperlattices. Here, we give some results about them.

I. Preliminaries

Definition 1.1:

Let H be a non-empty set. A Hyperoperation on H is a map ◦ from H□H to P*(H), the family of non-empty subsets of H. The Couple (H, ◦) is called a hypergroupoid . For any two non-empty subsets A and B of H and x Є H, we define A ◦ B = $\cup_{a\in A, b\in B}$ a ◦ b;

A ◦ x = A ◦ {x} and x ◦ B = {x} ◦ B

A Hypergroupoid (H, ◦) is called a Semihypergroup if for all a, b, c of H we have (a ◦ b) ◦ c = a ◦ (b ◦ c). Moreover, if for any element a ∈ H equalities

A ◦ H = H ◦ a = H holds, then (H, ◦) is called a Hypergroup.

Definition 1.2:

Let L be a non-empty set, □: L × L → p* (L) be a hyperoperation and ∧ : L × L →L be an operation. Then (L, ∧, □) is a join Hyperlattices if for all x, y, z Є L. The following conditions are satisfied:

1) x Є x □ x and x = x ∧ x
2) x ∧ (y ∧ z) = (x ∧ y) ∧ z and x □ (y □ z) = (x □ y) □ z
3) x ∧ y = y ∧ x and x □ y = y □ x
4) x Є x □ (x ∧ y) ∩ x ∧ (x □ y)

Definition 1.3:

An Ideal [1] P of a join hyperlattices L is Prime [2] if for all x, y Є L and x ∧ y Є P, we have x Є P and y Є P.

Proposition 1.4:

Let L be a join hyperlattices. A subset P of a hyperlattice L is prime if an only if L\P is a subhyperlattice of L.

Definition 1.5:

Let (L, ∧, □, ≤) be an ordered join hyperlattices and I C L be an ideal and F be a filter of L. We call I is a semiprime ideal if for every x, y, z Є L, (x ∧ y) Є I or (x ∧ z) Є I implies that x ∧ (y □ z) C I. Also, we call F is a semiprime filter if x □ y C F or x □ z C F implies that x □ (y ∧ z) C F.

II. Properties of semi prime ideals in ordered join hyperlattices [4]

Every Prime ideal I is semi prime [3]. Since if (x ∧ y) Є I or (x ∧ z) Є I, we have x Є I and y Є I or x Є I and z Є I.

If x ЄI, by x ∧ (y □ z) ≤ x we have, x ∧ (y □ z) C I

Otherwise, we have y, z Є I.

So, y □ z C I and x ∧ (y □ z) C I.

Proposition 2.1:

Let (L, ∧, ∨, ≤) be an ordered join hyperlattices and I be a semiprime ideal of L. Also, for any A, B C L, A ≤ B C I implies that A C I. Then, I_1 = {J Є Id(L); J C I} is a semiprime ideal of L. If L is a finite hyperlattice, I_2 = ∪ {J; J C I} is a semiprime ideal of L.

Proof:

LetJ_1, J_2 C I, then J_1 □ J_2 C I □ I.

Since, I is an ideal of L, we have I □ I C I.

Therefore, J_1 □ J_2 C I.

Let J_1 ∧ J_2 C I_1, J_1 ∧ J_3 C I_1 for any J_1, J_2, J_3 Є Id (L).

Then, let x' ∈ J_1 ∧ (J_2 ∨ J_3).

x' = x ∧ y for x Є J_1, y Є J_2 ∨ J_3.

Therefore, y = y' □ y'' for some y' Є J_2 or y'' Є J_3.

We have x ∧ y' Є $J_1 V J_2$ ⊆ I or x ∧ y'' Є J_1∧J_3 ⊆ I.

Since I is semiprime, we have

x ∧ (y' □ y'') ⊆ I and J_1 ∧ (J_2 ∨ J_3) ⊆ I.

If L is finite, we prove that I_2 is a semi prime ideal.

Let x, y Є I_2.

Thus, x Є J_1 ⊆ I or y Є J_2 ⊆ I.

Therefore, x □ y ⊆ J_1 ∨ J_2 ⊆ I.

Let x ≤ y Є J_1 ⊆ I.

Since, I is an ideal, we have

x Є I or x Є I_2.

Since L I finite, I_2 is a semiprime ideal of L.

Theorem 2.2:

Let L be a s-good (x □ 0 = x) bounded ordered join hyperlattices and I be an ideal and F be a filter of L such that I∩F = ∅ and for any A ⊆ F. If F is a semiprime filter, there exists a semiprime ideal J such that I ⊆ J and J∩F = ∅.

Proof:

Let F be a semiprime filter and θ be a congruence on L which is defined as a θ b if and only if

F:a = F:b where F:a = {x Є L; a □ x ⊆ F}.

Then, θ is an equivalence relation.

Now, we show that θ is compatible with **v** and V.

Let a θ b, since F is a semiprime filter, we have

F:a ∧ c = (F:a) ∪ (F:c)

= (F:b) ∪ (F:c)

= F:b ∧ c.

Thus, a ∧ c θ b ∧ c.

Let y ∈ F:a □ c.

Thus, y □ a □ c ⊆ F and therefore,

y □ c ⊆ F:a = F:b.

y □ c □ b ⊆ F and y ∈ F:c □ b.

Therefore, θ is compatible with □.

Clearly, θ is a strongly regular relation and therefore L/θ is a lattice.

Now, we claim that L/θ is a distributive lattice.

Let s θ x ∧ (y □ z) and

u ∈ F:s = F:x ∧ (y □ z).

A = u □ (x ∧ (y □ z)) ⊆ F.

Since L is bounded, we have A ≤ u □ (1 ∧ (y □ 1)) ≤ u □ (y □ 1).

So, we have u □ y ⊆ F or

u □ x ⊆ F.

By semi prime property of F, we have

u □ (x ∧ y) ⊆ F and since

u □ (x ∧ y) ≤ u □ (x ∧ y) □ (x ∧ z).

Therefore, u ∈ F:(x ∧ y) □ (x ∧ z) and

L/θ is a distributive lattice.

Also, in L/θ, we have Iθ ∩ Fθ = φ.

If there exists y ∈ Hθ ∩ Fθ, we have I θ F.

Thus, F:I = F:F and since 0 □ F = 0 ⊆ F, we have 0 ∈ F:I. 0 □ I = 0 ⊆ F which is a contradiction to I ∩ F = φ.

So Iθ ∩ Fθ = φ. Since I ∩ F = φ, there exists Pθ ∈ L/θ such that Iθ ⊆ Pθ where Pθ is a prime ideal.

Let us consider a canonical map h: L → L/θ by h(a) = θ(a).

Therefore, we have I ⊆ h −1 (Pθ) = P,

P ∩ F = φ and

P is a prime ideal of L.

Theorem 2.3:

Let (L, ∧, □, ≤) be an ordered join hyperlattices. L is a distributive hyperlattice if and only if for every ideal I and filter F of L such that I ∩ F = φ, there exist ideal J and filter G of L such that I ⊆ J, F ⊆ G, J ∩ G = φ, J or G is semi prime and for every x ∈ L, we have x ∈ J ∪ G

Proof:

Let L be a distributive hyperlattice. We know that, if (L, ∧, ∨) is a distributive hyperlattice if I and F are ideal and filter, respectively then I ∩ F = φ, then there exist ideal J and filter G of L such that

I ⊆ J, F ⊆ G, then J ∩ G = φ.

Now, we show that L is distributive.

Let x, y, z ∈ L and I be the ideal which is generated by

(x ∧ y) □ (x ∧ z) and F be a filter which is generated by x ∧ (y □ z).

Let, x ∧ (y □ z) ≰ (x ∧ y) □ (x ∧ z).

Therefore I ∩ F = φ.

Then, there exist ideal J and filter G such that I ⊆ J and F ⊆ G, J ∩ G = φ.

If J is semi prime ideal, since

x ∧ y ∈ J or x ∧ z ∈ J, we have x ∧ (y ⍰ z) ⊆ J.

Since x ∧ (y ⍰ z) ⊆ G, we have J ∩ G ≠ φ which is a contradiction.

If G is semi prime, we have x ∈ G or y ⍰ z ⊆ G.

If y ∈ G, since x ∈ G, we have x ∧ y ∈ G, and if z ∈ G, we have x ∧ z ∈ G,

which is a contradiction to J ∩ G = φ.

So neither y nor z are not in G.

If both y, z ∈ J, y ⍰ z ⊆ J.

This is contradiction with J ∩ G = φ.

So both y, z ∈ J is impossible.

Let y not belongs to J and z ∈ J.

We have x ∧ z ∈ J.

Since, x ∧ y ≤ (x ∧ y) ⍰ (x ∧ z) ∈ J,

We have x ∧ y ∈ J.

But x ∧ y ∈ G, and this is contradiction.

Then, we have x ∧ (y ⍰ z) ≤ (x ∧ y) ⍰ (x ∧ z).

Let (x ∧ y) ⍰ (x ∧ z) ≰ x ∧ (y ⍰ z) and I is an ideal which is generated by x ∧ (y ⍰ z), F is a filter which is generated by (x ∧ y) ⍰ (x ∧ z).

Similarly, we arrive at the contradiction and the proof is completed.

III. Conclusion

In this paper we have discussed about the semi prime ideals and their properties in ordered join hyperlattices.

IV. References

[1] http://mathworld.wolfram.com/Ideal.html

[2] https://en.wikipedia.org/wiki/Prime_ideal

[3]http://mathworld.wolfram.com/SemiprimeIdeal.html

[4]https://www.researchgate.net/publication/317743259_Ordered_join_hyperlattices