Performance Comparison of a Small Size Rotor with Self Induced Instability using Sliding Mode and Fractional order Controller

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Abstract— *Dynamic systems like gyro-pendulums and tippe-tops behave in a peculiar unstable manner due to material or internal damping. Dynamic systems beyond certain critical threshold spinning speed may become unstable due to rotating dampers. There has been several efforts in the past to stabilize such systems but most of the earlier efforts to stabilize such rotors had difficulty in stabilizing extremely small size, mini or micro rotors although one may find some efforts to stabilize medium to large size rotors using some heavy actuators like squeeze film dampers etc. The underlying problems with the small size rotors is that if the selection of the actuators is not proper than the actuator size may be heavier than the rotor itself therefore the choice of actuators remains always a challenge in a rotor with micro or mini size.*

Since in the present work the control, sensing and actuation all are embedded within a single embedded coupling due to obvious practical reasons as discussed in the paper the challenge is to see the entire dynamics from a moving frame or rotating coordinate system. Design and practical implementation of these smart structures always remains a big challenge.

Keywords **- internal damping, critical threshold spinning speed, micro or mini rotor, dry friction**

I. INTRODUCTION

Industrial Rotors after a certain threshold or critical spinning speed may become completely unstable due the presence of internal damping or material damping rotating along with the rotor. There has been several efforts previously to stabilize large industrial rotors with heavy actuators but efforts to stabilize small size rotating shafts remained always a challenge and considered a major industrial problem specially for automation in semiconductor industry. Flexible rotors have a coupled dynamics where the motion in one independent direction effects the other lateral direction and developing suitable control law for this always remains a challenge as one may expect complexity in decoupling of the coupled interacting states .

The paper also shows how the critical or threshold spinning speed of the rotor plays a pivotal role in designing the controller as because the control law has the tendency in general to stabilize the shaft by translating the threshold spinning speed way beyond the running speed of the rotor. This work clearly describes in favor of choosing the internal damping as the parameter which needs to be modulated by the controller as against the rotor stiffness and the external damping for obvious practical reasons.

Since the internal damping was purposely chosen as the modulating parameter to achieve stability of a small size rotor therefore while selecting the actuating mechanism one might be careful of not selecting a actuator whose size might outgrow the dimensions of the rotor itself therefore choosing a special type of actuator as the final controlling element properly embedded and aligned in a smart structure already coupled within a rotor system remains a difficult task.

Initially a un-tuned two degree PID like structure with orbital states as the principle feedback mechanism was used and later a tuned variable structure control is used to stabilize rotor of similar size and later conclusions were made on the basis of the simulation results obtained. It may be noted that initially the gains of the PID controller are chosen in a trial and error basis but later a sliding mode variable structure stabilizing controller was designed without essentially using any PID structure but completely based on internal state feedback control.

II. Instability Analysis

One might choose the overall rotor parameters like the internal damping in the rotating frame, as $\mu = \frac{R_i}{2}$ such that its effective value in all directions is 'R_i'. The external damping of the system is taken as $\alpha = \frac{R_a}{2}$ such that the effective damping coefficient in all direction is 'R_a'. The stiffness coefficient $\varepsilon = \frac{K_s}{2}$, such that '$K_s$' is experienced in all directions. If the overall dynamics of the rotor is considered then the dynamic equation can be given as below

$$m\begin{bmatrix}\ddot{X}\\ \ddot{Y}\end{bmatrix} = -K\begin{bmatrix}X\\ Y\end{bmatrix} - \begin{bmatrix}\dot{X}\\ \dot{Y}\end{bmatrix}R_1 + \begin{bmatrix}0 & \omega R_1\\ -\omega R_1 & 0\end{bmatrix}\begin{bmatrix}X\\ Y\end{bmatrix} \quad (1)$$

One might take this peculiar asymmetric force component given below for further analysis.

$$\begin{bmatrix}F_{xc}\\ F_{yc}\end{bmatrix} = \begin{bmatrix}0 & -\omega R_1\\ \omega R_1 & 0\end{bmatrix}\begin{bmatrix}X\\ Y\end{bmatrix}$$

the special characteristics of this circulating force component is that it cannot be actually derived from any potential components. One may then say that like most of the other force components found in nature this force is apparently non-potential $\bar{F}_c \neq -\bar{\nabla}\Phi$ where the function ϕ(x, y) can be defined as some potential function.
Due to the nature of the circulating force component one might also get a non vanishing curl from the said dynamics,

$$\bar{\nabla} \times \bar{F}_c = \left(\frac{\partial}{\partial x_i}\hat{\imath} + \frac{\partial}{\partial y_j}\hat{\jmath}\right) \times (-\omega R_i y\hat{\imath} + \omega R_i x\hat{\jmath}) = 2\omega R_i \hat{k}$$

This has a very important and a significant property as because the work done by the circulating component of this force 'F_c' will be path dependent. We can also say that the net work will be done if the point of applications traces a closed orbit. Therefore the net work done in tracing a closed orbit will be given as,

$$W_c = \oint \bar{F}_c . dr = \iint \bar{\nabla} \times \bar{F}_c . da = \iint (2\omega R_i \hat{k}).(da)\hat{k} = 2\omega R_i A$$

III. Proposed Embedded Coupling and Smart Structure

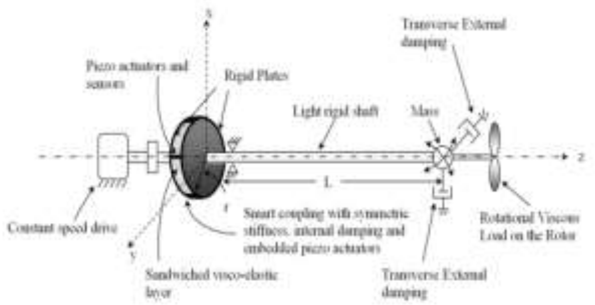

Fig 1: Proposed smart structure of the micro Rotor

If one considers the basic structure of the rotor from the figure Fig 2 then the rotor rotates along 'z' axis and it has flexibilities in both 'X' and 'Y' directions moreover the external damping is attached with the suspension system of the rotor . Both the isotropic stiffness and the internal damping has been taken as a lumped parameters coupled with the smart coupling for ease in modeling the system.

The integral part of the design includes a smart structure with attached sensor, controller and piezo type actuators within a single coupling .The essential feature of the smart coupling is that it rotates along with the rotor.

IV. Rotor Model Building

For doing the geometrical transformation one needs the dynamical model of the rotor which can be recalled as seen below

$$m\begin{bmatrix}\ddot{X}\\ \ddot{Y}\end{bmatrix} = -K\begin{bmatrix}X\\ Y\end{bmatrix} - \begin{bmatrix}\dot{X}\\ \dot{Y}\end{bmatrix}R_i + \begin{bmatrix}0 & \omega R_i\\ -\omega R_i & 0\end{bmatrix}\begin{bmatrix}X\\ Y\end{bmatrix}$$

Here 'θ' is the angle between the rotating and the fixed reference frame which has been used in the transformation
One might then finally get the transformed equation of the form below

$$\begin{bmatrix}\dot{X}_r\\ \dot{Y}_r\end{bmatrix} = \begin{bmatrix}cos\theta & sin\theta\\ -sin\theta & cos\theta\end{bmatrix}\begin{bmatrix}\dot{X}f\\ \dot{Y}f\end{bmatrix} + \begin{bmatrix}-sin\theta & cos\theta\\ -cos\theta & -sin\theta\end{bmatrix}\begin{bmatrix}Xf\\ Yf\end{bmatrix}\dot{\theta}$$

$$\dot{X}_r = (cos\theta)\dot{X}f + (sin\theta)\dot{Y}f + (-sin\theta Xf + cos\theta Yf)\dot{\theta}$$

$$\dot{Y}_r = (-sin\theta)\dot{X}f + (cos\theta)\dot{Y}f + ((-cos\theta)Xf + (-sin\theta)Yf)$$

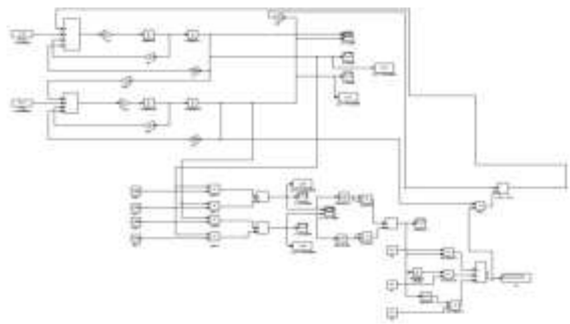

Fig 2 Simulink Model of the Rotor with the attached FOPID Controller

Since piezo actuators are available in small sizes therefore the choice of actuation system is done by embedding a set of piezo crystals properly aligned in the 'X' and 'Y' direction of the two degree of freedom rotor system . For very small size rotor system one might embed the actuator within the rotating smart structure itself.

V. A fractional order PID control for Stabilizing small size rotors based on Conventional tunings methods.

(a) Critical or threshold spinning speed calculation

The smartest way to calculate the critical or threshold spinning speed of the rotor is to compute the regenerative power and the dissipative power and equate the two to address the verge of instability condition. This equivalence of the regenerative and the dissipative power drawn by the shaft helps us to calculate the critical or threshold spinning speed of the rotor which is essential for the controller design.

The relationship for the critical or the threshold spinning speed of the rotor is given as

$$\omega_{th} = \sqrt{\frac{k}{m}\left(1 + \frac{R_a}{R_i}\right)}$$

The rotor beyond this spinning speed tends to become unstable. Here the critical speed depends on the shaft natural frequency $\sqrt{\frac{k}{m}}$, external damping 'R_a' and material or rotating damping 'R_i'.

(b) Comforts for choosing the internal damping as the modulating parameter for the overall stabilization of the rotor

One may increase the shaft stiffness by feeding back shaft displacement. This way of modulation by the controller on the shaft stiffness is impractical and moreover there would be limitations up to which the stiffness could be increased in flexible rotors with limited number of actuators.

The design of the controller can be done in such a way such that one may increase the shaft stationary damping 'R_a' in order to shift the shaft critical spinning speed way beyond the running speed 'ω' of the rotor. But some difficulties will be in this approach too let us say if the range$(\omega - \omega_{th})$ (that is the difference of spinning and instability onset speed) is too high then higher feedback gains are to be given and since 'Ra' already lies in the numerator of the critical speed relation one may find it difficult to generate such large scale external damping values by the controller. Fixing a high value of the external damping may pose certain difficulties as higher value of the external damping may increase the size of the actuation system itself.

Therefore for practical reasons one might modulate the internal damping 'R_i' thus one can re write the overall equivalent damping Reqv=R_i-R_c where R_{eqv} is effective value of the rotating damping. Therefore the relationship for the threshold spinning speed is than given as $\omega_{th} = \omega_n\left(1 + \frac{R_a}{R_i - R_c}\right)$ here 'Rc' is artificially created which is the control command . One may thus modulate the internal damping 'R_i' of the system by feeding back the forces proportional to the velocities in the rotating frame. The overall internal damping of the system may be reduced. This will translate 'wth' way beyond 'w' and stabilization of the rotor may be achieved.

The shaft drawn power may be put into a PID structure to propose a control law and the controller thus will generate signals to modulate the internal damping 'Ri' of the system such that the condition wth > w is attained. The control signal generated may be artificially given as the command parameter R_c. The role of Rc is to interact with the initial internal ramping 'Ri' of the rotor and modulate it through proper actuation system thus designed.

$$R_c = \alpha P + \beta\int_0^t Pdt + \sigma dP/dt$$

VI. PID structure based on orbital speed of the rotor

To achieve stability the equation can be written as R_i = R_i-R_c where 'R_c' can be taken as a design parameter which means that 'Rc' takes the structure of the control law . A control strategy is needed to design along 'R_c'. One possible way is to modulate directly the shaft drawn power that is the amount of power drawn by the shaft which renders it into unstable whirl already shown in the previous section . If 'P_s' is considered as the shaft drawn power which is the actual power responsible for making it unstable then the control law could be well designed as ,

$$R_c = \alpha P_s + \beta\int_0^t P_s dt + \gamma\frac{dP_s}{dt}$$

The following relation explains how the threshold running speed gets modulated

$$\omega_{th} = \omega_n\left(1 + \frac{R_a}{\left(R_i - \alpha P_s - \beta\int_0^t P_s dt - \gamma\frac{dP_s}{dt}\right)}\right)$$

Some disadvantage of the proposed controller is that it is already a frustrating task to measure the actual shaft drawn power 'Ps'. The disadvantage of directly measuring or sensing 'Ps' is that certain fraction of total power will be used for running the additional load and also some power will be lost to the environment via external damping therefore to determine the amount of power which actually renders the shaft unstable is a tedious task . Instead of taking the shaft drawn power 'Ps' as the sensing element one may resort to the orbital speed of the rotor itself. This is a good design as because the orbital speed is proportional to the area of the orbit taken by the rotor which is indirectly proportional to the shaft drawn power 'Ps'

One may sense the orbital area but even better response function would be the velocity response function $\dot{X}_r^2 + \dot{Y}_r^2$ as because the whirl orbiting speed is the natural frequency of the shaft without any damping.

Therefore the overall control structure could now be given as,

$$R_c' = (\alpha * V^2{}_{amp} + \beta * \int_0^t V^2{}_{amp} dt + \sigma * dV^2{}_{amp}/dt)$$

$$\text{where } V^2{}_{amp} = \left(\dot{X}_r * r/L\right)^2 + \left(\dot{Y}_r * r/L\right)^2$$

VII. Fractional PID controller tuning based on conventional frequency methods.

A generalized $PI^{\lambda}D^{\mu}$ controller of the form:
$C(s) = K_p + K_i / s^{\lambda} + K_d s^{\mu}$

Where λ and μ are the fractional orders of the integral and derivative parts of the controller, respectively. Since this kind of controller has five parameters to tune (K_p, K_d, K_i, λ, μ), up to five design specifications for the controlled system can be met, that is, two more than in the case of a conventional PID controller, where λ = 1 and μ = 1.

(a) Phase margin ϕ_m and gain cross-over frequency w_{cg} specifications.

Gain and phase margins have always served as important measures of the robustness. It is known that the phase margin is related to the damping ratio of the system and therefore can also serve as a performance measure. The equations that defines the phase margin and the gain crossover frequency are:
$|C(j\,wcg)G(j\,wcg)|_{dB} = 0$ dB; $\arg(C(j\,w_{cg})G(j\,w_{cg})) = -\pi + \phi_m$

(b) Robustness to variations in the gain of the plant.

The next constraint considered is:
$\frac{d \arg(F(s))}{dw}\Big|_{w=wcg} = 0$

(c) High- frequency noise rejection.

A constraint on the complementary sensitivity function T(jw) can be established:
$\left|T(jw) = \frac{C(jw)G(jw)}{1+C(jw)G(jw)}\right|_{dB} \le A$ dB,
$\forall w \ge w_t$ rad/sec => $|T(jw_t)| = A$ dB
With "A" the desired noise attenuation for frequencies $w \ge w_t$ rad/sec

(d) To ensure a good output disturbance rejection.

A constraint on the sensitivity function S(jw) can be defined as:
$$\left.\begin{array}{l}\left|S(jw) = \frac{1}{1+C(jw)G(jw)}\right|_{dB} \le B \text{ dB}, \\ \forall w \le w_s \text{ rad/sec} \Rightarrow |S(jw_s)| = B \text{ dB}\end{array}\right\}$$

With "B" the desired value of the sensitivity function for frequencies $w \le w_s$ rad/sec (desired frequency range)
After finding out the A, B, C, D matrices the gain parameters $[Kp, Ki, Kd, \mu, \lambda]$
are calculated using MATLAB with the help the 5 design specifications as mentioned earlier.
For optimization purpose, help of FOMCON toolbox has been taken. Now the gain parameters are used to stabilize the controller.
For the following designing specifications:
Phase Margin = 60 degree , Gain Margin =10 dB
Gain crossover frequency=0.10 rad/s
A=-20 dB at w □wt=10rad/s
B=-20 dB at t w □w =.01 rad/s

VIII. A tuned variable structure control design

Given a set of desired poles $\{\lambda_i ;\quad i = 1, 2, \ldots\ldots\ldots\ldots n\}$ the system can be restricted to a sliding surface following the fundamental steps below.

a) Initial goal is to check the existence of the transformation matrix $T(x)$.

$z = \tau(x), \qquad \tau(0) = 0$

For any system model
$\dot{x} = f(x) + g(x)u$
One may also check the inverse $x = T^{-1}(x)$.
One may conceive to design fundamentally the polynomial $T(x)$ and check weather transformation exists.

$$\dot{z}(t) = \frac{\partial T}{\partial x}\dot{x} = \frac{\partial T}{\partial x}(f(x) + g(x)u)$$ and

$$\frac{\partial T}{\partial x} f(x)\Big|_{x=T^{-1}(z)} = \begin{bmatrix} z_2 \\ z_3 \\ \vdots \\ z_n \\ \bar{f}_n(z_1, z_2, \ldots\ldots, z_n) \end{bmatrix}$$

$$\frac{\partial T}{\partial x} g(x)\Big|_{x=T^{-1}(z)} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} z_2 \\ z_3 \\ \vdots \\ z_n \\ \bar{f}_n(z_1, z_2, \ldots\ldots, z_n) \end{bmatrix} = \begin{bmatrix} T_2 \\ T_3 \\ \vdots \\ T_n \\ \bar{f}_n(z_1, z_2, \ldots\ldots, z_n) \end{bmatrix}$$

Considering the equation in matrix form,
Because $z = \tau(x)$□one may conclude

$$\frac{\partial \tau_i}{\partial x} f(x) = \tau_{i+1}, \qquad for\;\; i = 1, 2, \ldots\ldots, n-1$$

Or equivalently $\tau_{i+1} = \alpha_f \tau_i, \qquad i = 1, 2, \ldots\ldots, n-1$

Therefore introducing the Lee derivatives the final transformation $z = \tau(x)$ has the form

$$T = \begin{bmatrix} T_1 \\ L_f T_1 \\ \vdots \\ L_f^{n-2} T_1 \\ L_f^{n-1} T_1 \end{bmatrix}$$

b) If T exists then $[T, B]$ is invertible.

$$\begin{bmatrix} T & B \end{bmatrix}^{-1} = \begin{bmatrix} T^T \\ B^T \end{bmatrix}$$

c) One may find matrix $f \in R^{m\times(n-m)}$ such that eig $\left(T^T AT - T^T ABF\right) = \left(\lambda_1, \lambda_2, \ldots\ldots, \lambda_{n-m}\right)$

d) Matrix $W = (T - BF)$

e) Final computation of the sliding surface polynomial

$[W \ B]^{-1} = \begin{bmatrix} W^T \\ S \end{bmatrix}$

the last 'm' number of row's of $[W \ B]^{-1}$ could be used for the effective sliding surface $Sx = 0$.

The model equations are as follows:

$$m\binom{X''}{Y''} = -k\binom{X}{Y} - Ri\binom{X'}{Y'} + \begin{bmatrix} 0 & wRi \\ -wRi & 0 \end{bmatrix}\binom{X}{Y}$$

After equating, the above equation, it becomes

$$m\binom{X''}{Y''} = \begin{pmatrix} -kX - RiX' + wRiY \\ -kY - RiY' - wRiX \end{pmatrix}$$

$m\ddot{X}$=-kX-RiX'+wRiY

$m\ddot{X}$=-kY-RiY'+wRi

$$\ddot{Y} = -\frac{Ri}{m}Y' - \frac{k}{m}Y - \frac{wRi}{m}X + uY$$

$$\ddot{X} = -\frac{Ri}{m}X' - \frac{k}{m}X + \frac{wRi}{m}Y + uX$$

Where

k=spring constant=1000

m=mass=0.1

w=angular frequency=660

Ri= internal damping=0.02

σ=sigma

α=alpha

And here, $X = X_1, Y = Y_1$

$X' = X_2, Y' = Y_2$

$$\sigma = 74.4X_1 - 2.05X_2 - 2.93Y_1 + 3.93Y_2$$

$$\alpha = \frac{(2.05 \times 10^4 - 3.93 \times 132)X + (7 + 0.02 \times 2.05)X' - (2.05 \times 132 + 3.93 \times 10^4)Y -}{3.93Y' - 2.05X'}$$

The sliding surface is

u=αsgn(ϵ)

Where ϵ=constant =0.000001

The control logic is

$$\sigma < 0 = \begin{cases} u = \alpha + \epsilon \\ u = \alpha - \varepsilon \end{cases}$$

$$\sigma > 0 = \begin{cases} u = \alpha + \epsilon \\ u = \alpha - \varepsilon \end{cases}$$

IX. Simulation and comparison of results dynamics of an uncontrolled rotor system

The rotor is rotated at a speed beyond 'w_{th}'. There is no eccentricity within the rotor system. There is a initial inertia given to the rotor system in 'X' direction. The plots below shows that the orbit keeps increasing rapidly which means the anti-symmetric non potential nature of the circulating force drives the shaft unstable and the plot below also shows that the shaft drawn power increases monotonically at a very fast rate.

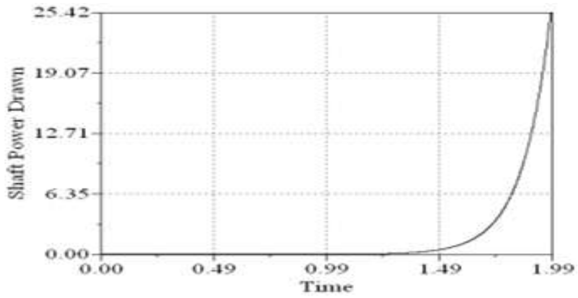

Fig 3. Shaft drawn power

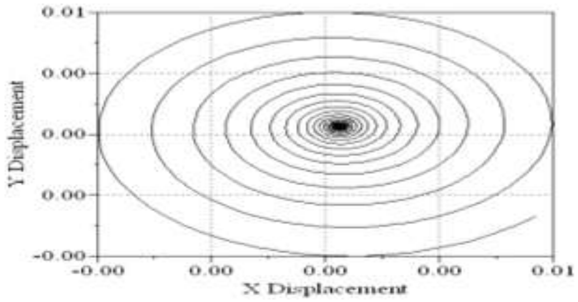

Fig 4. Instability shown in the shaft trajectory

X. Stabilization of the rotor system with typically tuned FOPID controller with the orbital response function as the principle feedback to the controller

The parameters for this set of simulations are Mass=0.1kg, Shaft stiffness=1000.0 N/m, Stationary Damping=0.2 Ns/m, Rotating Damping=0.02 Ns/m, Driving Speed=600.

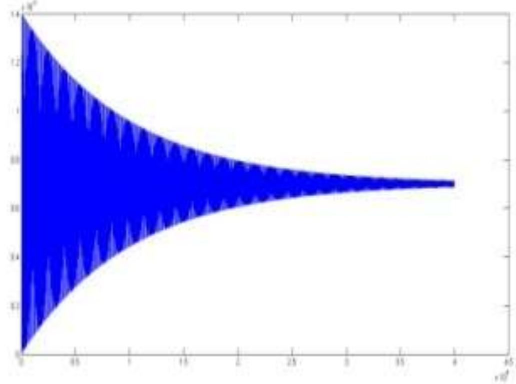

Fig 5. 'X' Displacement Rotor

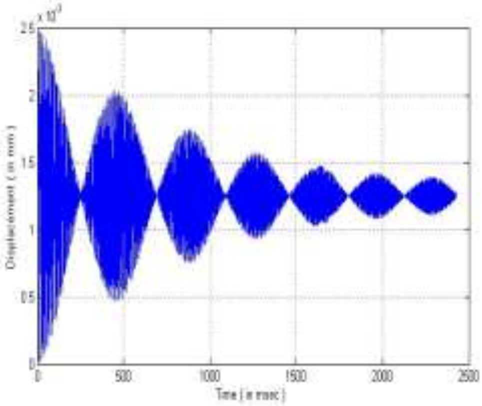

Fig 6. 'Y' displacement of Rotor

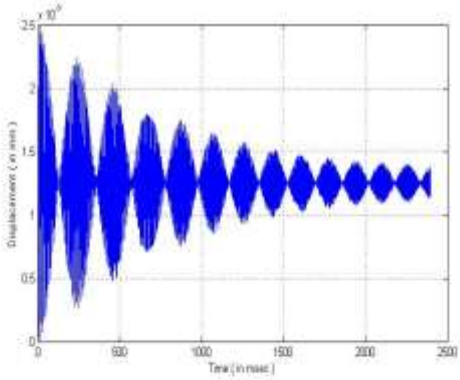

Fig 7. 'X' Displacement Rotor(1200) RPM

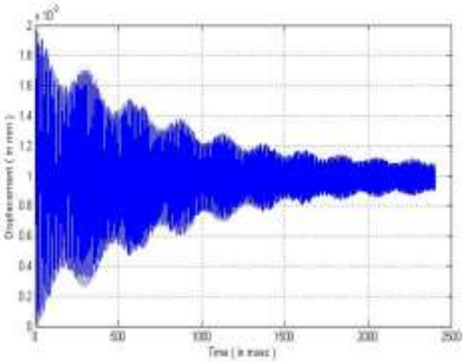

Fig 8. 'Y displacement at 1200 RPM

XI. Stabilization results with tuned variable structure control

The parameters for this set of simulations are Mass=0.1kg, Shaft stiffness=1000.0 N/m, Stationary Damping=0.2 Ns/m, Rotating Damping=0.02 Ns/m, Driving Speed=660 rad/sec.

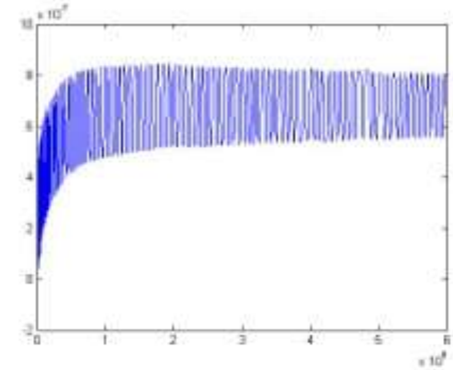

Fig 8. 'X' Displacement Rotor at 660 RPM

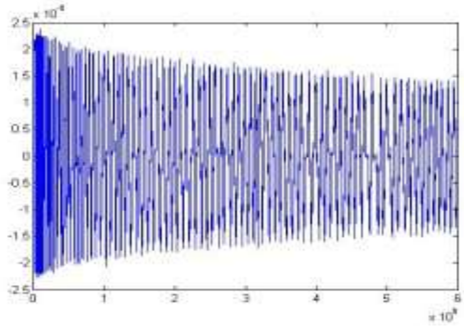

Fig 9. 'Y' Displacement Rotor at 660 RPM

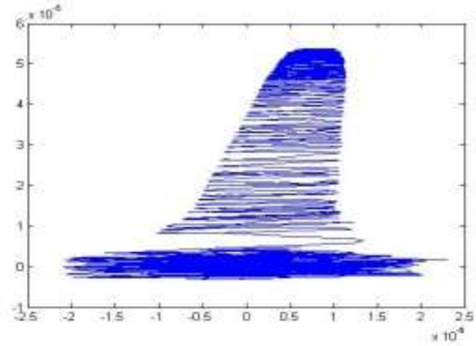

Fig 9 Stabilized phase trajectory of Rotor

One may see that initially the structure of the controller was taken like a PID with orbital states having two independent directions in 'X' and 'Y' the self tuning resulted in a stabilized state upto 10^{-3} mm in convergence but still the controller worked quite efficiently to stabilize rotors of such extreme diameters and finally a well tuned sliding mode controller could even improve the damping level to converge upto 10^{-7} mm .

XII. Conclusion

Instability issues for large size rotors due to internal damping or material damping of the rotor material is quite well known. Control strategies to stabilize such large size rotors with large mechanical structures and dampers are common but highly inefficient due to overhaul cost, maintenance and overall complexity of the system. Here in the paper firstly instability analysis of rotors in general is discussed due to material damping and later a elegant un-tuned PID algorithm is developed which is applicable for relatively small mini or micro size rotor.

. A widely tuned sliding mode controller raised the damping levels of the system such that it gave grossly a much better stabilization in terms of both convergence. It was observed that the convergence of a tuned variable structure control stabilized the 'X' and 'Y' displacement of the rotor in the order of 10^{-7}mm compared to normal PID like structure using orbital function which converged it to only 10^{-3} mm in "X' and 'Y' displacement.

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