Modulus of Elasticity of Unsaturated Soil

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Abstract –

Traditional soil mechanics practice has experienced significant changes during the past few decades. In part, this has been due to increased environmental concerns. The computational capability available to the geotechnical engineer has also strongly influenced practice. In keeping with the above factors, there has been a need to better understand the engineering behavior of the soil near to ground surface. This zone, known as the vadose zone, is subjected to a flux type boundary condition for many of the problems faced by geotechnical engineers. As a result, unsaturated soil mechanics has become a necessary tool for analysing common geotechnical and geo-environmental problems.

Key Words: Unsaturated Soil, Modulus of Elasticity, Vadose zone, Matric Suction, Net Normal Stress.

1. INTRODUCTION

The most common definition of an unsaturated soil is that it is one consisting of three phases, namely solid particles, pore water and pore air i.e. it is a soil for which the degree of saturation S, the volume of pore air divided by the volume of pore water, is less than unity, i.e. S<1.0. This could be termed a geometric or volumetric definition of unsaturation. However, there is also a definition based on pore water stress: The pore of fine-grained soils that dry out from a saturated state may retain at 1.0 even though the pore water pressure is considered negative with respect to atmosphere pressure.

Elastic moduli (Young's modulus and shear modulus) refer to the stiffness of materials in response to recoverable deformation. Young's modulus can be defined as the ratio of the applied normal stress to the responding normal strain. Shear modulus can be defined as the ratio of the applied shear stress to the responding shear strain. Young's modulus has been widely used as the basis for calculating settlements for a variety of foundation types bearing on compacted or over consolidated soils after construction. Shear modulus has been used as an effective parameter for examining stiffness of the base course and subgrade in pavement materials.

In the tropics, high temperature and rainfall are conducive to the weathering of rock formations and the formation of residual soil. Most residual soils are in an unsaturated conditions and it is now possible to reliably measure negative pore water pressure in the field. The behaviour of residual soil falls into realm of unsaturated soil mechanics. In fact, it is fair to say that completely dry and completely saturated natural soils are very rare, and the common condition of a soil in nature is one of unsaturation. Thus while determining the properties of soil it must be considered as unsatuated.

1.1 Unsaturated Soil

When the soil pores are filled by more than one fluid, e.g. water and air, the porous material is termed unsaturated with respect to the wetting fluid. The matric suction s is defined as: $s = (u_a - u_w)$

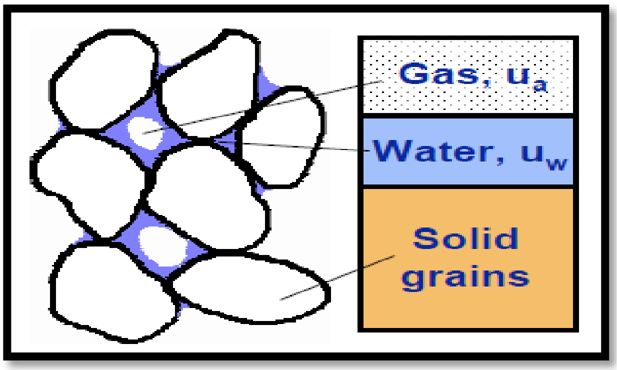

Fig- 1: Unsaturated soil

There is an increased awareness that the key to the estimation of unsaturated soil behaviour is the soil-water characteristics curve. The test duration for obtaining a soil-water characteristics curve is considerably much shorter than that to obtain other unsaturated soil properties. Many unsaturated soil properties can be obtained indirectly using the soil-water characteristics curve.

The universal acceptance of unsaturated soil mechanics depends largely upon how satisfactorily the stress state variables can be defined, justified, and measured. Historically, it has been the lack of certainty regarding the description of the stress state for an unsaturated soil that has been largely responsible for the slow emergence of unsaturated soil mechanics. In the last two decades, the identification of the stress state variables governing unsaturated soil behaviour has led to a major advancement in the theoretical framework of unsaturated soil mechanics. Laboratory testing of unsaturated soils has also seen major development with the advancement of data acquisition systems and instrumentation.

1.2 Modulus of Elasticity

Soil Young's modulus (E), commonly referred to as soil elastic modulus, is an elastic soil parameter and a measure of soil stiffness. It is defined as the ratio of the stress along an axis over the strain along that axis in the range of elastic soil behaviour. The elastic modulus is often used for estimation of soil settlement and elastic deformation analysis. Soil elastic modulus can be estimated from laboratory or in-situ tests or based on correlation with other soil properties.

1.3 Application of Modulus of Elasticity of Unsaturated Soil

Elasticity problems in soil mechanics deal with the deformation of the soil due to its own weight or due to external forces such as the weight of buildings. All settlement problems belong in this category. In order to solve these problems one must know the relationship between stress and strain for the soil. Young's modulus has been widely used as the basis for calculating settlements for a variety of foundation types bearing on compacted or over consolidated soils after construction.

2. UNSATURATED SOIL

The past two decades has seen a rapid development of unsaturated soil mechanics with the identification of the two stress state variables, matric suction and net normal stress, governing unsaturated soil behaviour. It can be easily illustrated using these stress state variables that unsaturated soil mechanics is more general and encompasses all the principles of saturated soil mechanics.

2.1 Characteristics of unsaturated soil

The zone between the ground surface and the water table is generally referred to as the unsaturated soil zone. This is somewhat of a misnomer since the capillary fringe is essentially saturated. A more correct term for the entire zone above the water table is the vadose zone. The entire zone subjected to negative pore–water pressures is commonly referred to as the unsaturated zone in geotechnical engineering. The unsaturated zone becomes the transition between the water in the atmosphere and the groundwater i.e., positive pore–water pressure zone. The pore–water pressures in the unsaturated soil zone can range from zero at the water table to a maximum tension of approximately 1,000,000 kPa i.e., soil suction of 1,000,000 kPa under dry soil conditions. The water degree of saturation of the soil can range from 100% to zero. The changes in soil suction result in distinct zones of saturation. The zones of saturation have been defined in situ as well as in the laboratory i.e., through the soil–water characteristic curve. Soils in situ start at saturation at the water table and tend to become unsaturated toward the ground surface.

The portion of the soil profile above the groundwater table is called the vadose zone (Fig - 2). This zone can be broadly subdivided into a portion immediately above the water table which remains saturated even though the pore- water pressures are negative, and a portion where the soil becomes unsaturated. The desaturation in the upper portion may be due to exceeding the air entry value of the intact soil or due to desaturation in the secondary structure (i.e., the fissures and cracks). Regardless of the degree of saturation of the soil, the pore-water pressure profile will come to a hydrostatic condition when there is no flux from the ground surface. One of the characteristics of the upper portion of the vadose zone is its ability to slowly release water vapour to the atmosphere at a rate dependent upon the permeability of the intact portions of soil. At the same time, the inflow of water can occur through the fissures under a gradient of unity. There appears to be no impedance to the inflow of water until the soil swells and becomes intact, or until the fissures and cracks are filled with water.

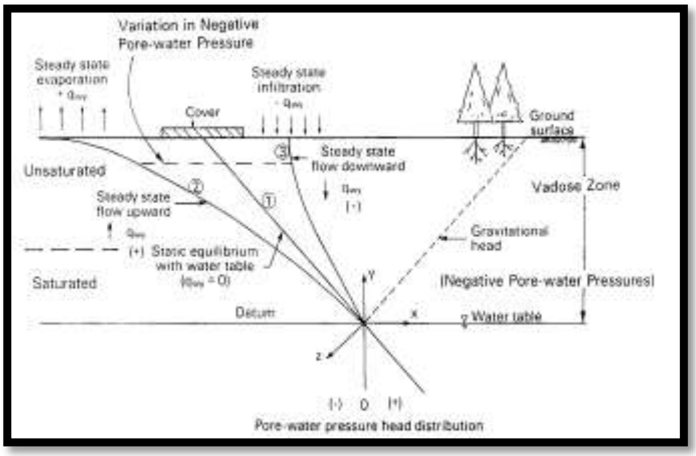

Fig – 2: Definition of Negative Pore-Water Pressure in Vandose Zone

Soils near to the ground surface are often classified as "problematic" soils. It is the change in the negative pore–water pressures that can result in adverse changes in shear strength and volume change. Common problematic soils are: expansive or swelling soils, collapsible soils, and residual soils. Any of the above-mentioned soils, as well as other soil types, can also be compacted, once again giving rise to a material with negative pore–water pressures.

Most soils have a water content less than porosity, which is the definition of unsaturated condition. A condition in which all easily drained voids (pores) between soil particles are temporarily or permanently filled with water is saturated condition. The capillary fringe of the water table is the dividing line between saturated and unsaturated conditions.

In most practical cases, the effect of unsaturation on the strength of a soil, i.e. the effect of matrix suction, will be small. Nevertheless the additional strength may have an important effect on the soil's load-carrying capacity. In other circumstances, an unsaturated soil may develop a surprisingly high strength. As an example of these two categories, a reduction in strength of 25 kPa in wet weather may be sufficient to cause a collapse of grain structure under load in a collapsing soil, whereas the same soil in dry weather may attain a strength well in excess of 250 kPa.

The relationship between effective stress and shear strength for an unsaturated soil can be significantly different from that of the same soil that has been in a saturated state over a similar range of effective stresses. Unsaturated soils appear to be stronger but more compressible than the same soil when saturated.

2.2 Importance of unsaturated soil

Unsaturated soil mechanics has rapidly become a part of geotechnical engineering practice as a result of solutions that have emerged to a number of key problems like - the fundamental theoretical behaviour of an unsaturated soil; the formulation of suitable constitutive equations; the ability to formulate and solve one or more nonlinear partial differential equations using numerical methods; the determination of indirect techniques for the estimation of unsaturated soil property functions, and in situ and laboratory devices for the measurement of a wide range of soil suctions.

Study of Soil properties considering it as unsaturated helps in overcoming the structural damage to housing and other structures, caused by heave of expansive clays. Unsaturated soils do not usually give rise to geotechnical problems, as long as they remain in their unsaturated state at an approximately constant water content. In recent years there have been a number of new construction techniques that have become common practice. Several of these techniques involve the use of soil in its unsaturated and/or compacted state.

Soil Nailing: The design of a soil nailing retaining structure assumes that load will come on to the anchors as a result of the influx of water into the soil behind the cover on the slope. It is the change in negative pore-water pressures in response to a ground surface flux which initiates the volume change necessary in order to load the anchors.

Geo-environmental Controls: The movement of moisture in the entire region surrounding a waste containment area is closely related to surface flux conditions and unsaturated soil behaviour. The mounding of the groundwater table below a waste containment area occurs in response to flow through unsaturated soils.

2.3 Properties of unsaturated soil

Before identifying relevant unsaturated soil properties, it is important to define the variables which can be used to represent the stress state of the soil. These are the variables which will allow engineering experience from various parts of the world to be readily compared. It will be the basis on which a transferable science can be built. The stress state is the single element which establishes geotechnical engineering as a science, set apart from an empirical art. As a result, it becomes the basis or the foundation block on which engineers build their engineering formulations.

2.3.1 Stress State Variables

There now appears to be a fairly general consensus that the stress state for an unsaturated soil should be described using two independent, normal stress variables. These two variables become a logical extension of the effective stress variable used for a saturated soil. The simplest way to visualize the need for two independent stress state variables is to realize that total stress changes and pore-water pressure changes do not produce equivalent responses in an unsatuated soil. This sets their behaviour apart from that of saturated soils. Since the pore-air pressure is constant (i.e., atmospheric) for most practical geotechnical problems, it is logical to independently reference the total stress and the pore-water pressure to the pore-air pressure. All descriptions of soil behaviour (i.e., volume change and shear strength) can now be referenced to the two stress state variables. There is also a smooth transition from the unsaturated case to the saturated case in that the pore-air pressure becomes equal to the pore-water pressure as the soil becomes saturated.

Table -1: Summary of classic saturated and unsaturated soil mechanics principles and equations

principles of equations	saturated soil	unsaturated soil
Stress state variables	$(\sigma - u_W)$	$(\sigma - u_a)$and $(u_a - u_w)$
Shear strength	$\tau = c' + (\sigma - u_w)$	$\tau = c' + (u_a - u_w) \tan \varphi^b + (\sigma - u_s) \tan \varphi'$

		$c = c' + (u_a - u_w)\tan\varphi$
Constitutive equations (Isotropic loading)	Void ratio and water content $de = G_s\,dw = a_v\,d(\sigma - u_w)$	Void ratio De=$a_t d(\sigma - u_a) + a_m d(u_a - u_w)$ Water content $d_w = b_t d(\sigma - u_a) + b_m d(u_a - u_w)$
Flow law for water (Darcy's Law) -Hydraulic Head	$v_w = -k_s \frac{\partial h_w}{\partial y}$ $h_w = y + \frac{u_w}{(\rho_w g)}$	$v_w = -k_w(u_a - u_w)\frac{\partial h_w}{\partial y}$ $h_w = y + \frac{u_w}{(\rho_w g)}$
Steady state seepage (Isotropic)	One dimensional : $\frac{d^2 h_w}{dy^2} = 0$ Two dimensional : $\frac{\partial^2 h_w}{\partial x^2} + \frac{\partial^2 h_w}{\partial y^2} = 0$	One dimensional : $k_w \frac{d^2 h_w}{dy^2} + \left(\frac{dk_w}{dy}\right)\frac{dh_w}{dy} = 0$ Two dimensional : $k_w \frac{\partial^2 h_w}{\partial x^2} + \left(\frac{\partial k_w}{\partial x}\right)\frac{\partial h_w}{\partial x} + k_w \frac{\partial^2 h_w}{\partial y^2} + \left(\frac{\partial k_w}{\partial y}\right)\frac{\partial h_w}{\partial y} = 0$

2.3.2 Volume Change Moduli

The independent stress state variables can be used to formulate the constitutive relations for an unsaturated soil (see Table 1). The volume change constitutive equation can be written in terms of a change in void ratio. However, more than one constitutive equation is required for the complete volume-mass characterization of an unsaturated soil since water content (or degree of saturation) change is independent of the void ratio change.

The water content constitutive equation has generally been used as the second constitutive relationship for an unsaturated soil. When the matric suction of the soil is zero, the change in void ratio and the change in water content are equivalent in terms of the response to a change in total stress. As a result, it is the water content versus matric suction relationship which becomes an important, additional relationship to quantify for an unsaturated soil. Equipment for the quantification of this relationship has been developed in the soil science discipline. The relationship is known as the "Soil Water Characteristic curve" (Fig - 3). It is the soil water characteristic curve which becomes of great value in quantifying unsaturated soil behaviour. In general, it is possible to estimate most unsaturated soil properties from saturated soil parameters and the Soil Water Characteristic curve. The methods for approximating the unsaturated soil properties are presently an area of fruitful research. In most cases, it appears that approximate estimates of the unsaturated soil properties are satisfactory for geotechnical practice. Several mathematical equations have been suggested to best-fit the water content versus matric suction experimental data. All equations have the form of a cummulative frequency distribution curve. It is suggested that one of these forms. Should be adopted for consistent use in geotechnical engineering.

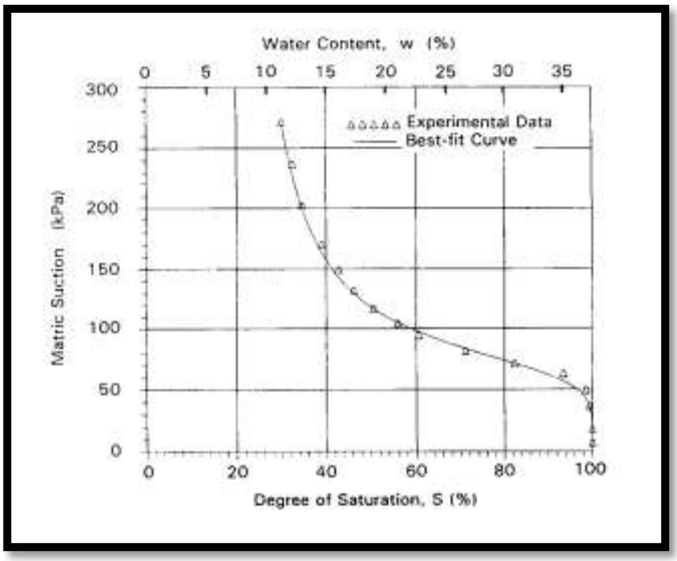

Fig – 3: Typical Soil Water Characteristics Curve Relating Matric Suction to the amount of water in the soil

2.3.3 Shear Strength Parameters

The shear strength equation for unsaturated soils has been formulated as a linear combination of the stress state variables incorporating shear strength parameters. As the testing of unsaturated soils has been extended over an ever increasing suction range, and for many soil types, there has become increasing evidence that the shear strength relationship involving suction can be nonlinear. In general, it is possible to linearize the relationship over a selected suction range. At the same time, the Soil Water Characteristic curve can be used to approximate the shear strength versus suction relationship. The angle indicating the change in shear strength relative to changes in matric suction begins to deviate from the effective angle of internal friction, as the matric suction desaturates the soil. As the suction reaches a value corresponding to the residual water content, the angle indicating change in shear strength relative to changes in matric suction appears to approach an angle of zero degrees (or it may even be negative).

2.3.4 Coefficient of Permeability

An unsaturated soil no longer has a constant coefficient of permeability and as a result, the engineer must characterize a permeability function for the soil. The function is dependent upon the stress state in the soil. In particular, the coefficient of permeability is a primary function of matric suction.

The permeability function can be empirically computed from a knowledge of the saturated coefficient of permeability and the Soil Water Characteristic curve. The computations are based on the assumption that water can only flow through the water portion of the soil. Therefore, an integration along the Soil Water Characteristic curve provides a measure of the quantity of water in the soil. For most geotechnical problems, this form of permeability function characterization is satisfactory. Several other functions have also been proposed but at present, Gardner's equation have quite wide acceptance.

2.3.5 Suction Stress

The type of force, which includes physicochemical forces, cementation forces, surface tension forces, and the force arising from negative pore-water pressure, may be conceptually combined into a macroscopic stress called suction stress. Suction stress characteristically depends on degree of saturation, water content, or matric suction. The method for obtaining suction stress is given from direct shear test and triaxial shear test of unsaturated soils. The relationship of suction stress and matric suction is called suction stress characteristic curve (SSCC). It is to be noted that suction stress is not zero for fine grain soils under saturated state.

Suction stress is the macroscopic effect of many different microscopic forces in unsaturated soils. Redundant parameters need not to be introduced to describe these microscopic forces, respectively. The failure envelop of unsaturated soils is unique at different matric suction and net normal stresses, based on the new effective stress framework. The cohesion arisen by tension among soil particles expresses its real concept. The relation of matric suction and suction stress can be uniquely expressed by the suction stress characteristic curve (SSCC), which is very important for describing the stress state, similar with SWRC for describing the soil-water state. The uncertainty of the parameters of shear strength of soils can be avoided in the effective stress frame.

3. MODULUS OF ELASTICITY

The modulus of elasticity (Young's modulus) E is a material property, that describes its stiffness and is therefore one of the most important properties of solid materials. Graphically we can define modulus of elasticity as a slope of the linear portion of the stress-strain diagram. Mechanical deformation puts energy into a material. The energy is stored elastically or dissipated plastically. The way a material stores this energy is summarized in stress-strain curves. Stress is defined as force per unit area and strain as elongation or contraction per unit length. When a material deforms elastically, the amount of deformation likewise depends on the size of the material, but the strain for a given stress is always the same and the two are related by Hooke´s Law (stress is directly proportional to strain):

$$s = E . e$$

where,
s = is stress [MPa]
E = modulus of elasticity [MPa]
e = strain [unitless or %]
From the Hook's law the modulus of elasticity is defined as the ratio of the stress to the strain:
E = [MPa]

3.1 Methods of Determination of Modulus of Elasticity

Following are methods to determine the elastic moduli of unsaturated soil:

1. Slow cycled triaxial tests
2. Conventional Plate load test
3. The cross-hole plate test
4. The screw plate test
5. The Menard pressuremeter test
6. Dry cake test
7. Three-Point Laboratory Test

3.2 Comparisons of different methods of assessing elastic modulus for unsaturated soils

Very few comparisons have been published of the various available methods of assessing compressibility or elastic moduli in situ for soils. One of the few such comparisons was made by Jones & Rust, (1989) for an unsaturated saprolitic weathered diabase. Their comparison, illustrated in Fig. 3.14a, shows that the Menard pressuremeter, plate bearing test and oedometer test give comparable results for E provided that rebound curves are used for the plate loading and oedometer tests.
The self-boring pressuremeter, that should give the lowest disturbance to the soil, also gives the highest values of E.

Also note from Fig. 3.14b that the N value (blow count) from the Standard Penetration Test (SPT) in the same profile has a very similar trend with depth.

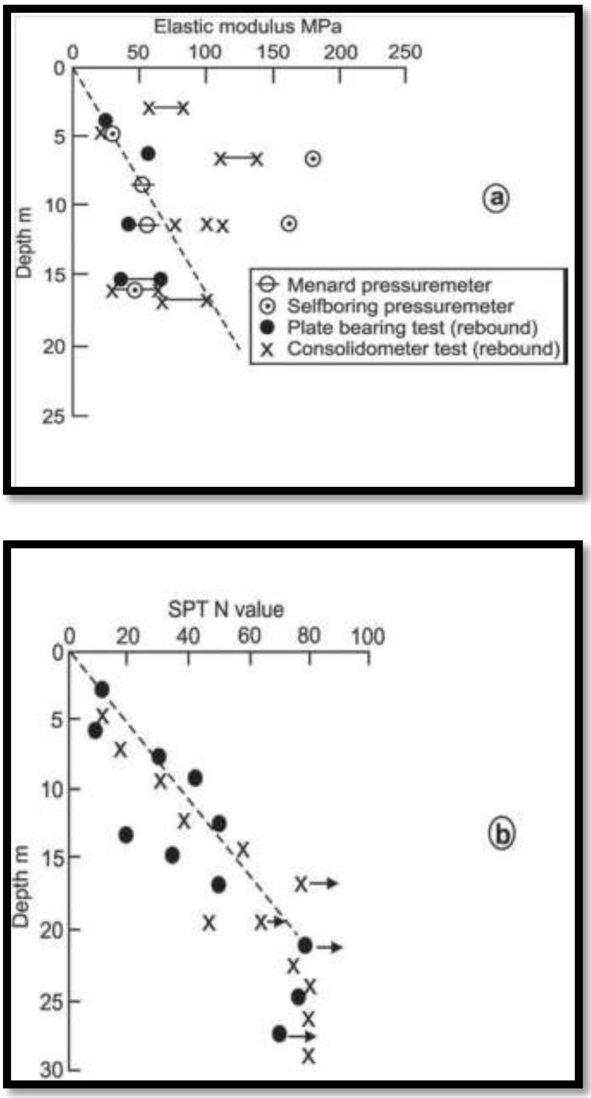

Chart 1: a: Comparison of E values derived by four different methods for a weathered diabase profile;
b: Variation of standard penetration test N with depth in the same profile.

From this it may be deduced that SPT results in unsaturated soils may also be used to obtain estimates of E. For this profile, the correlation is:

$$E = 1.6N \text{ MPa}$$

A popularly used correlation is:

$$E = N \text{ MPa}$$

which is rather more conservative, in terms of calculated settlement.

4. CONCLUSIONS

1. If a soil is completely dry then only air pressures will occur within its pores. If the soil is completely saturated, only water pressures can occur. Obviously the effects of the water and air phases in a soil depend upon the degree of saturation and should be considered while determining soil properties.

2. Since the unsaturated soil properties are functions of the stress state, it is necessary to make a series of soil property measurements on a particular soil. These measurements must be made under controlled stress states that involve both total stresses and matric suction.

3. The SPT results in unsaturated soils may also be used to obtain estimates of Modulus of Elasticity

4. Further study for developing cost effective, less time consuming and ralaible result giving methods for determination of elastic moduli and other properties of unsaturated soil is required.

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