Permanent Magnet Linear Motor using Discrete-Time Fast Terminal Sliding Mode Control

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Abstract - Permanent magnet synchronous motor (PMSM) depend on a 3-phase time-dependent voltage source (3-Phase AC supply voltages) that generates a magnetic flux in the air-gap of the machine. This generated magnetic flux interacts with the permanent magnetic flux on the rotor, to generate the required torque. The mathematical model of this motor is a non-linear time-varying system. To apply different control techniques, we transform this model to an equivalent linear time-invariant system. These transformations not only yield a linear time-varying model, but also, reduce the number of states in the model. Classical control techniques, such as PI control, can provide a speed tracking of this type of motors with some limitations. In general, the performance of the motor is limited in term of the range of speed and the range of applied load torque. Also, the performance is affected by parameter variations or the high frequency, un-modeled states. In this paper, a sliding-mode controller is used due to its insensitivity to the variations of the parameters. These types of controllers employ sliding surface, passing through the origin on the system trajectories plane. The proposed gain in this paper is a smooth function, depending on the surface value to eliminate the chattering phenomena (soft switching mechanism). The primary problem with this design is the steady state error when a full-load torque is applied motor. This is overcome by designing an observer to estimate the load torque.

Key Words: Digital control, Permanent magnet linear motor, Position control, Robustness, Terminal sliding mode.

1. INTRODUCTION

Permanent Magnet linear motor (PMLM) is a conversion device that does not require any intermediate switching mechanism to convert electrical energy into linear motion [1]– [4]. Due to its many advantages such as high speed, large pushing force, and high precision, PMLM has been successfully applied in industry, military, and some other motion occasions which require high-speed, low thrust, small displacement, and high-precision position control [5], [6]. Meanwhile, the study of PMLM has attracted much attention from various fields, such as electronic industry, control engineering, etc. From the viewpoint of control engineering, the model of PMLM is a typical nonlinear multivariable system. Furthermore, the control performance of PMLM is potentially affected by various nonlinear factors such as unknown load and friction. Recently, the control issue of PMLM has become an important topic in field of PMLM and how to improve the control performance of PMLM has obtained certain attention, see [7]–[9] and some references therein. To solve the control problem of PMLM, many nonlinear control methods have been employed in the literature [10]– [16]. The Permanent Magnet Synchronous Machines (PMSM) are high-performance electromechanical motion devices essentially superseding traditional dc servomotors, and fractional horsepower induction machine because of their high performance capability [10]. This type of device can widely be seen in our daily life; for instance, different household machines, vending machines, factories, and computers. For such a diverse range of applications, the performance requirements necessitate different speed and/or angular position specifications to meet designated operational goals. The necessity for high performance electromechanical systems increases as the demand for precision controls increases. Permanent magnet synchronous motors (PMSMs) constitute a significant electro-mechanical design option in many applications due to their high performance, high efficiency, high power density, and fast dynamics [2], [16] and [9]. In high performance drivers, servos, and power generation systems (up to 200 KW), three-phase permanent magnet synchronous machines (motors and generators) are the preferable choice. In high-power (from hundreds of KW to hundreds of MW range) generation systems, conventional three phase synchronous generators are used [10]. Given the complexity and the nonlinearity of PMSMs, substantial research has been reported detailing control and performance challenges. The classical proportional integral (PI) controller, due to its simplicity of implementation, is still the dominant choice in most applications; however, due to load disturbances, un-modeled states, parameter variations and friction forces, PI controllers are unable provide effective solutions to many practical problems [15]. To avoid these aforementioned problems, a nonlinear controller is used in this thesis. Sliding-mode control (SMC) has the striking feature that it is insensitive to modeling uncertainty [5]. Also, with the addition of an observer to estimate the applied load torque, it has the ability to compensate for load torque disturbances.

Considering the importance of PMLM and the superiority of fast TSMC, this paper aims to design a new kind of discrete time fast TSMC law for position control of PMLM. Specifically, a discrete-time model of PMLM is first obtained based on Euler's discretization. Then, by using traditional linear SMC method, a discrete-time linear SMC law is designed and the corresponding stability of closed-loop system is addressed. In the case of external disturbances, a compound strategy consisting of delayed estimation and dynamic compensation is employed to deal with the influence of disturbances. By performing rigorous analysis on the closed-loop system, the ultimate bounds for the sliding mode surface and steady-states are respectively obtained. To improve the control precision, based on the discrete-time fast TSMC method, an improved digital control algorithm, i.e., discrete-time fast TSMC algorithm, for PMLM is proposed. It is shown that the improved control algorithm can offer a higher control accuracy for sliding mode motion and steady-states of the closed-loop system no matter whether the disturbances are compensated or not. The main contribution of this work is to design a novel nonlinear digital control algorithm (i.e., the discrete-time fast TSMC algorithm) for the position control system of PMLM such that the closed-loop system's performance can be improved. By performing rigorous theoretical analysis, the explicit relationship between the ultimate bound for the tracking error and the fractional power from the terminal SMC law is given, which shows that the higher control accuracy can be achieved by choosing appropriate fractional power. Simulation and experimental results are finally provided to verify the effectiveness of the proposed control method and show the advantages of the designed methods by comparing with some existing ones.

1.1 Permanent Magnet Synchronous Motor

The motor under the test an 8-poles three-phase Permanent Magnet Synchronous Motor (PMSM). On the left, the equivalent electric circuit. On the right, the mechanical cross-section with an attached load. The machine has a permanent magnet round rotor, where the magnetic poles are aligned axially distributed around the circumference of the solid rotor. In synchronous motors, the process of energy conversion is accomplished by producing an electromagnetic torque from an interaction of a time-varying magnetic field developed in the machine air-gap and m the stationary magnetic field produced by the permanent magnets distributed on the m rotor [10].

In order to achieve a desired speed, the phase voltages must be varied as a function of the rotor angular displacement Θe. The angular speed ω_e of the synchronous motor tracks the supplied voltage frequency to the stator windings [10]. This necessitates the measurement or estimation of the angular displacement of the rotor, using encoders or estimators. In the case of a conventional PMSM, the motor typically operates at a fixed speed (synchronous speed), with dynamic changes to the operating speed expected by control of the source frequency of the 3-phase supply voltages.

1.2 Control Strategy

The motor supply voltage is a DC source. This voltage will be inverted using pulse width modulation (PWM) to provide a 3-phase AC source. The modulated voltages provide a variable amplitude and frequency sinusoidal source for the PMSM.

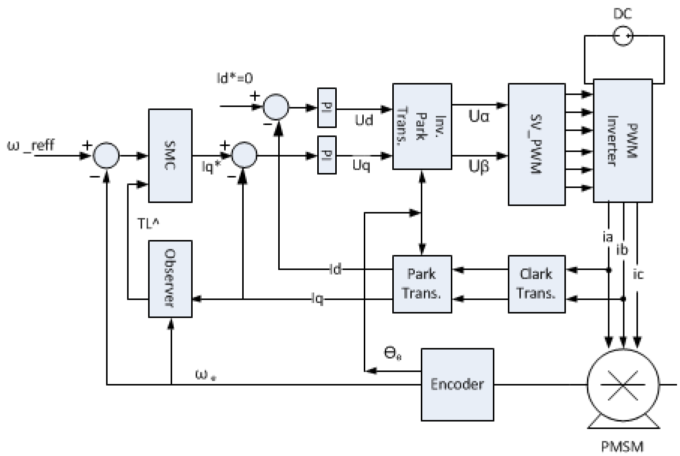

Fig 1: The Detailed Experiment setup View

2. DESCRIPTION OF SYSTEM MODEL AND CONTROL OBJECTIVE

2.1 Continuous-time model of PMLM

For a permanent magnet linear motor, the mathematical model is usually c approximately described by a second-order c system and is given in the form of [5]:

$\dot{x}_1(t)=x_2(t)$

$\dot{x}_2(t)=\frac{-KfKe}{Rm}x_{2(t)}\frac{+Kf}{Rm}u(t)\frac{-d(t)}{m}$

$y(t)=x_1(t)$ (1)

where $x1$ is the linear displacement, $x2$ is the linear velocity, $u(t)$ is the control signal, R is the resistance, m is the motor mass, kf is the force constant, ke is the back electromotive force, and $d(t)$ can be considered as the lumped disturbances including the friction and ripple force.

2.2 Control objective

The control objective of this paper to design a position tracking controller for PMLM such that the reference position trajectory can be tracked. Without loss of generality, assume that the reference signal is $xr(t)$, whose first-order and second order derivatives are bounded.

For the brevity, denote

$a=-\frac{kfke}{Rm}$,$b=\frac{kf}{Rm}$, $f(t)=\frac{d(t)}{m}$ (2)

under which the equation (1) is rewritten as:

$\dot{x}_1(t)=x_1(t)$

$\dot{x}_2(t) = -a\, x_2(t) + b\, u(t)$
$= -f(t)$
$y(t) = x_1(t)$ (3)
Define,
$e_1(t) = x_r(t) - x_1(t)$
$e_2(t) = \dot{x}_r(t) - x_2(t)$ (4)
as the tracking errors for linear displacement and linear velocity signal, where $xr(t)$ is the reference linear displacement, $\dot{x}r(t)$ is the reference linear velocity. It can be obtained from (3) that the dynamical equation for the tracking errors has the following form:

$\dot{e}_1(t) = e_2(t)$
$\dot{e}_2(t) = -ae_2(t) - bu + F(t) + a\dot{x}_r(t) + \ddot{x}_r(t)$ (5)

Based on the error equation, the control objective is to design a control law such that the tracking errors converge to zero. Although there have been some results on designing the position control laws for PMLM by using SMC method, see e.g., [15], [21], these designs are mainly based on continuous time SMC theory. Different from these results, in this paper, it is assumed that the control law $u(t)$ is digitally implemented through a zero-order-holder (ZOH), i.e., $u(t) = u(kh)$ over the time interval $[kh, (k + 1)h)$ with h being the sampling period, where k 2 {0, 1, 2, · · · , } = Z^+ S{0}. In other words, the main objective of this paper is to design a discrete-time SMC law for PMLM, which is more appropriate to digital implementation.

Before moving on, the following two assumptions on disturbances and one lemma are presented which will be used in the subsequent analysis.

Assumption 2.1: The disturbance $F(t)$ is assumed to be bounded, i.e., $|F(t)|$ _ d ·with a constant d ·.
Assumption 2.2: The derivative of disturbance is assumed to be bounded, i.e., $|\dot{F}(t)| \leq \delta^*$ with a constant δ^*.
Lemma 2.1: [29] Consider a scalar dynamical system
$z(k + 1) = z(k) - lz(k) + g(k)$. (6)
If $|l| < 1$ and $|g(k)| \leq \gamma$, $\gamma > 0$, then the state $z(k)$ is always bounded and there is a finite number K_ > 0 such that $|z(k)| \leq \gamma/|l|$, $\forall k \geq K$·.

2.3 Design of Discrete-Time SMC Law for PMLM

In this section, we will employ the method of discrete-time SMC to design a digital control algorithm for PMLM. Firstly, the method of Euler discretization is employed to obtain the discrete-time model of system (5), which is given as:
$e_1(k+1) = e_1(k) + he_2(k)$,
$e_2(k+1) = e_2(k) - hbu(k) - hae_2(k)$
$+h[a\dot{x}_r(k) + \ddot{x}_r(k)] + hF(k)$, (7)
where h is the sampling period. Based on the discrete-time model, we first design a traditional linear discrete-time SMC law and then propose an improved fast discrete-time terminal SMC law.

2.3.1 Designing a traditional linear discrete-time SMC law

For discrete-time system (7), by using the traditional discrete-time SMC method, the sliding mode surface is linear and is chosen as:
$s(k) = Ce(k) = e_2(k) + c_1e_1(k)$, (8)
with $0 < hc1 < 1$. As that in [28], by using an equivalent control method and directly solving
$s(k+1) = 0$, (9)

We get,

$e_2(k) - hbu(k) - hae_2(k) + h[a\dot{x}_r(k) + \ddot{x}_r(k)] + hF(k)$
$+c_1[e_1(k) + he_2(k)] = 0$ (10)
As a result, the equivalent control-based discrete-time SMC law is obtained as follows:
$u(t) = \frac{1}{hb}[(1 + c_1h - ha)e_2(k) + c_1e_1(k)$
$+h[a\dot{x}_r(k) + \ddot{x}_r(k)] + hF(k)]$ (11)

1) Case 1: Assumption 2.1 is satisfied and the disturbance is not compensated: In this case, since the disturbance information F(k) is unavailable, the final available controller should be
$u(k) = \frac{1}{hb}[(1 + c_1h - ha)e_2(k) + c_1e_1(k)$
$+h[a\dot{x}_r(k) + \ddot{x}_r(k)]]$ (12)
which results in the dynamical behavior of sliding mode state
as
$s(k+1) = Ce(k+1)$
$= hF(k)$ (13)
Under Assumption 2.1, the sliding mode state $s(k)$ is bounded
by1
$|s(k)| \leq d^*h = O(h), \forall k \in Z^+$ (14)
which means that the sliding mode state $s(k)$ has an $O(h)$ boundary layer.
In the sequel, we will analyze the dynamical behavior of output tracking error $e1(k)$. It follows from (7)-(8) that
$e_1(k+1) = e_1(k) + h[s(k) - c_1e_1(k)]$
$= (1 - hc_1)e_1(k) + hs(k)$ (15)
By Lemma 2.1, it can be concluded that the state $e1(k)$ is always bounded and the steady-state of tracking error $e1$ will be bounded by
$|e_1(\infty)| \leq \frac{h|s(\infty)|}{hc1} \leq \frac{d*}{c1} = O(h)$ (16)
That is to say that the system output tracking error $e_1(k)$ has an accuracy with O(h).

2) Case 2: Assumption 2.1 and Assumption 2.2 are satisfied and the disturbance is compensated: Since the disturbance information F(k) is unavailable, under Assumption 2.2, it can be estimated by using delayed estimation method as that in [28], i.e.,

$$\hat{F}(k)=F(k-1)$$
$$=\frac{1}{h}[e_2(k)-e_2(k-1)]+bu(k-1)+ae_2(k-1)$$
$$-[a\dot{x}_r(k-1)+\ddot{x}_r(k-1)] \quad (17)$$

Then the state $F(k)$ in the controller (11) can be substituted by the estimated value $\hat{F}(k)$, which leads to the available controller as follows

$$u(k)=\frac{1}{hb}[(1+c_1h-ha)e_2(k)+c_1e_1(k)$$
$$+h[a\dot{x}_r(k)+\ddot{x}_r(k)]+h\hat{F}(k)] \quad (18)$$

Under the discrete-time controller (18), the dynamical behavior of sliding mode state is given by

$$s(k+1)=c_e(k+1)$$
$$=h[F(k)-\hat{F}(k)]$$
$$=h[F(k)-F(k-1)] \quad (19)$$

Throughout of this paper, the big O notation is referred to that function $f(h)$ is said to be of order $g(h)$ as $h\to 0$ and denoted as $f(h) = O(g(h))$, if there exist $\delta > 0$ and $M > 0$ such that $|f(h)| < M|g(h)|$ for $|h| < \delta$.

which is bounded by

$$|s(k)|\le\delta^*h^2=O(h^2), \quad \forall k\in Z^+ \quad (20)$$

That is to say that the sliding mode state $s(k)$ has an $O(h2)$ boundary layer. By a similar proof as that in Case 1, the system output tracking error $e1(k)$ has also an accuracy with $O(h2)$.

Remark 3.1: From the previous analysis, it can be found that the ultimate bound for the system steady output tracking error is determined by both the steady state of sliding mode state and the structure of sliding mode surface. Motivated by this observation, in the next subsection, we will employ a nonlinear sliding mode surface to improve the control accuracy.

2.3.2 Designing an improved discrete-time FTSMC law

In this section, an improved discrete-time fast terminal SMC law will be designed to improve the accuracy of the system output tracking error. At the first step, the discrete-time fast terminal sliding mode surface is chosen as

$$S(k)=e_2(k)+c_1e_1(k)+c_2sig^{\alpha}(e_1(k)), \quad (21)$$

Where $sig^{\alpha}(e_1(k)) = sgn(e_1(k))\cdot|e_1(k)|^{\alpha}$, $0 < hc_1 < 1$, $c_2 > 0$, $0 < \alpha < 1$.

As that in the previous subsection, based on the equivalent control method, directly solving

$$S(k+1)=0, \quad (22)$$

leads to

$$u(k)=\frac{1}{hb}[(1+c_1h-ha)e_2(k)+c_1e_1(k)$$
$$+h[a\dot{x}_r(k)+\ddot{x}_r(k)]$$
$$+hF(k)+c_2sig^{\alpha}(e_1(k)+he_2(k))] \quad (23)$$

Similarly, the following two cases are respectively considered.

1) Case 1: Assumption 2.1 is satisfied and the disturbance is not compensated: In this case, the equivalent control-based discrete-time fast TSMC law is

$$u(k)=\frac{1}{hb}[(1+c_1h-ha)e_2(k)+c_1e_1(k)$$
$$+h[a\dot{x}_r(k)+\ddot{x}_r(k)]$$
$$+(k)+c_2sig^{\alpha}(e_1(k)+he_2(k))] \quad (24)$$

Under Assumption 2.1, the sliding mode state $s(k)$ is bounded
By

$$|s(k)|\le\lambda=d^*h=O(h), \quad k\in Z^+ \quad (25)$$

Next, the dynamical behavior of output tracking error $e1(k)$ will be considered. It follows from (7) and (21) that

$$e_1(k+1)= e_1(k)+h[s(k)-c_1e_1(k)-c_2sig^{\delta}e_1(k)]$$
$$=(1-hc_1)e_1(k)-hc_2sig^{\delta}e_1(k)+hs(k) \quad (26)$$

To analyze the stability of system (26), the following lemma is needed.

Lemma 3.1: [30] Consider the scalar dynamical system

$$Z(k+1)=z(k)-l_1sig^{\delta}z(k)-l_2z(k)+g(k) \quad (27)$$

where $l1 > 0$, $0 < l2 < 1$, and $0 < \gamma < 1$. If $|g(k)|\ \gamma$, > 0, then the state $z(k)$ is always bounded and there is a finite number $K^* > 0$ such that

$$|z(k)| \le\psi(\alpha)\cdot\max\{(\frac{\gamma}{l_1})^{1/\alpha}, (\frac{l_1}{1-l_1})^{\frac{1}{1-\alpha}}\}, \ \forall k\in k^* \quad (28)$$

where function $\psi(\alpha)$ is defined as

$$\psi(\alpha)= 1+\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}} \quad (29)$$

Theorem 3.1: For the error dynamical system (7) under the discrete-time fast TSMC law (24), if Assumption 2.1 holds, then the closed-loop system is stable and the ultimate bound for the output tracking error $e_1(k)$ could be of the order $O(h^2)$.

Proof: First, with the help of Lemma 3.1, it can be concluded from (26) that the state $e_1(k)$ is always bounded. In addition, it follows from (21) that

$$e_2(k)=s(k)-c_1e_1(k)-c_2sig^{\delta}(e_1(k)) \quad (30)$$

Since $e_1(k)$ is bounded and $s(k)$ is bounded from (25), then the state $e_2(k)$ is always bounded. That is to say that the stability of the closed-loop system can be guaranteed.
Second, as for the steady output tracking error, it follows from Lemma 3.1 that $e1$ is bounded by

Clearly, to get an optimal accuracy for the steady output tracking error, it is better to choose α= 1/2 such that

$$\frac{1}{\alpha}=\frac{1}{1-\alpha}=2, \quad (32)$$

which results in

$$|e_1(\infty)|\leq_{\varrho}=O(h^2) \quad (33)$$

Hence, compared with the linear sliding mode surface, the proposed fast terminal sliding mode surface can improve the accuracy of the steady output tracking error.
Next, we will consider the case when the disturbance can be compensated.
2) Case 2: Assumption 2.1 and Assumption 2.2 are satisfied and the disturbance is compensated: As shown in subsection III-A, the disturbance can be estimated and compensated by using delayed estimation method, which leads to the equivalent control-based discrete-time fast TSMC law.

$$u(k)=\frac{1}{h\bar{b}}[(1+c_1h-ha)e_2(k)+c_1e_1(k)$$
$$+h[a\dot{x}_r(k)+\ddot{x}_r(k)]+h\hat{F}(k)$$
$$+c_2 sig^{\alpha}(e_1(k)+he_2(k)] \quad (34)$$

Theorem 3.2: For the error dynamical system (7) under the discrete-time fast TSMC law (34), if Assumption 2.1 and Assumption 2.2 hold, then the closed-loop system is stable and the ultimate bound for the output tracking error $e_1(k)$ could be in the order of $O(h^3)$.
Proof: Under Assumptions 2.1-2.2, by following a same proof as previous case, it can be concluded that the sliding mode state s(k) is bounded by

$$|s(k)| \leq \delta^* h^2=O(h^2), \quad \forall k \in Z^+ \quad (35)$$

By using some similar statements as those employed in proving the stability of the closed-loop system in Theorem 3.1, the stability of the closed-loop system can be proved where the detailed proof is omitted for brevity. Let us analyze the dynamical behavior of output tracking error e_1. It follows from (26) and Lemma 3.1 that the steady output tracking error e_1 is bounded by

$$|e_1(\infty)|\leq p =\psi(\alpha).\max\{(\frac{\delta*h^3}{c_2 h})^{1/\alpha},(\frac{c_2 h}{1-c_1 h})^{1/1-\alpha}\}$$
$$= \psi(\alpha).\max\{(\frac{\delta*h^2}{c_2})^{1/\alpha},(\frac{c_2 h}{1-c_1 h})^{1/1-\alpha}\}$$
$$=\psi(\alpha).\max\{(O(h))^{2/\alpha},(O(h))^{1/1-\alpha}\} \quad (36)$$

Similarly, the optimal choice for the fractional power is α= 2/3, which results in

$$2/\alpha=1/(1-\alpha)=3, \quad (37)$$

which results in

$$|e_1(\infty)|\leq p\ O(h^3) \quad (38)$$

The above analysis indicates that the FTSMC method can offer a higher control accuracy than that of LSMC method if the disturbance can be estimated and compensated.

Remark 3.2: Note that in Theorem 3.1 and Theorem 3.2, only the steady-state performance of the closed-loop system is discussed. Actually, for the dynamic performance of the closed-loop system, since the fast terminal sliding mode surface, i.e., (21), is employed, a faster dynamic response can still be guaranteed when the system state is close to the equilibrium point. The rigorous theoretical analysis about this issue for continuous-time FTSMC has been given in [23]. However, for the discrete-time FTSMC, it is very hard to give a qualitative conclusion about the dynamic performance of the closed loop system. In the simulation section, the corresponding comparisons (i.e., Table III) are given to demonstrate that the discrete-time FTSMC can also offer a good dynamic performance.

Remark 3.3: Note that the work [32] has studied the continuous-time FTSMC for position control problem of PMLM. However, the main differences of the design of FTSMC in continuous-time case and discrete-time case lie in two aspects. 1) In practice, more and more controllers are implemented based on digital computers in practice, e.g., the implementation of the proposed control algorithm in this paper is based on the digital signal processor (DSP, TMS320F2812). Thus, the designed discrete-time fast terminal SMC law in this paper can be directly implemented. 2) Although the continuous-time FTSMC law can be digital implementation through different discretization methods (such as Euler's discretization), the stability analysis for the closed-loop system (i.e., the control plant is continuous-time but the controller is in the form of discrete-time) is not provided in most of cases (actually, the stability analysis is extremely difficult in this case, see for example, [33]). In this paper, the design of discrete-time FTSMC law is based on the discrete-time model and the corresponding stability analysis is successfully provided by developing new analysis methods for discrete-time systems.

The detailed design procedure for designing a discrete-time FTSMC algorithm for PMLM is summarized as follows:

- **Step 1:** Obtain a discrete-time model of PMLM by using some discretization techniques.
- **Step 2:** Choose a discrete-time fast terminal sliding mode surface in the form of (21).
- **Step 3:** Design a discrete-time FTSMC algorithm in the form of:

(24), without disturbance compensation;
(34) with (17), with disturbance compensation.

Fig. 2 shows the block diagram of the discrete-time FTSMC for PMLM with disturbance compensation.

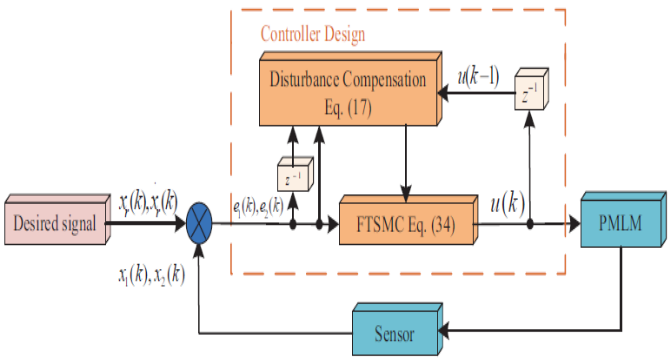

Fig. 2: The block diagram of the discrete-time FTSMC for PMLM.

4. SIMULATION RESULTS AND EXPERIMENTAL RESULTS

In this section, numerical simulations and experimental results are given to verify the effectiveness of the proposed control methods.

4.1 Simulation results

The system's parameters of PMLM considered in simulations are given in Table I.

Table I. System's parameters

Descriptions	Parameters
motor mass	m = 5.4 kg
Resistance	R = 16.8 ohms
force constant	K_f = 130 N/A
back electromotive force	k_e = 123 V/m/s

The disturbance is composed of two parts, i.e., friction force and ripple force. Specifically, let

$d = F_{fric} + F_{ripple},$ (39)

where F_{fric} is the friction force and F_{ripple} is ripple force. The friction force is defined as:

$F_{fric} = [f_c + (f_s - f_c)e^{-(\dot{x}/\dot{x}_s)^2} + f_v \dot{x}),]\text{sign}(\dot{x}),$ (40)

where f_c = 10 N is the Coulomb friction coefficient, f_s = 20 N is the static friction coefficient, f_v = 10 N is the static friction coefficient and $\dot{x}_s$ = 0.1 is the lubricant parameter. The ripple force is given as:

$F_{fipple} = A_1 \sin(\omega x) + A_2 \sin(3\omega x) + A_3 \sin(5\omega x),$ (41)

with A_1 = 8.5, A_2 = 4.25, A_3 = 2.0 and ω = 314 rad/s.

In this section, a *step signal* with an amplitude of 200 mm and a *sinusoidal signal* with an amplitude of 5 mm and the frequency of 1 rad/s, i.e., $x_r = 5\sin(t)$ are respectively considered as the desired displacement.

To achieve the position tracking control, three kinds of control algorithms are employed, i.e., the proposed discrete-time FTSMC, LSMC, and standard PID control. In simulations the sample period h is chosen as 0.005(sec) to be consistent with the experiment. To make a relatively fair comparison, the control parameters of three kinds of control algorithms are repeatedly tested to obtain optimal parameters such that there is a good tradeoff between the dynamic performance and the steady-state performance of the closed-loop system. The controllers' parameters are given in Table II.

Table II. Controllers' parameters

Control algorithm	Control gains
PID	k_p = 300, k_i = 50, k_d = 2
LSMC	c_1 = 3
FTSMC	c_1 = 1.5, c_2 = 1.5

Case 1: the disturbance is not compensated

In this case, assume that the disturbance satisfies Assumption 2.1, but it cannot be estimated and compensated. In addition, the fraction power α of proposed FTSMC law (24) is chosen as α = 1/2.

1) Step response: the amplitude of step signal is chosen s 200 mm. It can be found that the proposed discrete-time FTSMC can offer a faster dynamic response and a smaller steady-state tracking error.

2) Tracking a sinusoid signal: a sinusoidal signal with amplitude of 5 mm and frequency of 1rad/s is considered. Under the three kinds of control methods, the response curves are given in Fig. 3. In this case, it can be found that the proposed discrete-time FTSMC can significantly reduce the steady-state error.

Case 2: the disturbance is estimated and compensated

It is assumed that the disturbance satisfies Assumption 2.1and Assumption 2.2. Hence, we can use the delay estimation method provided in (17) to estimate and compensate the disturbance. More specifically, the discrete-time LSMC (18) and the proposed discrete-time FTSMC (34) with α = 2/3 are employed to solve the position tracking problem of PMLM.

1) Step response: the step signal is selected the same as that in Case 1 and the response curves of PMLM's displacement are plotted in Fig. 5. By comparing with Fig. 2, it can be found from Fig. 4 that the disturbance compensation strategy is effective. Furthermore, it can be seen from the numerical simulations that the improved discrete-time FTSMC algorithm

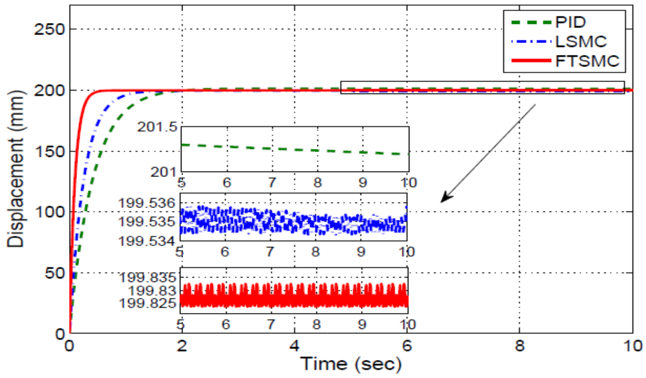

Fig. 3: The response curves for PMLM's displacement under step response in the absence of disturbance compensation.

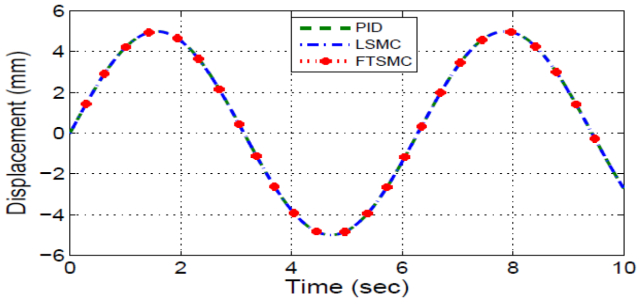

Fig. 4: The response curves for tracking a sinusoidal signal In the absence of disturbance compensation

2) Tracking a sinusoid signal: in the presence of disturbance compensation, the response curves of PMLM's displacement for tracking a sinusoid signal under different control algorithms are provided in Fig. 5. On one hand, by comparing with Fig. 4, it can be found that the disturbance compensation strategy is effective. On the other hand, the proposed discrete time FTSMC can significantly reduce steady-state error by comparing with the other two control algorithms. For the convenience to compare the dynamic performance of the closed-loop system under different control algorithms, Table III gives the detailed dynamic performance index (i.e., the rise time $tr(s)$ and settling time $ts(s)$) under step response.

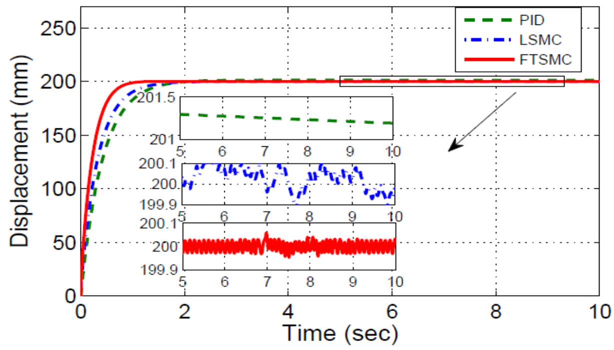

Fig. 4: The response curves for PMLM's displacement under step response in the presence of disturbance compensation

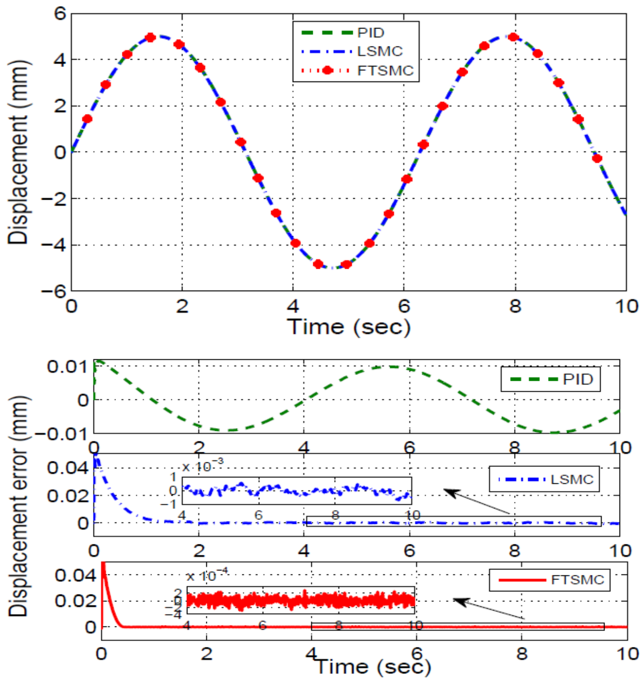

Fig. 6: The response curves for tracking a sinusoidal signal in the presence of disturbance compensation, (a) displacement, (b) Displacement error.

From Table III, it can be found that the proposed discrete time FTSMC algorithm can offer a good dynamic performance whether the disturbance is compensated or not.

Table III. The comparisons of dynamic performance of closed-loop system under the step response.

Algorithms		Rise time (sec)	Settling time (sec)
Case 1	PID	0.892	1.515
	LSMC	0.790	1.460
	FTSMC	0.653	1.112
Case 2	PID	0.892	1.515
	LSMC	0.741	1.305
	FTSMC	0.487	0.800

5. CONCLUSIONS

This paper has investigated the position tracking problem for PMLM via an improved discrete-time SMC method. By employing a nonlinear discrete-time sliding mode surface (i.e., discrete-time terminal sliding mode surface) instead of the traditional linear sliding mode surface, it has been shown that the steady-state performance for the closed-loop system can be improved. In addition, the explicit relationship between the ultimate bound for the tracking error and the fractional power from the terminal SMC law is theoretically given, which provides guidance on how to choose the optimal fractional power in practice. Simulation and experimental results have been performed to verify the theoretical analysis results and show the advantages of the present method over some existing ones as traditional linear SMC approach and PID method. Although only the position control problem of PMLM is considered in this paper, the developed nonlinear control algorithm is applicable for analysis and control of some other practical systems which can be modeled as second-order systems.

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