Use of Genetic Algorithm to Develop Economic design of X-Bar Chart

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Abstract *–Control chart is a key tool in Statistical Process Control. This chart is one type of statistical tool which is used to monitor the quality of a process. It gives a visual representation of the status of the process indication whether the process is under control or not. It is used for finding any variation present in any process. Control charts display the variation in a process, so that anyone can easily determine whether the process is within control or it is out of control. For the design of X-bar control chart we need to find the optimal values of sample size, sampling frequency and width of control limit. In our work, we made a computer program in MATLAB based on Genetic Algorithm for finding the optimal values of above three parameters so that the total expected cost is minimized. Our result showed that Genetic Algorithm provides better result as compared to others reported in the literature.*

Keywords – X Chart; MAT Lab; SPC; Algorithm.

I. INTRODUCTION

One very important factor for any business or manufacturers or service provider to understand is that nothing stays the same as time changes. With time a lot of factors change, quality changes, variation occur, a lot of factor come and go as process continues, and therefore we end up with some variation [1]. But for a product if there is a lot of variation, the customer or user will not be satisfied with that. Hence for this purpose we need some sort of control device, which will inform us when there is too much variation, i.e. there will be some kind of feedback mechanism. It will look at the result of the output, compare with the desired result or nominal level of quality and if the deviation is too large it will trigger a control action [2]. This is the basic principle of any type of control. In case of statistical process control what we do is we let this control be activated when the data shows an exceptional behavior, then we apply some sort of corrective measures or decisions to minimize these variations.

II. MATHEMETICAL MODELING

(a) MONTGOMETRY: Montgomery [15] optimized the Duncan [16] model. The design of X-bar control chart depends on three parameters, they are

i. n (sample size),

ii. h(Interval of Sampling)

iii. K (Control limit Width).

For optimizing the value of Cost function we need to find the optimal value of these three parameters [3].

Duncan considered being the initial process mean for the in-control process. However assignable causes occurs which results in the shift of the process from the mean. They either get shifted from to or, Where:

σ is the standard deviation of the process and

δ is the shift parameter [4].

For finding the control limits of the control chart we need to add k times of the standard deviation to process mean or subtract k times the standard deviation from the process mean, i.e.

UCL= $_0$ + kσ/√n and

LCL= $_0$ - kσ/√n, where k is control limit coefficient

The assignable causes which occur are assumed to be occurring at a rate of per hour and according to Poisson's ratio [6].

Production Rate:

A production cycle has four different periods:

1) Period when the process is in-control
2) Period when the process is out-of-control
3) Sampling time and interpreting time and
4) Time for finding assignable cause

The in control period is given by 1[7]. It is also assumed that for an assignable cause occurring between j^{th} and $(j+1)^{th}$ samples, the expected time of occurrence is given by Equation 1

$$\tau = \frac{1-(1+\lambda h)e^{-\lambda h}}{\lambda(1-e^{-\lambda h})} \qquad (1)$$

The probability for detection of any assignable cause for a sample is given by:2

$$1-\beta = \int_{\infty}^{-k\sigma\sqrt{n}} \phi(z)dz + \int_{-k\sigma\sqrt{n}}^{\infty} \phi(z)dz \qquad (2)$$

Here β is the Type- II error. This is the type of error in which the process is actually out-of-control, but the control chart says that it is in –control and $\phi(z) = (2\Pi)^{-1/2} \exp(-z^2/2)$ is the standard normal density [7]. Also the probability of Type –I error, i.e. the probability that the control chart will indicate out-of-control process but actually it is in control is given by the equation 3.

$$\alpha = 2\int_{k}^{\infty} \phi(z)dz \qquad (3)$$

They showed that expected length of the out-of control period is given by $h/(1-\beta)-\tau$. Sampling and interpretation time is given by a constant g, hence length for this portion of cycle is given by gn. Time for finding a assignable cause is given by D.

The equation for expected length for the cycle is shown in equation 4[8].

$$E(T) = \frac{1}{\lambda} + \frac{h}{1-\beta} - \tau + gn + D \qquad (4)$$

Let Vo be the income for an hour for in –control operation and V_1 be the income for an hour for an out-of –control operation. The cost spent when we take a sample of size n is given by ($a1+a_2*n$), where a_1 is the fixed cost and a_2 is the variable cost of sampling. For finding an assignable cause the cost is a_3 and for a false alarm is a_3.

The number of samples which are taken before a shift is [9]:

$$\alpha \sum_{j=0}^{\infty} \int_{jh}^{(j+1)h} je^{-\lambda t} dt = \frac{\alpha e^{-\lambda t}}{1-e^{-\lambda t}} \qquad (5)$$

For a cycle the net income is given by the equation: 6

$$E(C) = V_0 \frac{1}{\lambda} + V_1\left(\frac{h}{1-\beta} - \tau + gn + D\right) - a_3 - \frac{a_3 e^{-\lambda h}}{1-e^{-\lambda h}} - (a_1 + a_{2n})\frac{E(T)}{h} \qquad (6)$$

Hence the net income per hour can be found out by dividing E(C) with E (T). So we obtain [10]

$$E(A) = \frac{E(C)}{E(T)}$$

Putting the value E(C) and E (T) from equation 6 and Equation 4 respectively and obtained[11] :

$$E(A) = \frac{V_0\left(\frac{1}{\lambda}\right) + V_1\left[\frac{h}{(1-\beta)-\tau+gn+D}\right] - a_3 - a_3'\alpha e^{-\lambda h}/(1-e^{-\lambda h})}{\frac{1}{\lambda + h/(1-\beta) - \tau + gn + D}} \frac{a_1 + a_2 n}{h} \qquad (7)$$

If a_4 be the penalty cost per hour for production in out-of-control state. Let $a_4=V_0-V_1$, Then E(A) can also written as:

$$E(A) = V_0 - \frac{(a_1 + a_2 n)}{h} - \frac{a_4\left[\frac{h}{(1-\beta)-\tau+gn+D}\right] - a_3 - a_3'\alpha e^{-\lambda h}/(1-e^{-\lambda h})}{\frac{1}{\lambda + h/(1-\beta) - \tau + gn + D}} - \frac{a_1 + a_2 n}{h}$$

(8)

$$E(A) = V_0 - E(L)$$

$$E_L = \frac{(a_1 + a_2 n)}{h} + \frac{a_4\left[\frac{h}{(1-\beta)-\tau+gn+D}\right] - a_3 - a_3'\alpha e^{-\lambda h}/(1-e^{-\lambda h})}{\frac{1}{\lambda + h/(1-\beta) - \tau + gn + D}}$$

(9)

Thus we see that the loss cost per hour E(L) is a function of only three parameters n,h and k. Hence we need to optimize these three parameters for finding the optimal value for loss cost function [12-16].

Deventer and Manna:

Deventer and manna took [17] took the Lorenzen and Vance [18] model, they said that a process start in an in –control state said then shifts to an out-of-control state, the time between this is known a production time. They showed the expected cycle time to be

$$E(T) = \frac{1}{\theta} + (1-\gamma_1).S.\frac{T_0}{ARL_0} - \tau + ng + h(ARL_1) + T_1 + T_2$$

(10)

The various costs involved per cycle are as follows:

1) Cost involved by producing defective items,
2) Cost due to false alarm,
3) Cost involved in repairing assignable variation.

Hence total quality cost per cycle is given by Equation 10

$$E(C) = \frac{C_0}{\theta} + C_1(-\tau + ng + h(ARL_1) + \gamma_1 T_1 + \gamma_2 T_2) + \frac{sY}{ARL_0}W$$
$$+(a+bn)\left(\frac{\frac{1}{\theta} - \tau + ng + h(ARL_1) + \gamma_1 T_1 + \gamma_2 T_2)}{h}\right)$$

(11)

Here the above function is dependent on only three variables n,h and k. we can get the total expected cost per unit time dividing the total quality cost by the expected cycle time. It has shown in below equation:

$$E(L) = \frac{E(C)}{E(T)}$$

$$E(L)=\frac{\frac{C_0}{\theta}+C_1(-\tau+ng+h(ARL_1)+\gamma_1T_1+\gamma_2T_2)+\frac{sY}{ARL_0}W+(a+bn)(\frac{\frac{1}{\theta}-\tau+ng+h(ARL_1)+\gamma_1T_1+\gamma_2T_2)}{h})}{\frac{1}{\theta}+(1-\gamma_1).S.\frac{T_0}{ARL_0}-\tau+ng+h(ARL_1)+T_1+T_2}$$

(12)

We have seen that the the above equation depends on only three variable n, h and k.

All other parameters W, Y, b, a, δ, ϴ, C_1, C_0, g, T_1, T_0, T_2, γ_2 and γ_1 are fixed quantities.

There are two more parameters which we find, they are ARL_0 and ARL_1, but they are dependent on α and β. Since ARL_0 is equal to 1/α and ARL_1 is equal to 1/(1-β). They in turn are dependent on n and k. Hence overall E(L) is a function of n, h and k, and we need to optimize the value of this three parameters for optimal value of E(L).

While the process is considered to be in an in-control state, the number of samples expected is given by [18]:

$$s=\sum_i^\infty ip \qquad (13)$$

where *iP* the assignable cause which occurs between i^{th} and $(i+1)^{st}$sample.

$$s=\frac{1}{e^{\theta-h}-1} \qquad (14)$$

The expected time of occurrence within this interval is given by the Equation 14

$$\tau=\frac{1}{\theta}-\frac{h}{e^{\theta-h}-1}$$

Hence by optimizing n, h and k we will find the optimal value of E(L).

III. GENETIC ALGORITHM

Genetic algorithm is a form of algorithm which helps in finding the optimal solution of a problem from a solution space. In this algorithm, initially a possible set of solutions are created which are referred as population. This population then evolves to find a better solution. The general form of the algorithm is given below [18]:

1. First a population is created comprising of random solutions.
2. Then we have to repeat the following steps until termination criteria is met:
   (a) Random selection of two individual from population. More fit the individual more is its chance of selection.
   (b) Cross-over between the two to get a better one.
   (c) New individuals have a random chance to mutate. However this chance is very small, because we do not want the individuals to chance completely.
   (d) Replace old solutions with new one.
3. Finally the one with the highest fitness value is selected as the solution.

Population is a set of solutions. As the algorithm progresses new individuals replace the old ones, new are born and old die. In the population a single solution is termed as an individual. And how good solution the individual provides is termed as its fitness function. More fit an individual is, more is its chance of getting selected for cross- over. Two new individuals are produced by the cross-over of two old individuals. There is also some chance of mutation.

Some question and their asnswer :

There are certain questions like [19];

1. How to represent an individual?
2. How to calculate the fitness of individual?
3. How to select individuals for breeding?
4. How to achieve cross-over?
5. How to achieve mutation of individual?
6. What should be the population size?
7. Termination criteria?

The answer of the above questions varies from problem to problem. But the last two questions can be discussed generally.

Population size can be anything. It can be small and also can be large. Larger the population size, more number of solutions will be available. Hence more number of variations in the population can be achieved by cross-over. This means that better solution can be obtained if the population size is large, rather than which can be achieved if the population size is small. Hence we should take the population size as large as possible. But larger the population size more time will be needed for the algorithm to run.

In the algorithm we saw the ending criteria are much undefined. It is because there are a lot of ways in which we can stop the algorithm. One way is to specify number of generation and many others. All the solution of the other question generally depends on the problem.

An illustrative example:

We will try to find the solution to our above questions with the help of a simple example. Let's take a maximization problem

$f(x, a, b, c) = x^3 + a^2 - b^2 - c^2 + 2bc - 3xa + xc - ab + 2.$

How to represent an individual?

One of the simplest processes to do it is to have an array of four values. But larger the individual more number of variations can be achieved; hence better solution can be obtained. Researchers like Beasley [19] and Holland[20] in their work showed that when we represent individuals by bit strings, best result can be obtained. Let's see some values and understand how we can represent them. We will simply take bits for each variable and finally add all the four values together and get a single bit string.

Let X=12,a=5,b=8 & c=11.

Then we represent it as:

1100 0101 1000 1011

How to calculate the fitness of individual?

Now we know how to represent an individual. Now our aim is to calculate the fitness of individual. In this we need to know about two terms, 'evaluation' and 'fitness' functions. The basic difference between the two is that, evaluation is an absolute quantity and fitness is a relative quantity. Fitness tells us how an individual is better than rest [20].

For our case we can calculate the value of '*f*' which will serve as our evaluation function. Let our population is shown by,

0001 0110 1000 0000

0111 0110 1110 1011

0110 1001 1111 0110

1010 1110 1000 0011

Individual	X	A	b	c	F
0001011010000000	1	6	8	0	-91
0111011011101010	7	9	14	11	239
0110100111110110	6	9	15	6	-43
1010111010000011	10	14	8	3	671

Table 4.1 Values for Population

To calculate fitness function there are many ways. We can use ordinal ranking method, in which individuals are listed according to their value of fitness function. The best value has the highest rank and so on. We can also use averaging method. In averaging method we divide the evaluation values with the average evaluation value. For calculating the average value we added 100 to all the evaluation value. The ordinal and averaging values are given in Table.

Table 4.2: Fitness Function

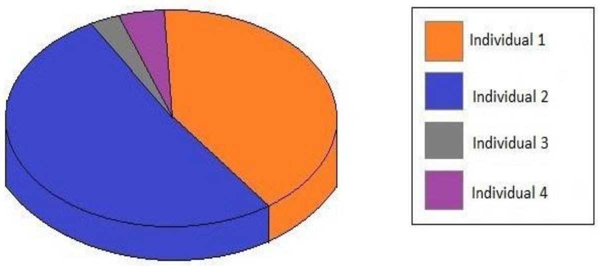
Individual	evaluation	Ordinal	averaging
0001011010000000	-91	1	0.03
0111011011101011	239	3	0.81
0110100111110110	-43	2	0.19
1010111010000011	671	4	2.62

How to select individuals for breeding?

Normally individuals with higher fitness function have higher probability of getting selected. However there is no hard and fast rule for selection [17].

One way is to use the ordinal method. Which give more chance to individuals with more fitness function? In our example the fourth individual will get a chance of

Figure 1 Roulette wheel



40% for selection, the second individual will get a chance of 30% for selection. Similarly the third and first will get a chance of 20% and 10% for selection respectively. Another process can be by using average fitness value.

There are various others ways of selection also, like roulette wheel selection or rank selection methods.

(a)Roulette wheel selection:

In roulette wheel selection individuals are selected according to their fitness value. More the value of their fitness, more area will they cover in the roulette wheel as shown in Fig. 4.1. And hence will have more chance of getting selected.

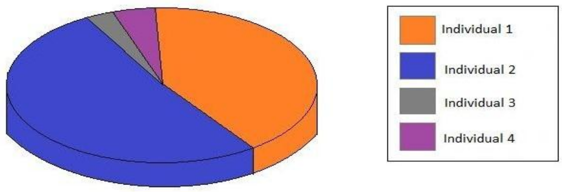


Fig 2 Situation before ranking

(Graph fitness)

But it has some problem, as in this case we can see that individual 1 and 2 will have maximum chance of getting selected, whereas individual 3 and 4 may not get selected at all.

(b) Rank selection

The above problem can be solved using rank selection method. In this first the rank of individual are [18] decided, and then individual are assigned a new fitness value. The worst individual will have fitness value 1, the second worst will have fitness value 2 and so on. In our above case individual 3 will have fitness value 1, and individual 4, 1 and 2 will have fitness value 2, 3 and 4 respectively. This situation has been shown in Fig. 2 and Fig.3 respectively. Now all the individuals will have a fair chance of selection. But the problem with it is that it will lead to slower convergence.

How to achieve cross-over?

Once our selection is complete, i.e. we have selected individuals, they are then crossed-over or breeding is done. Two

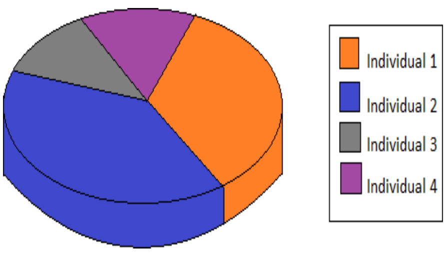


Fig.3. Situation after ranking

(graph of order numbers)

New individuals are created from parents. There are many ways in which cross-over can be performed. Within the individuals, two locations are randomly chosen. This location refers to substrings. Then swapping of substrings is performed between them, and thus two individuals are created. Let's take the previous four individuals again:

0001 0110 1000 0000

0111 0110 1110 1011

0110 1001 1111 0110

1010 1110 1000 0011

Suppose the second individual and the fourth individual has been selected for cross-over. This also goes with the fact that they have the highest fitness function. However we should not forget that selection is a complete random event. The fourth to fourteenth bits have been selected. Then there is a swapping between the two individuals.

0111 0110 1110 1011

011***1 0110 1110 10***11

0110 1110 1000 0011

1010 1110 1000 0011

101***0 1110 1000 00***11

1011 0110 11101011

We should go on doing cross- over until the whole population is replaced with new individual. In our case there is a need for another cross-over. Now let's take that first and fourth individual have been selected randomly. One thing we should understand that, any individual can get selected more than one time, while some may not be selected for a single time. All this process is completely random. Let's perform one

More cross-over [14]:

0001 0110 1000 0000

0001 0110 1000 0000

0001 0110 1000 0011

1010 1110 1000 0011

1010 1110 1000 0011

1010 1110 1000 0000

So, now our new population is:

0110 1110 1000 0011

1011 0110 1110 1011

0001 0110 1000 0011

1010 1110 1000 0000

How to achieve mutation?

Mutation is very small, because we do not want to change the individual drastically, we want only small change. For achieving mutation some random flipping of bit is done,i.e. somewhere 0 is changed to 1, and somewhere 1 is changed to 0.

IV. METHODOLOGY, RESULTS AND DISCUSSION

In our present work we have done the economic design of control chart using genetic algorithm. For this purpose we have taken an example which has been already been solved by Montgomery [15], and compared our result with that of theirs. In next part of our work we compared our result with that obtained by Deventer and Manna [17].

Numerical example-1:

Here we have taken an numerical example from Montgomery [15]. Glass bottles are to be made, thickness of wall is an important criterion in this purpose. If it's very thin, then bottle will burst due to the internal pressure. For reducing the loss cost, the company wants an economic design of X-bar chart. Various known parameters in the process are as follows:

a_1 = \$1, a_2 = \$0.10, a_3 = \$25, a_3' = \$50, a_4 = \$100, = 0.05, δ = 2.0, g = 0.0167 and D=1,

Where :

a_1= is the fixed cost,a_2= is the variable cost,a_3 is the cost of investigating an action signal,a_3' is the cost of investigating a false alarm,a_4= is the cost of operating in out-of-control state for one hour is the mean frequency of process shift, δ is is the size of shift, g is tha is the time in hour for measurement, and D is the average time to investigate any out of control signal. For our work we have developed a MATLAB program based on genetic algorithm to best solution.

Result and discussion for numerical example-1:

By our program we have calculated the optimum value of n, h and k and also calculated the minimum value of cost function. For this purpose we took range of n from 1 to 15, taking only integer value, h from 0.1 to 1 and k from 0.1 to 5 respectively. The result obtained is shown in Table 5.1. By observing the result we found that minimum value of cost function is obtained for n=5 and the corresponding h and k values are 0.815 and 2.982 respectively. The minimum value of cost function found is10.3675. We also observed that the value we obtained by genetic algorithm is superior to that obtained by Montgomery [15]

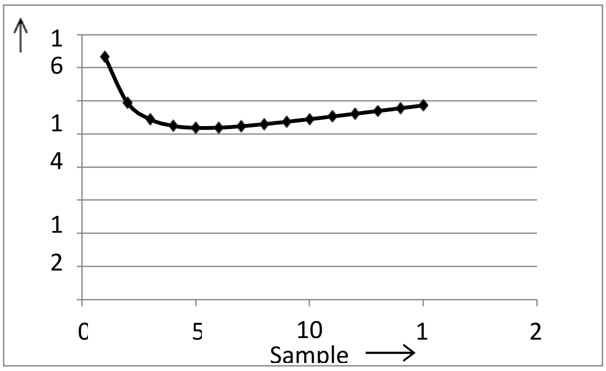


Fig:4 optimum value vs sample size

Our result obtained was found to be superior to Montgomery.

Numerical example-2:

Now going to the next part of our work we took an example from Deventer and Manna [17]. He in his work found out the optimal value of n, h and k for minimizing loss cost function. For this purpose also we developed a MATLAB program based on genetic algorithm and compared the results [18]. As we already found that cost function reduces with increase in the number of generation, hence we omitted that work here. Since most of the X-bar chart in practice are based on 3-sigma limit, here in one part of our work we kept the width of control limit fixed, i.e. k=3, and found the value of cost function. In our next part we found the optimal value of n, h and k for calculating the optimal cost function.

Parameters provided are:

a= \$0.50, b = \$0.10, g = 0.05 hrs, δ = 1, Θ = 0.01, T_1 = 2, Y = \$50, W = \$25, C_0 = \$10/hr, C_1 = \$100/hr, $T_0 = T_2 = 0$, γ_1 = 1, γ_2 = 1, $ARL_0 = 1/\alpha$, $ARL_1 = 1/(1-\beta)$; where –

a is the fixed cost of sampling, b is the variable cost, g is the time in hour, δ is the shift size, Θ is the mean frequency of process shift, T_1 is the time to investigate a action alarm, Y is the cost to investigate a false alarm, W is the cost to investigate a true signal, C_0 is the hourly cost for operating in the in-control state, C_1 is the hourly cost for operating in the out-of –control state, T_0 and T_2 is the value that process continues during search and repair, γ_1 and γ_2 is the indicator that the process continues during search and repair respectively, ARL_0 is the minimum value (lower bound) on the in-control, & ARL_1 is the minimum value (upper bound) on the out-of-control [15].

Result and discussion for numerical example-2:

In the first part of our work we took value of k=3, since in actual practice x-bar chart are based on 3-sigma limit. We vary the value of n from 1 to 20, taking only integer value [20]. The range of h was from 0.5 to 2.5. The result obtained is shown in Table 5.4. and relation between sample size and cost is shown in Fig.5. The minimum value of cost function was found to be 14.9424, this value was obtained for n=14 and h=1.751 and k=3, since we fixed the value of k to 3.

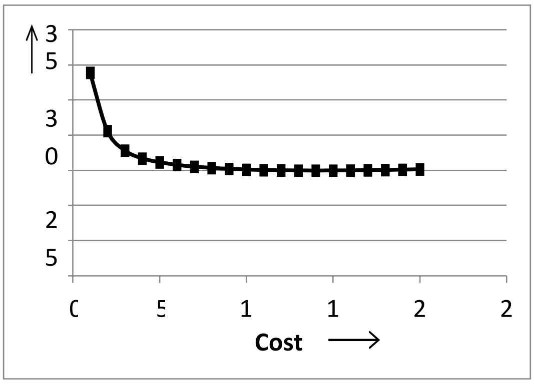


Fig: 5. Sample size versus Optimal cost

For the next part of our work we also found out the optimal value of k along with optimal value of n and h, for calculating the optimal value of E(L), i.e. cost function. For doing this we take the range of n from 1 to 20, h from 0.5 to 2.5 and k from 2 to 3. We found out that the minimum value of E(L) is 14.83 for n=12, h=1.843 and k=2.624. We also found out that the result obtained by us is superior to that obtained by Deventer and Manna [17]. Optimal value of cost function obtained by him was 14.90 while that obtained by us is 14.83.

V. CONCLUSIONS:

In our work we have made the economic design of X-bar control chart and using Genetic Algorithm. Following are the conclusion which we arrived based on the results obtained.

1) Genetic Algorithm provided superior result than that provided in the literature.
2) For Montgomery problem minimum value of cost function was found to be 10.3675, and was obtained for n=5, h=0.815 and k=2.982.
3) By increasing the number of generation the cost function reduces, i.e. we are able to find more optimal solution.
4) For Deventer and Manna problem the minimum value of cost function was found to be 14.9424 and this value was obtained for n=14 and h=1.751 and k=3, since we fixed the value of k to 3.
5) For the next part of our work we also optimized the value and k and found the minimum value of E(L) to be 14.83 for n=12, h=1.843 and k=2.624.

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