Kinematic Synthesis of In-Line Ten Link Gear Slider Mechanism of Variable Topology with 12R-1G-1P Joints

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Abstract- *This article presents the synthesis of an in-line ten link gear slider mechanism of variable topology type for two finitely separated positions. The synthesis is carried out in two phases, considering in-line ten link gear slider mechanism as in-line nine link gear slider mechanism in each phase. Motion generation, one of the tasks of kinematic synthesis is the design criterion. This process is illustrated with an example. The synthesized mechanism comprises of 12-revolute, 1- gear and 1- slider joint. Thus, variable topology method focuses on a non-iterative and simplified way of mechanism synthesis.*

Keywords: Kinematic Synthesis; In-Line Ten Link Gear Slider; Variable Topology

1. INTRODUCTION

In kinematic synthesis of mechanisms, synthesizing a mechanism dimensionally to determine the parameters becomes one of the aspects of design. When number of links and degree of freedom increases, the synthesis becomes difficult. Therefore, Multi- Loop mechanisms, Adjustable Mechanisms and Variable Topology Mechanisms are some of the methods to synthesize mechanisms that can be considered. In this paper a planar in-line ten link gear slider mechanism with degrees of freedom two is considered for synthesis using variable topology method.

A planar in-line ten link gear slider mechanism of variable topology type is a mechanism working in two phases with single degree of freedom in each phase. A link adjacent to permanently fixed link of in-line ten link gear slider mechanism is temporarily fixed and the resulting mechanism becomes an in-line nine link gear slider mechanism with degree of freedom one in each phase.

This paragraph summarizes the literature and works carried out on variable topology mechanisms. Balli and Chand [1] worked on variable topology with synthesize of five bar mechanism using an analytical method for motion between extreme positions. Balli and Chand [2] utilized variable topology with complex number to synthesize a five bar motion and path generation for motion between extreme positions.

In order to synthesize a variable topology mechanism or motion between two dead centers, Balli and Chand [3] have suggested an analytical method. Gadad *et al.,* [4] focused on seven link mechanism of variable topology for function generation using dyad and triad synthesis. Daivagna and Balli [5] worked on off-set five-bar slider with variable topology. Daivagna and Balli [6] designed a mechanism with variable topology for seven-bar slider. Daivagna and Balli [7] synthesized a five-bar slider mechanism for two positions with variable topology. Prashant and Balli [8] reviewed on mechanisms with variable topology.

A mechanism is a mechanical device that has the purpose of transferring the motion or force from an input link to an output link [9]. The mechanism consists of links or bars connected by joints to form a closed loop. The mechanism may also consist of lower pairs, higher pairs or combination of both pairs. Mechanisms with five or more links are available with two or more degrees of freedom. Methods to synthesize these mechanisms are also suggested which are complex. In order to cope up with the difficulties faced by kinematicians, such mechanisms can be made to operate in two or more phases as suggested by variable topology method and thus, synthesis can be carried out easily.

2. VARIABLE TOPOLOGY MECHANISM

The intention of variable topology is to make the process of synthesis simple by reducing two degrees of freedom in-line ten link gear slider mechanism into a single degree of freedom in-line nine link gear slider mechanism in two phases. In order to t carry out various tasks in different phases more effectively the mechanism may be evolved.

The complex number method, one of the analytical synthesis techniques is presented as an ideal tool for modeling linkages with groups of standard form of equations for motion, path and function generation [9]. Hence, the method of variable topology using complex number draws the attention of design engineers.

2.1 In-Line Ten Link Gear Slider Mechanism

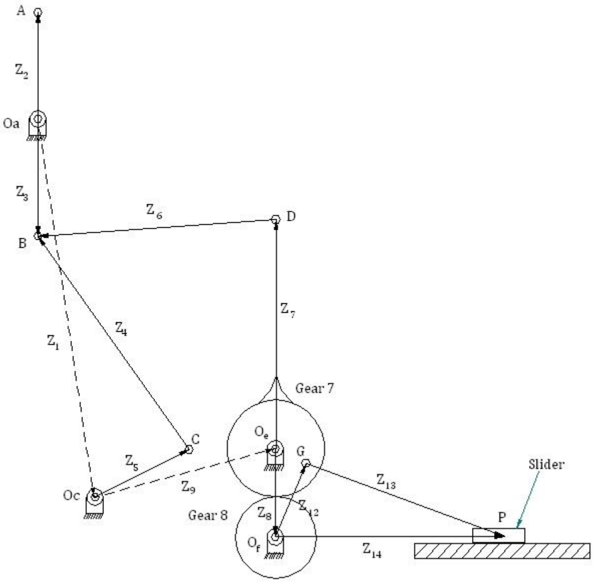

Fig.1 In-Line Ten Link Gear Slider Mechanism

Fig.1 shows an in-line ten link gear slider mechanism in which O_aA and O_cC act as input links to the mechanism. O_aO_c is the ground link fixed. The input links are connected to links AB and CB which in turn are connected to link DB which acts as a coupler. This coupler is connected to gear 7 at point D which is an extension of gear 7 paired to gear 8. The extension O_eD is fixed to gear 7 and rotates along with it. The gear pair is connected by a rigid link O_eO_f. The end points of rigid link act as pivot points on which both the gears rotate. When motion is given to any of the input links, it is transferred to the coupler through links AB and CB to the link O_eD which will rotate according to the desired input motion provided. The vector O_fG represents the rotation of gear 8. When gear 7 rotates with an angle γ then gear 8 will rotate with an angle ε in opposite direction with respect to gear 7. As gear 8 rotates, extension of this gear O_fG also rotates through angle ε which is linked to a connecting rod GP. Whereas P is the slider attached to the connecting rod. The associated angle of rotation with connecting rod is Ω. When the motion is exhibited by the mechanism, slider slides on the surface with linear displacement P_{12}.

With variable topology method, in-line ten link gear slider mechanism can be made to work in different Phases to carry out the different tasks by the slider. This can be achieved by making one of the input links to be active and another input link to be stationary or temporarily fixed.

2.2 In-Line Ten Link Gear Slider Mechanism with Variable Topology in Phase I

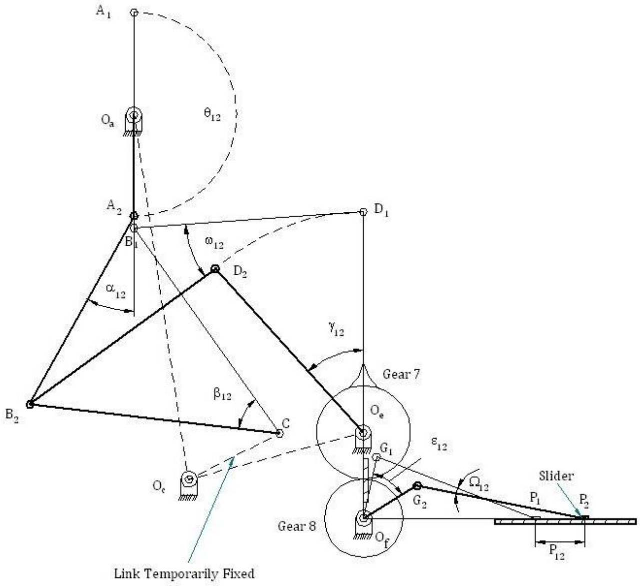

Fig.2 In-Line Ten Link Gear Slider Mechanism Phase I, Position 1: $O_aA_1B_1CD_1O_eO_fG_1P_1$, Position 2: $O_aA_2B_2CD_2O_eO_f G_2P_2$

Fig.2 shows the in-line ten link gear slider mechanism with variable topology in Phase I. In this phase, link O_cC is temporarily fixed and O_aA is the input link. In this Phase Position 1 is treated as starting point and Position 2 is treated as end point. When the input link O_aA is rotated by an angle $\theta_{12} = -180^0$ CW from initial position O_aA_1, the link occupies the new position O_aA_2. Thus, the position of link AB changes from A_1B_1 to A_2B_2. The angle of motion associated with this link will be α_{12}. In the same way, the positions of link DB will change from D_1B_1 to D_2B_2. Since O_cC is temporarily fixed, position of link CB will change from CB_1 to CB_2. The associated angle of motion for the links will be ω_{12} and β_{12} respectively. As the coupler moves, the gear 7 changes it's position from O_eD_1 to O_eD_2 with rotation angle γ_{12}. Due to this, gear 8 rotates by ε_{12} and changes its position from O_fG_1 to O_fG_2. The connecting rod linked to extension O_fG of gear 8 moves by an angle Ω_{12} and changes its position from G_1P_1 to G_2P_2. As a result of this motion, slider P changes its position from P_1 to P_2, which is the linear displacement of slider given by P_{12}.

2.3 In-Line Ten Link Gear Slider Mechanism with Variable Topology in Phase II

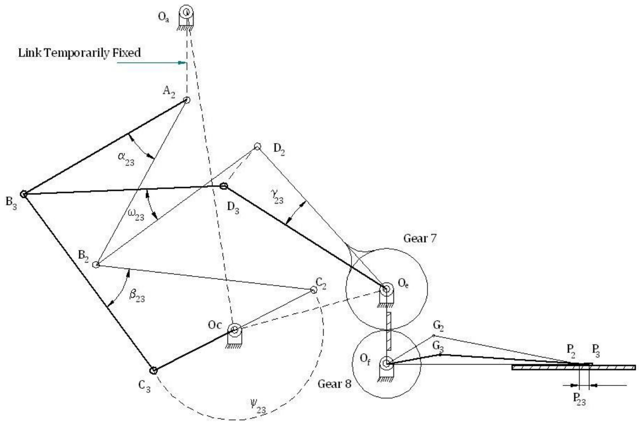

Fig.3 In-Line Ten Link Gear Slider Mechanism Phase II, Position 2: $O_cC_2B_2A_2B_2D_2O_eO_fG_2P_2$ Position 3: $O_cC_3B_3A_2B_3D_3O_eO_f G_3P_3$

Fig.3 shows the in-line ten link gear slider mechanism with variable topology in Phase II. In this phase, link O_cC_2 which was temporarily fixed in Phase I, is released to move from the initial position 2 to position 3, that is, from O_cC_2 to O_cC_3 by an input angle of ψ_{23}=-180^0 CW and link O_aA_2 is fixed temporarily. As explained earlier, the ending position of Phase I will become starting position of Phase II. The suffixes given to links and angles denote the positions of the links and angles. Hence, the complete motion of mechanism can be visualized in terms of three positions occurring in different Phases. Positions 1&2 of the mechanism are related to Phase I and Positions 2&3 are related to Phase II with starting and ending points respectively.

3. SOLUTION CRITERIA

a. Phase I

In Phase I, the displacement vector B_1B_2 and linear displacement vector of slider is prescribed to be δ_{12} and P_{12}. The link lengths Z_2, Z_7, Z_8, Z_{12} and Z_{14} are prescribed. Input crank angle θ_{12}, rotation of gear 7 γ_{12} are prescribed, rotation of gear 8 ε_{12} and movement angle of connecting rod Ω_{12} are prescribed. The free choices available are α_{12}, β_{12} and ω_{12}. With these conditions the parameters to be determined in Phase I are, link lengths Z_3, Z_4, Z_6 and Z_{13}. Hence, the number of unknowns is four and free choices are three. Thus, the solution reduces to ∞^4 in Phase I.

b. Phase II

In Phase II, δ_{23} is the displacement vector and Z_5 is the input link which will be determined. Hence, α_{23} and β_{23} are free choices available. Number of unknowns is two and solution reduces to ∞^2 in this Phase. Thus, the total number of solutions will be ∞^6 for the overall mechanism.

4. SYNTHESIS OF IN-LINE TEN LINK GEAR SLIDER MECHANISM OF VARIABLE TOPOLOGY TYPE

4.1 Synthesis of In-Line Ten Link Gear Slider Mechanism in Phase I

The mechanism moves from Position 1 to Position 2 in this Phase.

The dyad equation for Phase I [9] is, (refer Fig.2)

$$Z_2\,(e^{i\theta 12}-1) + Z_3\,(e^{i\alpha 12}-1) = \delta_{12} \text{ ------------------------------ } (1)$$

In Eq. (1) the prescribed parameters are link length Z_2, displacement δ_{12} =B_1B_2, the input crank angle θ_{12}. Free choice available is α_{12} and Z_3 is the only unknown parameter which will be determined.

Hence, the unknown parameter is determined by the Eq. (2)

$$Z_3 = \frac{\delta_{12}-Z_2(e^{i\theta 12}-1)}{(e^{i\alpha 12}-1)} \text{ -- } (2)$$

The dyad equation for rest of the links,

$$Z_6\,(e^{i\omega 12}-1) + Z_7\,(e^{i\gamma 12}-1) = \delta_{12} \text{ ---------------------------- } (3)$$

In Eq. (3) the prescribed parameters are link length Z_7, angle γ_{12} and displacement δ_{12}. Free choice available is ω_{12} and Z_6 is only unknown parameter and will be determined by Eq. (4).

$$Z_6 = \frac{\delta_{12}-Z_7(e^{i\gamma 12}-1)}{(e^{i\omega 12}-1)} \text{ -- } (4)$$

The dyad equation,

$$\text{Let } Z_4\,(e^{i\beta 12}-1) = \delta_{12} \text{ --------------------------------------- } (5)$$

In Eq. (5) the prescribed parameter is displacement δ_{12} and free choice available is β_{12} and Z_4 is the only unknown parameter and will be determined by Eq. (6).

$$Z_4 = \frac{\delta_{12}}{(e^{i\beta 12}-1)} \text{ -- } (6)$$

Writing the dyad equation for in-line slider crank mechanism

$$Z_{12}\,(e^{i\varepsilon 12}-1) + Z_{13}\,(e^{i\Omega 12}-1) - (\rho-1)\,Z_{14} = 0 \text{ ---------------- } (7)$$

In Eq. (7) the prescribed parameters are Z_{12}, Z_{14} and the stretch ratio ρ. The assumed angular movements are ε_{12} and Ω_{12}. Only unknown Z_{13} will be determined by the Eq. (8).

$$Z_{13} = \frac{(\rho-1)Z_{14}-Z_{12}(e^{i\varepsilon 12}-1)}{(e^{i\Omega 12}-1)} \text{ ----------------------------------- } (8)$$

The stretch ratio is given by Eq. (9)

$$\rho = \frac{Z_{14}+P_{12}}{Z_{14}} \text{ -- } (9)$$

In Eq. (9) the Z_{14} is the distance from center of gear 8 to initial position of the slider. Further, P_{12} is the displacement of slider from Position 1 to Position 2.

Hence, the link lengths Z_3, Z_4, Z_6 and Z_{13} are determined in Phase I.

4.2 Synthesis of In-Line Ten Link Gear Slider Mechanism in Phase II

The mechanism moves from Position 2 to Position 3 in Phase II (refer Fig. 3). The displacement vector δ_{23} can be determined from Eq. (10).

The equation of motion to find the displacement is,
Let, $(Z_3e^{i\alpha_{12}})\ (e^{\alpha_{23}}-1) = \delta_{23}$ ------------------------------ (10)

The value $(Z_3e^{i\alpha_{12}})$ determined in the Eq. (10) of Phase I, is retained in Phase II of the synthesis process. Free choice available is α_{23} and the only unknown is δ_{23} which is to be determined.

The dyad equation of motion is,
$(Z_4e^{i\beta_{12}})\ (e^{i\beta_{23}}-1) + Z_5\,(e^{i\psi_{23}}-1) = \delta_{23}$ ------------------- (11)

In the Eq. (11), ψ_{23} the input crank angle is prescribed and β_{23} is the free choice made. The value $(Z_4e^{i\beta_{12}})$ determined in Phase I is retained and only unknown parameter Z_5 is to be determined from Eq. (12).

$$Z_5=\frac{\delta_{23}-(Z_4e^{i\beta_{12}})(e^{i\beta_{23}}-1)}{(e^{i\psi_{23}}-1)} \text{ ----------------------------------- } (12)$$

Hence, link lengths Z_5 and displacement δ_{23} are determined in Phase II. Considering all the possibilities, the solution space reduces to ∞^2 in Phase II. The complete solution space of the synthesis is the summation of solutions in both Phases which results in ∞^5.

The following are the loop closure equations

To determine Z_1,
$Z_2+Z_3-Z_4-Z_5-Z_1=0$ -- (13)

To determine Z_9
$Z_5+Z_4-Z_6-Z_7-Z_8-Z_9=0$ ------------------------------------- (14)

To determine Z_{14}
$Z_{12}+Z_{13}-Z_{14}=0$ -- (15)

Thus, all the required parameters are determined.

The conventions followed to indicate links and angles in Phase I and Phase II are listed in Table 1 and the summary of Phase I and Phase II for 2 FSP synthesis of motion generation is provided in Table 2.

Table 1 Conventions to indicate links and angles in Phase I and Phase II

Link (vector representation and angle between two different positions of link)	**Phase I (Position 1: $O_aA_1B_1CD_1O_eO_fG_1P_1$, to Position 2: $O_aA_2B_2CD_2O_eO_fG_2P_2$)**	**Phase II (Position 2: $O_cC_2B_2A_2B_2D_2O_eO_fG_2P_2$ to Position 3: $O_cC_3B_3A_2B_3D_3O_eO_fG_3P_3$)**
O_aO_c, Z_1 Fixed Link	Fixed Link	Fixed Link
O_aA, Z_2, θ	θ_{12}	Temporarily Fixed
AB, Z_3, α	α_{12}	α_{23}
CB, Z_4, β	β_{12}	β_{23}
O_cC, Z_5, ψ	Temporarily Fixed	ψ_{23}
DB, Z_6, ω	ω_{12}	ω_{23}
O_eD, Z_7, γ	γ_{12}	γ_{23}
O_eO_f, Z_8	Rigid	Rigid
O_fG, Z_{12}, ε	ε_{12}	ε_{23}
GP, Z_{13}, Ω	Ω_{12}	Ω_{23}
Slider Displacement	P_{12}	P_{23}
Displacement Vector: δ	$B_1B_2=\delta_{12}$	$B_2B_3=\delta_{23}$
Sign Convention	Counter Clockwise (CCW) motion (positive)	Clockwise motion (CW) (negative)

Table 2 Summary of Phase I and Phase II for 2 FSP Synthesis of Motion Generation

Sl. No.	**Description**	**Phase I**	**Phase II**
1	Link fixed	O_cC	O_aA
2	Prescribed Parameter	α_{12}, β_{12}, ω_{12} δ_{12}, Z_2, Z_7, Z_8, Ω_{12} Z_{12}, Z_{14}, ρ	α_{23} β_{23}, ω_{23}, δ_{23}
3	Free Choice	θ_{12}, γ_{12}, P_{12} ε_{12},	ψ_{23} γ_{23}
4	Unknown	Z_3, Z_4, Z_6, Z_{13}	Z_5
5	No. of Solutions	∞^4	∞^2
6	Total No. of Solutions	∞^6	

5. AN ILLUSTRATION

Synthesize an in-line ten link gear slider mechanism as shown in Fig.1 with variable topology for the following specifications:

Specifications for Phase I:
$\alpha_{12}= -30^0$CW

$\beta_{12} = 49^0$CCW
$\omega_{12} = 30^0$CCW
$Z_2 = 0+38i$
$\delta_{12} = -40-65i$
$Z_8 = 0-32i$
$Z_7 = 0+82i$
$Z_{12} = 5+22i$
$Z_{14} = 62+0i$
$\Omega_{12} = 10^0$ CCW

Specification for Phase II:
$\alpha_{23}= -27^0$CW
$\beta_{23}= -45^0$CW
$\omega_{23}=-180^0$CW
$\delta_{23} =-26.9+25.9i$

Solutions:

Solution for Phase I Synthesis:

From Eq. (2), δ_{12}, Z_2 and α_{12} are prescribed. Free choice made is $\theta_{12} = -30^0$ CW. Solving the equation determines the value $Z_3= -00.522-80.156i$
Magnitude: $|Z_3|$= AB = 80.1

From Eq. (4), δ_{12}, Z_7 and ω_{12} are prescribed. Free choice made is $\gamma_{12} = 30^0$ CCW. Solving the equation determines the value $Z_6= -84.0-15.2i$
Magnitude: $|Z_6|$= DB = 85.3

From Eq. (6), δ_{12} and $\beta_{12} = 49^0$ CCW are prescribed. Solving the equation determines the value $Z_4= -51.30+76.388i$
Magnitude: $|Z_4|$= CB = 92.0

From Eq. (8), Z_{12}, $Z_{14,}$ Ω_{12} and stretch ratio ρ are prescribed. Angular movements of links ε_{12} and linear displacement P_{12} are free choice. Solving the equation determines the value $Z_{13} = 59.0-25.8i$
Magnitude: $|Z_{13}|$= GP = 64.3

Hence, the link lengths Z_3, Z_4, Z_6, and Z_{13} are determined in Phase I.

Solution for Phase II Synthesis:

From Eq. (8), $\alpha_{23}= -27^0$ CW and displacement $\delta_{23} = -26.9+25.9i$ are prescribed.

From Eq. (10) ψ_{23} is free choice, $\beta_{23} = -45^0$ CW and δ_{23} are prescribed. Solving the equation determines the value $Z_5 = 30.8+17.65i$
Magnitude: $|Z_5|= O_cC = 35.4$
Hence, link length Z_5 is determined in Phase II
From Eq. (13) and (14)
$Z_1= 19.1-136.1i$
Magnitude: $|Z_1|= O_aO_c = 137.5$
$Z_9= 63.5+27.15i$
Magnitude: $|Z_9|= O_cO_f = 69.0$

Thus, the determined parameters of in-line ten link gear slider mechanism are,
$|Z_1|= O_aO_c = 137.5$
$|Z_2|= O_aA = 38$
$|Z_3|$= AB = 80.1
$|Z_4|$= CB = 92.0
$|Z_5|= O_cC = 35.4$
$|Z_6|$= DB = 85.3
$|Z_7|= O_eD = 82$
$|Z_8|= O_fO_e = 32$
$|Z_9|= O_cO_f = 69.0$
$|Z_{12}|= O_fG = 22.5$
$|Z_{13}|$= GP = 22.5
$|Z_{14}|= O_fP = 62$
$|P_{12}|= P_1P_2 = 18$

6. CONCLUSION

This article concludes that an in-line ten link gear slider mechanism can be synthesized using variable topology method. The synthesized mechanism includes a pair of gears which are utilized as output of the mechanism. Slider crank mechanism attached to the output gear, serves to be one of the important aspects of design. The extreme positions of the slider in each phase can be considered to carry out different tasks. Thus, the variable topology method stands to be one of the prominent methods in synthesis process of in-line ten link gear slider mechanism.

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