

COMPUTATIONAL ANALYSIS OF MICROCHANNEL HEAT SINK

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Abstract- Microchannel heat sinks are currently projected as twenty first century cooling solution. In the present numerical study, heat transfer enhancement in microchannel using extended surface has been carried out. Rectangular microchannel and cylindrical micro fins are used in current study. Three different configurations of extended surface microchannel; Case 1 (Full length fins in complete microchannel), Case 2 (Full length fins at the upstream.) and Case 3 (Full length fins at the downstream.) are compared with plain rectangular microchannel. The analysis is performed for single-phase fluid deionized ultra- filtered water with temperature-dependent properties, for low Reynolds number range of 150–350. Higher Nusselt number is obtained for the case 2, whereas low pressure drop is obtained for the case 3.

Keywords- Rectangular microchannel, cylindrical micro fins, plain rectangular microchannels

I-INTRODUCTION

The relentless pursuit of smaller, more powerful electronic components has resulted in the packing of nearly 4.3 billion transistors on a 541 mm² footprint – a nearly 7 fold increase from what was the densest chip a decade ago. As the packing density increases, there will be a corresponding rise in terms of heat flux due to waste heat dissipated. One of the viable methods is the direct application of liquid cooling through the incorporation of micro-channels, which may either be single or two-phase. For intermediate heat removal capabilities, the single-phase liquid cooling is more attractive as it is reliable and simpler to implement.

Microchannel cooling technology was first put forward by Tuckerman and Pease. They circulate water in microchannel fabricated in silicon chips, able to reach the heat flux of 790W/cm² without a phase change on penalty in pressure drop of 1.94 bar. These microchannel heat sinks are fabricated by the help of micromachining technology mainly by copper[1] or silicon[2][3] as base material. The thermal energy developed during the relentless operation of electronics chips is to be dissipated by incorporating efficient heat sinks on the chips. During the yesteryears; it has been observed that the chip failures are caused due to temperature rise in the circuits. This happens because of accumulation of heat. Hence the micro channel embedded chips are the solution in ultra-compact electronics gadgets.

Zhao et al carried out a numerical studies on geometry features of pin fin microchannel and the characteristics of flowing and heat transfer in two types of MMC structures which are respectively optimized through porosity with different pin-fin distribution and pin-fin located angle were investigated. Moreover, better effect on heat transfer could be achieved at a 30 degree around of rectangular pin fin located angle in our work.

Jadhav et al. compared the performance of microchannels with three different pin fin layouts with the baseline model. The investigation is done for different coolant inflow velocities, keeping the channel and pin fin dimensions constant, and the results are presented in terms of the Nusselt number and pressure drop. From the results obtained in this study, it is concluded that a staggered pin fin orientation in the microchannel enhances the thermal performance better than inline arrangement.

It has been found that for every 2 °C temperature rise, the reliability of a silicon chip will be decreased by about 10 %. The major cause of an electronic chip failure is due to temperature rise (55%) as against other factors which accounts 20 % vibration, 19 % humidity and 6 % dust. So it's a great challenge for the packaging engineers to remove the heat from the electronics chips very effectively.

II-NEED FOR MICROCHANNELS

The present electronic cooling systems primarily use air as the coolant. Air has the advantages of good compatibility with the microelectronic circuit environment, low auxiliary system support requirements, high reliability of the cooling system, low initial cost, low operating and maintenance cost and long development history and experience. But the main concern for air cooling systems is their low heat dissipation potential because of low specific heat value of air. The low heat transfer coefficient in air cooling necessitates the use of heat spreader to increase heat transfer surface area. With the spreader, the air cooled heat sink experiences three thermal resistance components. They are: thermal resistance of the bonding material which bonds the spreader with electronic chip, thermal resistance of the spreader and the thermal resistance due to convection between the fin and the air. The spreader resistance can be reduced by using spreader of good conducting material. The convective resistance can be reduced by increasing the air flow rate. But reducing the resistance of the bonding material still poses problems, because of the requirement of a thick base of high thermal conductivity material for the heat spreader and a good bond between silicon and heat spreader. Another factor which limits further reduction in the bonding material thickness is the difference in thermal expansion coefficients for copper and the silicon substrate. Reducing the thickness of the bonding material causes much higher thermal stresses in the silicon substrate, leading to its mechanical failure. In such situations, microchannels may be directly etched on the back of the silicon substrate, which eliminates the contact resistance and associated thermal expansion problem. In addition to this, the high aspect ratio of microchannels further increases the convective heat transfer surface area and convective heat transfer coefficient, which in turn reduce the convective resistance.

Equation (1) shows Newton's law of cooling which governs convective heat transfer from a surface.

$$Q = hA(T_s - T_a) \dots\dots(1)$$

Generally, the temperature limits may be fixed and therefore the heat transfer rate can be increased by increasing the product hA .

Microchannels are channels whose characteristic dimensions are in the range of 10 μm - 1000 μm (Gad-el-Hak 2006). Thus, deep narrow microchannels etched at the backside of a silicon substrate, increase the surface area for heat transfer. The flow in the microchannels is generally laminar in nature, because of the smaller characteristic dimensions involved. For a fully developed internal flow, the Nusselt number (Nu) is constant.

The Nusselt number is given in Equation (2) as

$$Nu = hD/k \dots\dots\dots(2)$$

Rearranging for h results in Equation (3)

$$h = Nu.k/D \dots\dots\dots(3)$$

It is seen that h is inversely proportional to the diameter of the microchannel, at constant Nusselt number. The microchannels possess the advantage of very high h value (of the order of few thousand $\text{W}/\text{m}^2 \text{ }^\circ\text{C}$) because of their smaller diameter values. Hence for microchannels, the product hA is very large which results in larger convective heat transfer. This makes microchannel a suitable candidate among various high heat removal options of the order of 100 W/cm^2 or more.

III-SIMULATION MODEL

This paper examines the effect of varying height of pin fins in a different arrangement. Following four configurations are considered in this paper:

- i) Plain rectangular microchannel without fins.
- ii) Case 1, Full length fins in complete microchannel.
- iii) Case 2, Full length fins at the upstream.
- iv) Case 3, Full length fins at the downstream

The base model of rectangular microchannel are taken from the experimental work of Qu and Mudawwar[1], who construct the 21 array of microchannel with $231\ \mu\text{m}$ width and $713\ \mu\text{m}$ height by etching the copper base. Deionized ultra-filtered water is used as a fluid in the microchannel. The properties of fluid are temperature dependent and the thermo physical properties of copper are considered as constant. Figure 1 shows the cross sectional view of rectangular microchannel, where on the base wall (B) a constant uniform heat flux of $100\text{w}/\text{cm}^2$ is applied and the top wall(A) is supposed to be adiabatic. For the ease of simulation single microchannel with fluid along with two half neighboring fins is considered.

1.1 Assumptions-

Following are the assumptions considered for the numerical model:

1. Laminar flow under steady-state conditions.
2. No phase change during the flow.
3. No-slip boundary condition.
4. No viscous dissipation.
5. Radiation heat transfer is not applicable.
6. Thermal conductivity, density, and specific heat are considered as temperature-dependent properties of the fluid.
7. Thermo-physical properties are constant for solid substrate.
8. Developing flow at inlet boundary.
9. Adiabatic boundary condition is assumed for top wall and substrate wall along the direction of flow

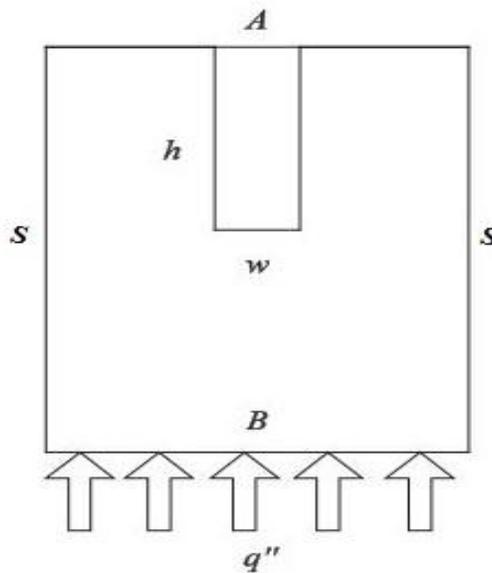
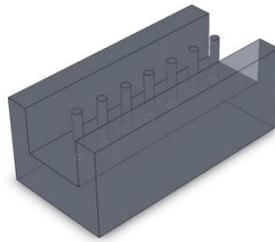
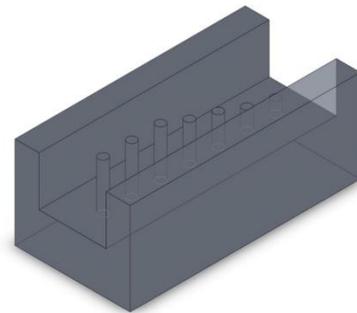


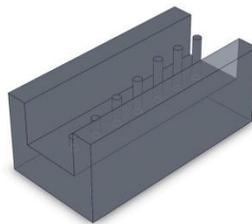
Figure 1. Cross sectional view of geometry.



(a) Case 1



(b) Case 2



(b) Case 3

(c) Figure 2: Different configurations of microchannel (a) Case1, (b) Case2, (c) Case3

Table 1: Dimension of fins for different cases

Configuration	Fins dimension							
	d (m)	height of fins, h(m)						
		Fin 1	Fin 2	Fin 3	Fin 4	Fin 5	Fin 6	Fin 7
Case 1	0.00008	0.000713	0.000713	0.000713	0.000713	0.000713	0.000713	0.000713
Case 2	0.00008	0.000713	0.000713	0.000713	0.000535	0.000357	0.000257	0.000157
Case 3	0.00008	0.000157	0.000257	0.000357	0.000535	0.000713	0.000713	0.000713

1.2 Boundary Conditions- Based on the assumptions, following governing equations are required for fluid and heat transfer analysis.

Continuity Equation

$$\nabla \cdot (\rho_f \vec{V}) = 0 \dots\dots\dots(4)$$

Momentum equation:

$$\nabla \cdot (\rho_f \vec{V} \cdot \nabla \vec{V}) = -\nabla P + \nabla \cdot \mu_f [(\nabla \vec{V} + \vec{V}^t) - \left(\frac{2}{3}\right) \nabla \vec{V}] + \rho_f \cdot \vec{g} \dots\dots\dots(5)$$

where \vec{V}^t is the transpose matrix.

Energy equation:

For Fluid

$$\nabla \cdot (\rho_f c_p \vec{V} T) = \nabla \cdot (k_f \nabla T) \dots\dots\dots(6)$$

For Solid

$$k_s \nabla^2 T = 0 \dots\dots\dots(7)$$

At inlet the fluid temperature and velocity applied are constant as in equations (8) and (9) respectively

$$T = T_{f,in} \dots\dots(8)$$

Where $T_{f,in} = 288 K$

$$u = u_{in} \dots\dots\dots(9)$$

Where u_{in} is taken as the average velocity for each channel in equation (7) as per the equation used in Qu and Mudawwar [1].

$$u_{in} = \frac{\dot{V}}{nA_c} \dots\dots\dots(10)$$

At the interface boundary, heat flux equation (8) and temperature equation (9) are used to obtain an equilibrium between solid and fluid

$$k_s \nabla T_{s,r} = k_s \nabla T_{f,r} \dots\dots\dots(11) \quad T_{s,r} = T_{f,r} \dots\dots\dots(12)$$

On bottom wall of

microchannel heat sink

$$q'' = -k_s \frac{\partial T_s}{\partial y} \dots\dots\dots(13)$$

The flow obtained at the outlet is fully developed both hydraulically and thermally

$$\frac{\partial u}{\partial x} = 0, \frac{\partial v}{\partial y} = 0, \frac{\partial w}{\partial z} = 0 \dots\dots\dots(14)$$

$$\frac{\partial^2 T}{\partial x^2} = 0 \dots\dots\dots(15)$$

Fluid flow and heat transfer characteristic are defined with following variables

$$Re = \frac{\rho_{f,in} u_{in} d_h}{\mu_{f,in}} \dots\dots\dots(16)$$

The average Nusselt number is given by the following

$$Nu = \frac{q''_r d_h}{k_f (T_{r,m} - T_{f,m})} \dots\dots\dots(17)$$

Where

$$q''_r = \frac{q''_{AB}}{A_r} \dots\dots\dots(18)$$

IV- DESCRIPTION OF PROBLEM

The experimental work done by Qu and Mudawar, (2001) on the test apparatus is modelled and simulated in this present study. Water is moving through a straight rectangular smaller scale channel implanted in a test module. 21 rectangular smaller scale openings were machined into micro- channel surface by an accuracy machining procedure. The miniaturized scale openings were equidistantly divided inside of the 1-cm heat sink width and had the cross-sectional measurements of 231 μ m wide and 712 μ m profound. There are 21 parallel rectangular small scale directs in the module.

V-HEAT SINK

A heat sink is a passive heat exchanger that transfers the heat generated by an electronic or a mechanical device to a fluid medium, often air or a liquid coolant, where it is dissipated away from the device, thereby allowing regulation of the device's temperature. In computers, heat sinks are used to cool CPUs, GPUs, and some chipsets and RAM modules. Heat sinks are used with high-power semiconductor devices such as power transistors and optoelectronics such as lasers and light emitting diodes (LEDs), where the heat dissipation ability of the component itself is insufficient to moderate its temperature.

A heat sink is designed to maximize its surface area in contact with the cooling medium surrounding it, such as the air. Air velocity, choice of material, protrusion design and surface treatment are factors that affect the performance of a heat sink. Heat sink attachment methods and thermal interface materials also affect the die temperature of the integrated circuit. Thermal adhesive or thermal grease improve the heat sink's performance by filling air gaps between the heat sink and the heat spreader on the device. A heat sink is usually made out of aluminium or copper.

A heat sink transfers thermal energy from a higher temperature device to a lower temperature fluid medium. The fluid medium is frequently air, but can also be water, refrigerants or oil. If the fluid medium is water, the heat sink is frequently called a cold plate. In thermodynamics a heat sink is a heat reservoir that can absorb an arbitrary amount of heat without significantly changing temperature. Practical heat sinks for electronic devices must have a temperature higher than the surroundings to transfer heat by convection, radiation, and conduction. The power supplies of electronics are not 100% efficient, so extra heat is produced that may be detrimental to the function of the device. As such, a heat sink is included in the design to disperse heat.

To understand the principle of a heat sink, consider Fourier's law of heat conduction. Fourier's law of heat conduction, simplified to a one-dimensional form in the x-direction, shows that when there is a temperature gradient in a body, heat will be transferred from the higher temperature region to the lower temperature region.

Application:-

1. Cooling electronics devices like microprocessors, DSPs, GSPs
2. Refrigeration
3. Heat engines

In common use it is metal object brought in to contact with an electronic component's hot surface- though in most cases, a thin thermal interface material mediates between the two surfaces. Microprocessors and power handling semiconductors are examples of electronics that need a heat sink to reduce their temperature through increased thermal mass and heat dissipations (primarily by conduction and convection and to a lesser extent by radiation).

VI-TEST

Grid Independency Test-

Grid independence study has been carried out prior to detailed analysis in order to eliminate the error due to coarseness of grids. Complete computational domain has been discretized using unstructured grids of tetrahedral volume elements. Finer meshing has been adopted for the finned surface and for the water liquid domain regions, as parameters like temperature, pressure and velocity are more sensitive in these regions. The results obtained for different grid systems have been compared and it is observed that the solutions by the last two grid systems for all cases are very close to each other, outlet temperature and pressure drop variations are less than 0.3%. Hence, in order to save computation time intermediate grid system II has been used for all four cases in final CFD analysis.

VII-RESULT AND DISCUSSION

Good agreement is obtained between the current numerical results and experimental data from Qu and Mudawar¹⁸ for plain microchannel. A comparison of both numerical results and experimental results are shown in Figure 3. Pressure drop at various Reynolds number for heat flux, $100\text{W}/\text{cm}^2$ are compared in Figure 3(a). In Figure 3(b), the comparison of temperature observed in thermocouple and the temperature from numerical simulation for the same position as that of thermocouple, along the length of microchannel heat sink, for the two cases of heat flux are shown. It is clearly observed that the numerical scheme prepared is in good agreement and hence the same may be further applied for evaluating micro- channel heat sink in the laminar regime. Any deviation from the laminar regime causes the deviation from meeting the convergence criteria imposed for obtaining the solution.

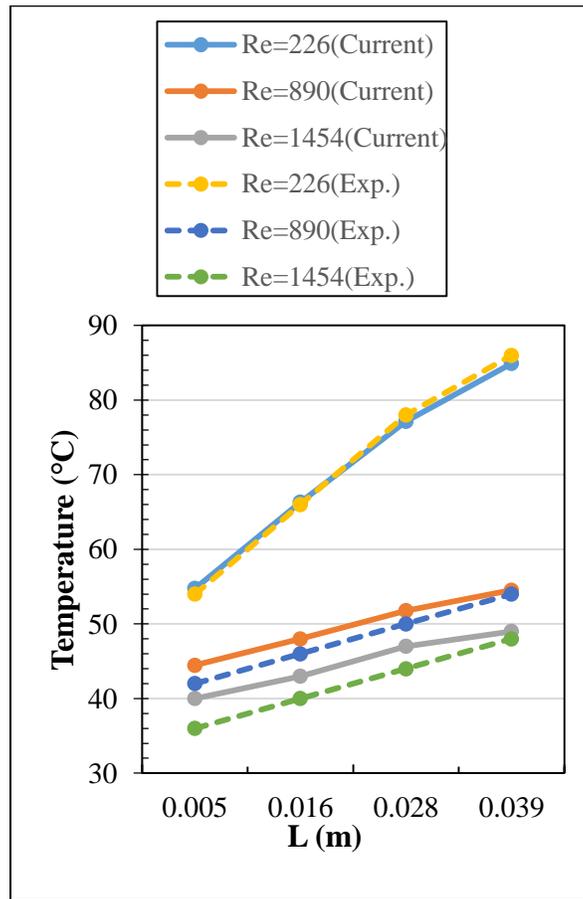


Figure 3: Comparison of numerical result and experimental results of Qu and Mudawar for temperature.

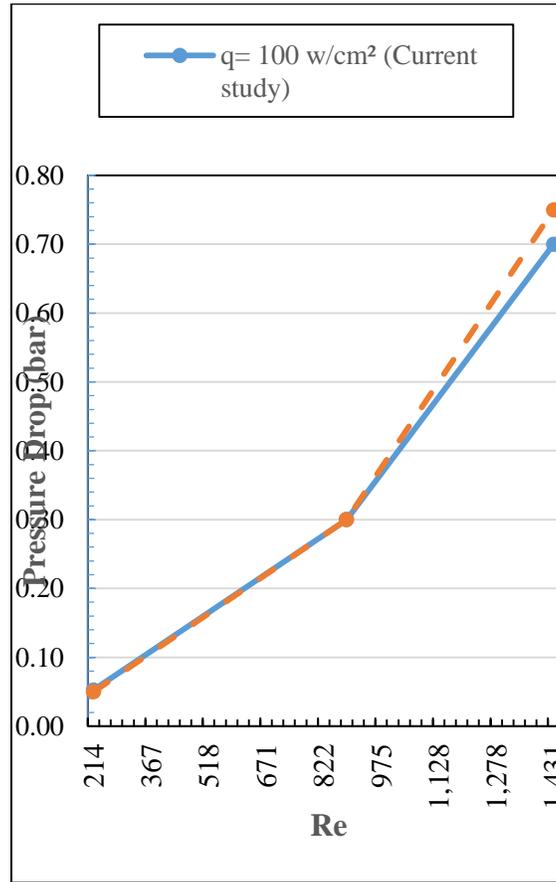


Figure 4: Comparison of numerical result and experimental results of Qu and Mudawar for pressure drop

Temperature Distribution:

In Figure 4, the average interface temperature is plotted for various Reynolds numbers. The average interface temperature is obtained as in the following equation

$$T_{\Gamma} = \frac{1}{\Gamma} \int_{\Gamma} T_{\Gamma} d\Gamma$$

The interface temperature decreases rapidly with increasing Reynolds number, as bulk of fluid taking up heat and removal, increases. The interface temperature decreases for pin finned microchannel compared to plain microchannel, but not much variation is observed at low Reynold’s number.

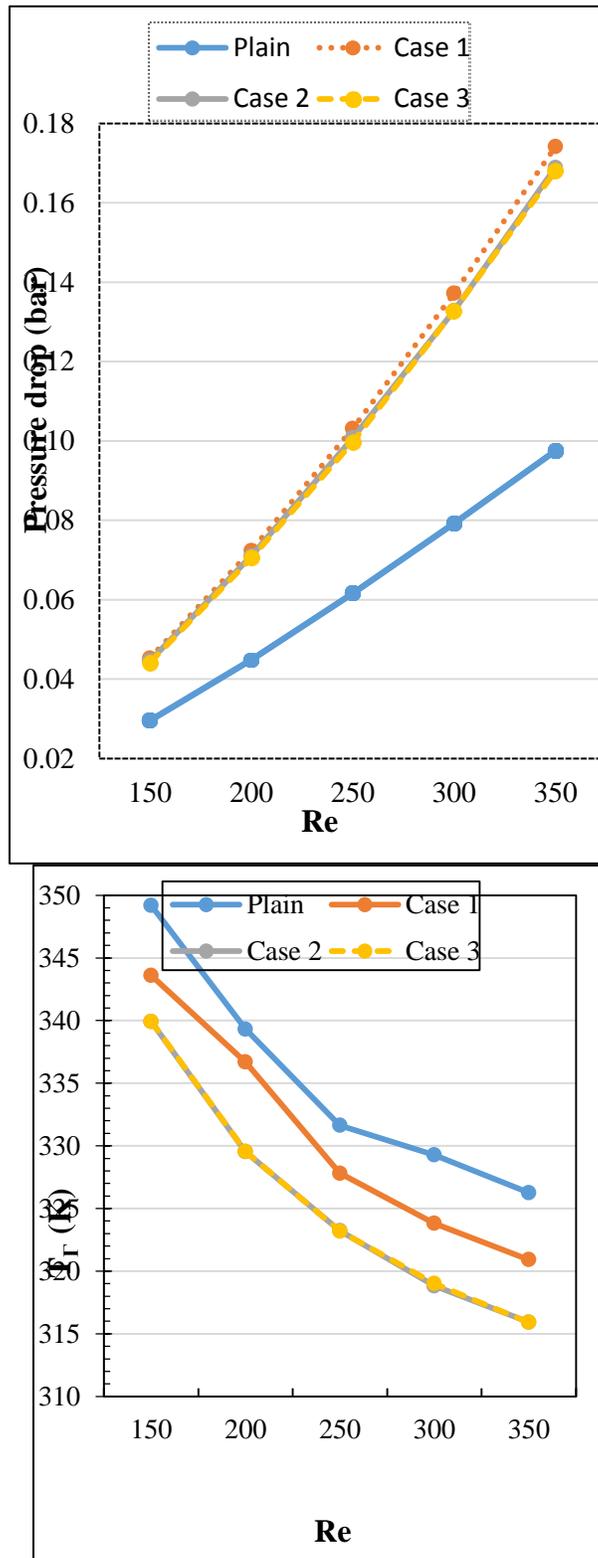


Figure 5: Variation of average interface temperature with Reynolds number.

Pressure Drop Characteristics:

Figure 6 shows the pressure drop for all the cases, including plain microchannel. As expected, the pressure drop is lowest in the plain microchannel, but higher in all the other cases. The increase in pressure drop occurs due to the increase in resisting surface for the flow. However, the pressure drop for case 3 is lower than for all corresponding case 2 because in Case 2 longest fins are entrenched at the lower temperature region where they offer large resistance to the fluids having higher viscosity whereas pin fins with maximum height are employed in higher temperature region of fluid having low viscosity offering low resistance to fluid in Case 3. The slope of pressure drops up to the value of Reynold's number 200 for all the case **Fig 6: Fig 6: Variation of average interface pressure drop with Reynold's number.**

Except Case 1 are nearly same. On increasing the Reynold's number the slope of Case 1 is more than Case 2 and 3 because of offering more resistance.

VIII- CONCLUSIONS

In the present analysis, 3 cases of microchannel with pin fins are considered and compared with plain rectangular microchannel having flux of 100 W/cm^2 at the underside of heated base wall of domain with Reynold's number starting from 150 to 350.

The important conclusion based on this study are:

1. The magnitude of interface temperature decreases gradually after the implementation of pin fins compared to plain case and also decreases with an increase in Reynold's number. Case 3 has minimal interface temperature at higher Reynold's number and nearly same to case 2 while the case 1 has highest mean interface temperature in all pin fins cases.
2. The capability of heat transfer is improved by the entrenchment of fins at channel base wall because of the continuous redevelopment of flow and induced mixing. Value of Nusselt number is maximum for case 2 at low Reynold's number. Case 3 has slightly less value of Nusselt number than case 2 whereas the case 1 has the lowest Nusselt number among all the pin fin cases due to extra flow resistance offered.
3. The Pressure drop of all pin fin microchannel cases is higher than plain microchannel and increases with increase in Reynold's number. Case 1 has a higher pressure drop because it has a maximum area of fins to offer resistance followed by the case 2 and 3 which has nearly same pressure drop.

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