EFFECTIVENESS ANALYSIS OF A CROSS-FLOW HEAT EXCHANGER FOR COMPLEX FLOW ARRANGEMENTS USING MATHEMATICAL MODELLING

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Abstract - *The cross-flow heat exchangers are employed in variety of commercial, domestic and industrial applications such as air conditioning, power generation, refrigeration, petrochemical, petroleum, and other industries because of the wide range of design possibilities, simple manufacturing technology, low maintenance and low cost. The performance of the heat exchanger is a significant effect on the performance of cooling systems. The determination of ε-NTU relation by mathematical model for cross-flow heat exchangers with complex flow arrangements is presented in the current work. Tube element approach is used in this model. In this approach, the coil along the path of the tube fluid is discretized, to find the outlet temperatures of the heat exchanger. Tube side fluid temperature is supposed to be constant in each cross section of the element since the heat capacity ratio* $C^*=C_{min} / C_{max}$ *in the element tends toward zero. Therefore temperature is regulated by the effectiveness of a local element corresponding to a condenser or evaporator type element. A mathematical model, Numerical discretization, and algorithm for calculating the effectiveness of a cross-flow heat exchanger are described in this paper. The proposed model represents valuable research tool for efficiency of the heat exchangers in experimental studies and theoretical work.*

Key Words: Mathematical model, Effectiveness, cross flow heat exchange, ε-NTU

1. INTRODUCTION

To estimate the performance of the heat exchanger, if the inlet temperatures are known, ε-NTU, the effectiveness-number of transfer units (ε-NTU) method is used which optimizes the algebra involved to predict the performance of complex flow arrangements. For the design calculations and experimental studies of a heat exchanger ε -NTU relationships in the algebraic form are commonly used. The mechanism of pressure drop and heat transfer is complex for compact heat exchangers. Hence, the analytical derivation of ε-NTU is challenging work. The application of the appropriate heat transfer correlation to the sizing or rating of a heat exchanger is performed only after the use of correct ε-NTU relations.

In the present paper, a review of several designs of the plate-fin and tube heat exchangers is conducted. Air is commonly passed between the fin plates for this kind of heat exchanger. Domanski [1] developed a discretization model based on a tube-by-tube approach. Every tube that has associated fins acts as a heat exchanger. Bensafi et al. [2] suggested a model that would make heat exchangers discretizes into tube elements. Heat transfer coefficients and local values of properties are used. A computational procedure requiring data on the circuit and coil geometry and operational parameters such as mass flow rate and temperature and pressure is also presented. The cooling coils were analyzed by log mean temperature difference (LMTD) method in this model. Vardhan and Dhar [3] introduced a model that discretizes the coil into nodes along the tube-side path and performs repetitive movement between the inlet and outlet of the tube element while at the same time updating the values of the air stream properties. The effectiveness is determined by the mixed-unmixed cross-flow relationship between ε- NTU (Kays and London,[4] used by each element with the airside specified by the minimum heat capacity. Corberán and Melón [5] established a model that discretized the tubular path with a UA-log mean temperature difference local approach for testing the R134a evaporation and condensation. A theoretical comparison with experimental results indicates the most effective correlation for the numerical simulations. Bansal and Purkayastha [6] were designed to simulate the effectiveness of alternative refrigerants in heat exchangers of heat pump /vapor compression refrigeration systems using a similar discretization model based on the ε -NTU procedure.

In the present work, a mathematical model for evaluating ε -NTU relationships with complex flow arrangements for cross-flow heat exchangers. The model is based upon the tube element method. Accordingly, the outlet heat exchanger temperatures are obtained by discretizing the coil along the tube fluid path. Each element consists of a piece of tubing that has its fins. The size of the element is small enough to provide a reasonable ratio of heat capacity for the external fluid. In the cross-section of the device, the heat capacity ratio C*=Cmin / Cmax tends to be zero, and the temperature of the tube-side fluid is assumed constant. Therefore the temperature in the element is controlled by local effectiveness corresponding to that where one of the fluids changes phase (constant temperature). A mathematical model, Numerical discretization, and algorithm for calculating the effectiveness of a cross-flow heat exchanger are described in the methodology section.

2. METHODOLOGY

2.1 Mathematical Modeling

The effectiveness of a heat exchanger is defined as the ratio between the actual heat transfers to the maximum possible heat transfer.

$$\varepsilon = \frac{q}{q_{max}} = \frac{C_h (T_{h,i} - T_{h,o})}{C_{min} (T_{h,i} - T_{c,i})} = \frac{C_c (T_{c,o} - T_{c,i})}{C_{min} (T_{h,i} - T_{c,i})} \quad (1)$$

The effectiveness can be expressed as a function of the nu mber of transfer units NTU, the heat capacity ratio C *; and the heat exchanger flo w arrangement.

$$\varepsilon = f(NTU, C^*, \text{flow arrangement}) \quad (2)$$

The dimensionless number (NTU) used for analysis of the heat exchanger can be defined as

$$NTU = \frac{UA}{C_{min}} \quad (3a)$$

$$\text{Heat capacity rate ratio } c^* = \frac{C_{min}}{C_{max}} \quad (3b)$$

Where C_{min} / C_{max} is equal to C_c / C_h or C_h / C_c, depending on the relative magnitudes of the heat capacity of the hot and cold fluid

2.1.1 Governing Equations for the Cross-Flow Heat Exchanger

The governing equations described in this section are established in accordance with Kays and London [4] for cross flow heat exchangers with one fluid mixed and the other unmixed. The temperature variation for a mixed unmixed heat exchanger with a single pass cross-flow is shown in figure 1. In this case, the hot fluid (tube side) is supposed to be heated, and the cold is unmixed. However, the model developed for this case also applies when the hot fluid is unmixed and cold fluid is mixed. The subscript for the hot and cold fluid in all equations must be interchanged for this case.

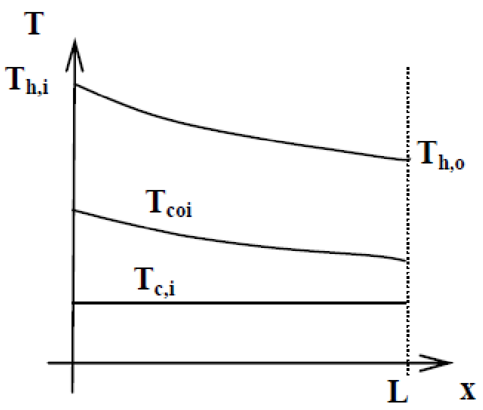

Fig. 1: Variations in temperature of fluid in the longitudinal direction

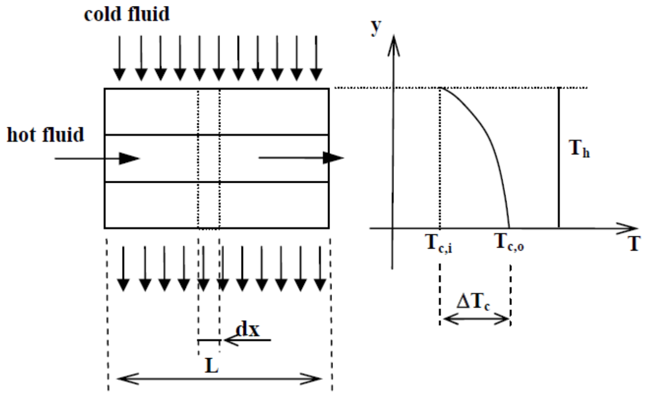

Fig. 2: Variation in cross-flow air temperature in a differential element of the heat exchanger

The variations in hot and cold fluid along the length (L) of the heat exchanger are shown in figure 1. Figure 2 shows the temperature variation of the cold fluid and hot fluid on a differential section (dx).In this infinitesimal section, the cold mass flow rate is low and therefore the hot fluid temperature is constant. The energy balance for the differential length, dx, for the hot and cold fluids can be written as

$$\delta_q = -C_h dT_h \quad (4a)$$

$$\delta_q = dC_c \Delta T_c \quad (4b)$$

Where $\Delta T_c = (T_{c,o} - T_{c,i})$ is the variation in temperature of the cold fluid for the differential length, dx.

Assume that at the differential section dx, the cold mass flow rate is low and then the heat capacity of hot fluid C_h is constant. The heat capacity ratio of the differential section is given by

$$dC^* = \frac{dC_c}{C_h} \quad (5)$$

For a condenser or an evaporator, C*=0, because when the temperature of one fluid is constant its specific heat and heat carrying capacity is infinity .Therefore the equation for effectiveness become $s\varepsilon = 1 - e^{-NTU}$. So, a condenser-type of effectiveness expression is applicable for Eq. (5) and a given temperature (Fig.2), A parameter Γ that expresses the 'local effectiveness' in the differential section dx can be defined as (Kays and London [4]) using the effectiveness definition (Eq. 1).

$$\Gamma = \frac{\Delta T_c}{(T_h - T_{c,i})} = 1 - e^{-\frac{UdA}{dC_c}} \quad (6)$$

Assume that both cold flow and heat transfer area A distributions are uniform, the following relations are valid

$$\frac{dC_c}{dA_{fr}} = \frac{C_c}{A_{fr}} = constant \quad (7a)$$

$$\frac{dC_c}{dA} = \frac{C_c}{A} \quad (7b)$$

Thus along the tube length L

$$\Gamma\text{=}1\text{-}e^{\frac{-UA}{Cc}}\text{=constant} \quad (8)$$

Combining equations (4.a),(4,b) and (6) and separating the variables results in

$$\frac{dT_h}{T_h - T_{c,i}} = -\Gamma dC^* = -\Gamma \frac{C_C}{C_h}\frac{dA_{fr}}{A_{fr}} \quad (9)$$

In Eq. (9), it is seen that C_c, C_h, and A_{fr} are the total magnitude and are not variables. As previously mentioned, the above formulation is true for a single pass cross-flow heat exchanger with one mixed fluid and the other unmixed. For this type of heat exchanger, by integrating eqn(9) ,ε-NTU relation can be obtained. However, most heat exchanger for engineering applications has a complex geometry with more than one row and several circuits. Hence it is not easy to implement the above method. Due to the non-validity of the Eqs(7b) and (8),the derivation and integration of an equation like Eq. (9), is complicated. This results in a numerical method for the overall heat exchanger area and to a variation of the cold (unmixed) fluid temperature distribution, $T_{c,i}$ in each row of the heat exchanger section.

2.2 Numerical Model

A new simulation model was developed to solve this problem, based on the above method. This new model consists of applying the previous equations to a control volume called the element. These equations are valid since each element functions as a single pass cross-flow heat exchanger with one mixed and the other unmixed fluid. These elements are obtained by discretizing the heat exchanger into 3D control volumes (Fig. 3). This form of discretization produces elements defined by the triplet (i, j , k), where the indices 1≤i≤Ne, 1≤j≤Nt and1≤k≤Nr respectively, represent the element on a particular-tube, the tube, and the row. The model performs an element by element iterative analysis based on the application of the parameter Γ (Eq. 8) for each element, allowing simulation of cross flow heat exchangers with different flow arrangements (one mixed fluid and the other non-mixed, both mixed and un mixed) by using governing equations (Section 2.1.1).

Thus the technique of simulation proposed in this paper must be divided into two parts. The first is the use of governing equations (section 2.1.1) to obtain the simulation model described in this section for a small mixed / unmixed cross-flow heat exchanger (element). The second is to simulate the cross-flow with one fluid mixed and the other unmixed, both fluids mixed, or both fluids unmixed for all elements through the proposed algorithm (section 2.3)

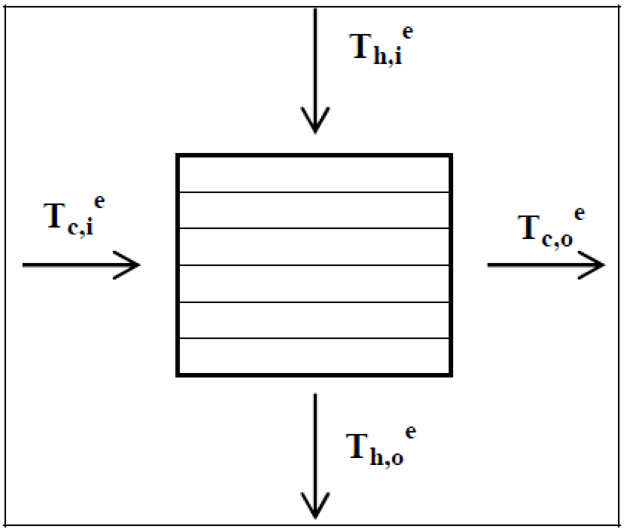

Fig. 3: Element (i, j, k) with one fluid mixed and the other unmixed.

Here we consider the elements or volumes of control are small. The integration of the governing equation (section 2.1.1) is the base of the proposed simulation model. This condition supports the validity of the model developed for the simulation. So the temperature of the hot mixed fluid varies linearly with an exponential fluctuation of the unmixed cold fluid. The average fluid temperature of the hot element is calculated as

$$T_h^e\text{=}0.5(T_{h,i}^e + T_{h,0}^e \;) \quad (10)$$

Where superscript e is linked to a specific element (I, j , k). Integration of Eq. (4a) the resulting element in

$$q^e = -C_h^e(T_{h,0}^e - T_{h,i}^e) \quad (11)$$

From the heat balance for the cold fluid, through integration of Eq. (4b), and using Eq. (7a), the following expression can be obtained:

$$q^e = \Delta T_h^e \int_e dC_c = \Delta T_h^e \int_e \frac{C_c}{A_{fr}} dA_{fr} = \Delta T_h^e C_C^e \quad (12)$$

Where

$\Delta T_c^e = T_{c,o}^e - T_{c,i}^e$ represents the variation in cold fluid temperature in the element. The algebraic equation system is closed by integration of Eq. (6) into the element that results in

$$\Gamma^e\text{=}\frac{\Delta T_c^e}{(T_c^e - T_{c,i}^e)} = 1 - e^{\frac{-(UA)^e}{Cc^e}} \quad (13)$$

For a Single –pass cross flow heat exchanger, the last terms of eqn (13) and eqn (8) are equivalent. But for an another form of cross –flow heat exchanger it is not same since analytical integration for complex flow arrangements in eqn (6) lead to an eqn(8) is not practical anymore because in eqn 7(b) it is assumed that heat transfer area A distributed uniformly. This work consists of a closed system of five equations and five unknowns for each element ,ie q_e, Γ_e, $T_h{}^e$, , $T_{h,0}{}^e$ and $\Delta T_c{}^e$, and $T_{h,i}{}^e$, $T_{c,i}{}^e$, $(UA)^e$, $C_c{}^e$, , and $C_h{}^e$, are the given parameters from the eqn(10) to eqn (13) represent in the simulation model. The iterative procedure is needed to solve the system for the heat exchanger. The next section explained the algorithm for this process

2.3 Algorithm for the Computation of Effectiveness

The simple and complex geometries involving multi-pass parallel and counter cross-flow heat exchangers with multiple circuit configurations can be simulated using the proposed model. The procedure allows the measurement of various parameters such as relationships between distribution of cold and hot fluid temperature, coefficients of friction and heat transfer ε-NTU, difference in mean temperature, heat transfer field, and ε-NTU. However this work will mainly be related to gain ε-NTU graphs for several arrangements for the cross-flow heat exchanger. For model derivation the heat exchanger and tube curves are assumed to be adiabatic, the mixed fluid inlet conditions are homogeneous for each part, and the unmixed fluid is distributed uniformly.

The computational algorithms (shown below) were built to obtain the relationships between ε-NTU. These algorithms presume the hot fluid is mixed together, and the cold is not mixed. However, it is also true, as previously assumed, when the hot fluid is unmixed and the cold is mixed in. In this case all related equations of the algorithms interchange the subscripts (hot and cold).

Algorithm for Calculation of Effectiveness

Step 1: Read a heat exchanger geometry file.
Step 2: inputs NTU, C^*
Step3: Choose C_{min}=(C_c or C_h).
Step 4: Introduce $T_{c,i}$, $T_{h,i}$, and UA values.
Step 5: Compute $(UA)^e$
Step 6 :$(UA)^e = \frac{UA}{N_e N_t N_r}$
Step 7: Compute $C_c{}^e$ and $C_h{}^e$
Step 8: If $C_{min} = C_c$ then go to step 9
Step 9: $C_c{}^e = \frac{UA}{NTU N_e N_t}$ and $C_h{}^e = \frac{UA}{NTU c^* N_c}$
Step 10: If $C_{min} = C_h$ then go to step 11
Step 11: $C_c{}^e = \frac{UA}{NTU c^* N_e N_t}$ and $C_h{}^e = \frac{UA}{NTU N_c}$
Step 12: Compute temperature distribution iteratively

Compute: $\Gamma^e = \frac{\Delta T_c{}^e}{(T_c{}^e - T_{c,i}{}^e)} = 1 - e^{\frac{-(UA)^e}{C_c{}^e}}$
Compute initial temperature distribution
Do
Compute the sum $S = \sum_{i,j,N_r} T_{c,0}{}^e$
Compute temperature distribution following circuiting
For 1 to N_c
Update temperatures for cold fluid side While not(end of a circuit)
End While
End For
Go to step 13
Step 13: Compute the new value of the sum $S_{new} = \sum_{i,j,N_r} T_{c,0}{}^e$ while $\frac{|S_{new} - S|}{S} < Tolerance$
Step14: Compute effectiveness of heat exchanger

$$T_{c,0} = \frac{1}{N_t N_e} \sum_{i,j,N_r} T_{c,0}{}^e$$

$$T_{h,0} = \frac{1}{N_c} \sum_{each\,last\,circuit\,element} T_{h,0}{}^e$$

$q = \sum_{i,j,k} q^e$ or $q = -C_h(T_{h,0} - T_{h,i})$ or $q = C_c(T_{c,0} - T_{c,i})$ go to step 15
Step 15: Compute

$$\varepsilon = \frac{q}{q_{max}} = \frac{C_h(T_{h,i} - T_{h,0})}{C_{min}(T_{h,i} - T_{c,i})} = \frac{C_c(T_{c,0} - T_{c,i})}{C_{min}(T_{h,i} - T_{c,i})}$$

Step 16 Stop

3. CONCLUSIONS

In the present work a mathematical model was developed for a cross flow heat exchanger with complex flow arrangement. This model was applied to analyze the effectiveness of the cross flow heat exchanger. The governing equation developed in this study is solved numerically. An algorithm for computing the effectiveness of a cross flow heat exchanger is also presented in this work. A simulation element-by element model discretizes the entire heat exchanger into smaller ones along the tube fluid path based on a concept of local effectiveness. It determines the overall effectiveness of the heat exchanger by calculating the temperature distribution of both fluids. The model developed is a valuable research technique for experimental and theoretical studies on the performance of heat exchangers due to its predictive capability.

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