Investigating the Physics of Cylindrical Motion

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Abstract

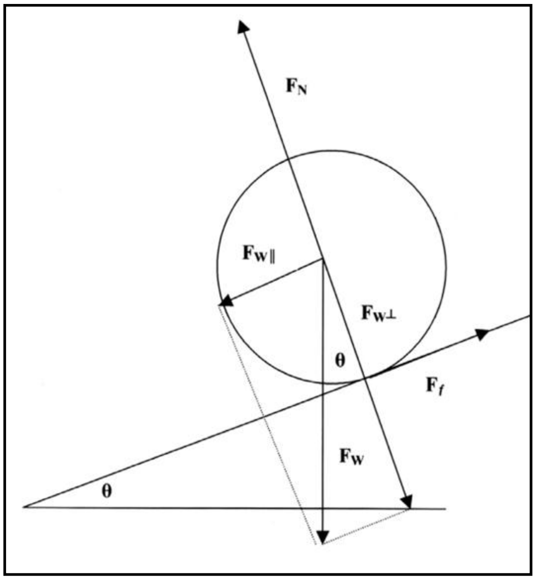
Research on the physics of motion has come a long way since Newton's infamous observation of a falling piece of fruit. Indeed, Newton's laws of gravitation and motion pervade all areas of mechanics research today. One context in which Newton's laws of motion have been extensively studied is cylindrical motion. This paper explores the factors that affect cylindrical motion down an inclined plane via an extensive literature review. In particular, I examine the angle of the inclined plane, mass and radius of the cylinder, and surface composition of the cylinder. My findings are as follows: As the angle of the incline increases (up to a certain angle), the cylinder moves with a greater velocity down the inclined plane. At a high enough angle, the cylinder rolls and slips at the same time, which causes it to have a greater acceleration that when there is only rolling. The motion of the cylinder remains unaffected by the mass and radius of the cylinder. Having a viscous layer around the cylinder plays an important role in lubricating the motion down the inclined plane. Studying this topic is of paramount importance due to the huge number of important real-life applications it carries, such as the motion of cartwheels, moving heavy objects on a ramp, and more. This literature review provides an in-depth analysis of the different factors that affect the motion of a cylinder down an inclined plane in the presence and absence of slippage, and thus providing a strong basis for future, more in-depth studies.

Introduction & Background Information

Researchers have been studying motion in a variety of ways for centuries. In fact, some of the earliest studies on motion date back to as early as 1583, when Galileo Galilei hypothesized that an object would never stop moving on an "infinitely" smooth surface. Later, around 1687, Galileo's hypothesis was further developed by Sir Isaac Newton, who worked in many areas of physics and mathematics. Newton was a master at turning theories into practice; he turned Galileo's theory into a Physics practice known as Newton's first law of motion. Although there is no evidence, it is widely believed that the inspiration behind Newton's first, second, and third laws of motion emerged when an apple fell on his head while he was sitting in a garden (*Isaac Newton*, 2022). This caused him to theorize about gravity, which plays a critical role in his laws of motion. Newton's three laws of motion are defined as follows: (1) An object at rest remains at rest, and an object in motion remains in motion at constant speed and in a straight line unless acted on by an unbalanced force; (2) The acceleration of an object depends on the mass of the object and the amount of force applied; and (3) Whenever one object exerts a force on another object, the second object exerts an equal and opposite force on the first object. In 1686, Newton's three laws of motion were presented in the "Principia Mathematica Philosophiae Naturalis". Afterwards, they were used for a wide variety of applications. For example, Orville and Wilbur Wright applied Newton's Laws of Motion to the flight of their aircraft.

One context where Newton's Laws of Motion have been extensively studied is in cylindrical motion. A cylinder is defined as a "three-dimensional solid that contains two bases covered by a curved surface". When the cylinder is placed on an inclined plane, the component of the force of gravity that is parallel to the surface is unbalanced. The presence of the unbalanced force will cause the object to accelerate down the inclined plane. Figure 1 depicts a cylinder that is freely rolling down an inclined plane *(Chakrabarty et al.)*

Figure 1 – Cylinder on an Inclined Plane



With:
F_f: Friction force acting on the cylinder due to surface friction with board
F_N: Normal force from the board acting on the cylinder perpendicular to the plane.
θ: Angle of elevation of the inclined plane
F_W: Weight of cylinder (mg)
$F_{W\parallel}$: Component of the weight of the cylinder parallel to the plane (mgsin θ)
$F_{W\perp}$: Component of the weight of the cylinder perpendicular to the plane (mgcos θ)

The source of the rolling motion of the cylinder is its frictional force, as it creates a torque about the center of mass of the cylinder, which causes it to roll. Since the frictional force is proportional to the normal reaction force, it can be deemed that: $F_f = \mu_r F_N = \mu_r mgcos\theta$, wherein F_f is the frictional force, μ_r is the coefficient of rolling friction, and F_N is the normal reaction force. With reference to this equation, as the value of θ increases, there will be a decrease in the value of $cos\theta$. Thus, the frictional force will gradually become smaller. Hence, hypothetically, there will be a point when the frictional force is too small to make the cylinder roll as fast as theoretically expected. This leads to slipping; Slipping occurs as the instantaneous velocity of the rolling body is not zero at its point of contact with the medium that supports it. In other words, the body rolls and 'slides' simultaneously in the direction of its motion.

For normal rolling motion (non-slipping conditions), linear velocity (v) and angular velocity (ω) are related by the equation $v = r\omega$. In this case, where there is an absence of slippage, the following conclusions about the energy from the cylindrical motion can be drawn, stemming from the law of conservation of energy.

With:
m: mass of the cylinder
g: acceleration due to gravity
Δ*h*: change in vertical height of the inclined plane
v: velocity of the rolling cylinder
ω: angular velocity
I: moment of inertia
r: radius of the cylinder

When considering linear kinetic energy:

$$E_{potential} = E_{kinetic} + E_{rolling}$$
$$mg\Delta h = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$
$$mg\Delta h = \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{r}\right)^2$$
$$mg\Delta h = \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v^2}{r^2}\right)$$
$$mg\Delta h = \frac{1}{2}mv^2 + \frac{1}{2}(\frac{1}{2}mr^2)\left(\frac{v^2}{r^2}\right)$$
$$mg\Delta h = \frac{1}{2}mv^2 + \frac{1}{4}mv^2$$
$$mg\Delta h = \left(\frac{3}{2}\right)\frac{1}{2}mv^2$$
$$\frac{1}{2}mv^2 = \frac{2}{3}mg\Delta h$$

Equation 1

$$\boxed{E_{kinetic} = \frac{2}{3}E_{potential}}$$

When considering rotational kinetic energy:

$$E_{potential} = E_{kinetic} + E_{rotational}$$
$$mg\Delta h = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$
$$mg\Delta h = \frac{1}{2}m(r\omega)^2 + \frac{1}{2}I\omega^2$$
$$mg\Delta h = \frac{1}{2}mr^2\omega^2 + \frac{1}{2}I\omega^2$$
$$mg\Delta h = I\omega^2 + \frac{1}{2}I\omega^2$$
$$mg\Delta h = 3\left(\frac{1}{2}I\omega^2\right)$$
$$\frac{1}{2}I\omega^2 = \frac{1}{3}mg\Delta h$$

Equation 2

$$\boxed{E_{rotational} = \frac{1}{3}E_{potential}}$$

With reference to Equations 1 and 2, it can be deemed that the gravitational potential energy utilized in the cylinder's descent will always be in the ratio of one-third rotational kinetic energy to two-thirds linear kinetic energy, irrespective of the angular or linear velocity. Therefore, the ratio $E_{kinetic} : E_{rolling} = 2 : 1$ can be derived.

Here, the basic model of a cylinder rolling down an inclined plane has been described which accounts for friction, normal force, and weight. In reality, however, there are many other factors that can impact cylindrical motion in addition to these three effects. The primary goal of this paper is to investigate some of the other factors affecting the motion of a cylinder down an inclined plane. After conducting some preliminary research, I hypothesized certain variables that would have an impact on the cylindrical motion down an inclined plane, and this literature review further evaluates them. The factors that I shall be examining are the angle of inclination, mass and radius of the cylinder, and surface composition of the cylinder and incline. In the following sections, I elaborate on each of these factors. Such a study aims to help researchers form a strong foundation for future studies on cylindrical motion and assists in drawing correlations between the different variables that I have separately explored in this paper.

Angle of the Inclined Plane

The angle of inclination is defined as the angle formed by the x axis and a given line (slope) and is calculated counterclockwise from the positive x axis. When the angle of inclination is low, the cylinder will roll down in the absence of slipping. When slipping is absent, the equation $v = R\omega$ is fulfilled, wherein:
v → velocity of the object
R → radius of cylinder
ω → angular velocity

Angular velocity is defined as the time rate at which an object rotates, or revolves, about an axis, or at which the angular displacement between two bodies changes. In the absence of slippage, $\frac{dv}{dt} = \frac{Rd\omega}{dt}$. Hence, the equations for the motion of the cylinder can be derived as → $a = \frac{dv}{dt} = \frac{g \sin\theta}{1+\alpha}$ AND $\mu = \frac{\alpha \tan\theta}{1+\alpha}$ when there is a low angle of inclination, wherein 'a' is the acceleration of the cylinder, and 'g' is the acceleration due to gravity.

When the angle of inclination is high enough, the cylinder will slip and roll down at the same time. Slipping occurs when the object's center of rotation moves faster than velocity ratio, the rotation can't 'keep up', and the object slides over the surface. When the cylinder slips and rolls down the inclined plane at the same time, $v > R\omega$. In the presence of slipping, the equations for the motion of the cylinder are as following:

$\frac{dv}{dt} = g \sin\theta - \mu g \cos\theta$ AND $\frac{d\omega}{dt} = \frac{\mu g \cos\theta}{\alpha R}$

With reference to the equations of cylindrical motion in the presence of slippage as shown in the line above, if μ (coefficient of friction) exceeds the coefficient of sliding friction, then the ball will slip. Thus, there is a limiting angle θ_k, which is given by the formula $\tan\theta_k = \frac{(1+\alpha)\mu_k}{\alpha}$. Using this equation, the cylinder will roll down the inclined plane based on the corresponding value of μ_k. For example, if μ_k is 0.2, then the ball will only slip if θ is 35° or larger.

Based on an experiment conducted by Cross (2021), it was found that μ increases as the value of θ increases, until slipping occurs. If the persisting frictional force is large enough, then the ball can roll in the absence of sliding, which thus causes the acceleration down the incline to be reduced by a factor of $1 + \alpha$. In the presence of slipping, at high incline angles, the ball will roll and slip at the same time, which causes it to have a greater acceleration down the incline, compared to when there is solely rolling. However, the angular acceleration of the cylinder acts differently. The angular acceleration is 0 in the absence of friction. If there is no slipping, then the angular acceleration increases with an increase in the inclination (as θ increases) until slippage starts to occur. Once slippage commences to occur, then the angular acceleration begins to decrease as the value of θ further increases, and the linear acceleration increases.

The angular acceleration of the cylinder is proportional to its torque, which is the tendency of a force to turn or twist and is given by $FR = \mu mgR \cos\theta$. Thus, as the inclination (θ) increases to 90°, the torque decreases to 0. However, μ can only reach a maximum value of μ_k. At a small value of θ, the value of μ is small as well since only a little friction is required to make the cylinder roll. As the value of θ increases, the value of μ increases at a rate faster than the rate of decrease of $\cos\theta$. Thus, the torque keeps increasing until a point where the cylinder starts to slip. Once θ reaches a higher value, the value of μ remains constant which thereby causes the torque to decrease as the value of $\cos\theta$ is decreasing.

In the case where the cylinder rolls across a horizontal plane (0°incline), there would hypothetically be constant deceleration due to the presence of rolling friction, considering that the coefficient of friction remains constant. Rolling friction is the force resisting the motion of a rolling body on a surface. The equations $F_f = ma$ and $Ia = T_f - R_e F_f$ can be used to deduce the deceleration, where F_f is the frictional force required for rolling, m is the mass of the cylinder, a is the acceleration, and I is the moment of inertia with respect to the diameter.

Figure 2 – The effective radius R_e of the ball corresponds to the distance between its mass center and the instantaneous rotation axis (dotted line).

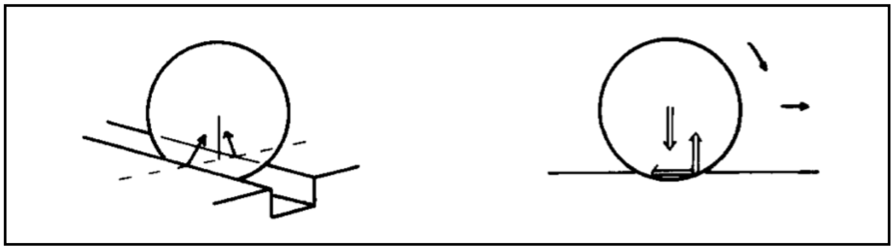


Figure courtesy of A. Doménech (1985)

With reference to the Figure 2, since $a = \alpha R_e$ (where α is the acceleration of the cylinder's mass center), and $N = mg(\frac{R_b}{R_e})$, the acceleration of the cylinder's mass center can be given by $a = g(\frac{(\frac{\rho}{R_e})(\frac{R_b}{R_e})}{1+(\frac{2}{5})(\frac{R_b}{R_e})^2})$, where ρ is the coefficient of rolling friction.

Mass and Radius of the Cylinder

The equation of the torque on a cylinder with respect to its point of contact with the inclined plane is given by the equation $\tau = mgR\sin\theta$ (where τ is the torque on the cylinder, m is the mass of the cylinder, R is the radius of the cylinder), which then leads to a change in the angular momentum which is given by $\frac{dL}{dt} = \tau$, where $L = I\omega$ gives the angular momentum. ($L \rightarrow$ angular momentum, $\omega \rightarrow$ angular velocity, $I \rightarrow$ moment of inertia). By combining the two torque equations above, I get $\frac{d\omega}{dt} = \frac{1}{I_{cp}}\frac{dL}{dt} = \frac{\tau}{I_{cp}} = \frac{mgR\sin\theta}{I_{cp}}$. The relation $v = \omega R$ (where v is the velocity of the cylinder) can be used in the above expression for angular acceleration ($\frac{d\omega}{dt}$) to derive an expression for the linear acceleration, which is: $a = \frac{dv}{dt} = R\frac{d\omega}{dt} = \frac{mR^2 g\sin\theta}{I_{cp}}$, where g is acceleration due to gravity. Thus, the acceleration of the rolling object can be written as $a = Cg\sin\theta$, ($C = \frac{mR^2}{I_{cp}}$). As a result, the acceleration for the cylinder becomes: $a_{cylinder} = \frac{5}{7}g\sin\theta$. This expression takes neither mass nor radius into account. Thus, it can be deemed that the mass and radius of the cylinder does not have an impact on the motion of the cylinder.

Surface Composition of the Cylinder

Sliding friction, also known as kinetic friction, is the friction between two bodies that are in sliding contact. The coefficient of sliding is defined as "a value that measures the force of sliding friction for a particular surface type" (Johnson and Linde, 2022). The coefficient of sliding helps to determine whether a cylinder would slide or roll down an inclined plane (if the coefficient is 0, then the cylinder slides; if the coefficient is 1, then the cylinder rolls). With reference to an experiment conducted by J. Bico and J, Ashmore-Chakrabarty (2009), when the cylinder is fully immersed in a viscous fluid, the fluid forms a thin film around the cylinder in order to change the composition of the cylinder's surface. Then, when the cylinder is placed on an incline plane, the coefficient approximately reaches a value of 0.25, contradicting the higher value of 0.6 when the cylinder is not dipped in the viscous liquid. As the velocity of the cylinder increased, it was observed that the sliding ratio was close to unity (equal to the original quantity) at lower velocities, and there was transient solid friction between the cylinder and the incline, which could be extrapolated from the partially erratic motion of the cylinder. Based on this experiment, it was found out that thicker lubricating layers resulted in increased sliding. Thus, it can be concluded that viscous liquids (composition) play an important lubricating role in the motion of a cylinder. Overall, the cylinder moved down the incline (coated with viscous fluid) with constant velocity. As the cylinder rolled- it slid as well, which verifies the lubricating role that was played by the viscous layer.

Discussion & Conclusion

In this literature review, I explored how different attributes of cylinders and inclined planes can impact the motion of a cylinder down an inclined plane. The factors I considered include angle of the inclined plane, mass and radius of the cylinder, and surface composition of the cylinder. The aim of this paper was to provide a mathematical explanation of how and to what extent these factors impact the motion of a cylinder down an inclined plane according to the latest literature in this area.

A limitation of this literature review is that I was unable to physically carry out experiments and get quantitative results for more in-depth analysis and observations, which is what I would have ideally liked to do. Moreover, due to the unavailability of a good online simulation of cylindrical motion down an inclined plane, I was unable to use a simulation to obtain data as well. This topic has a lot of future scope, and I could have explored the impact of other variables on the cylindrical motion, such as density, adhesion, deformation, and the height of the inclined surface. I would have also liked to investigate the distance reached by the cylinders after they have rolled to the bottom of the inclined surface. Moreover, I would have liked to investigate different types of motion of the cylinder, for example – linear motion, rotational motion, translational motion, and non-uniform motion. Furthermore, there is great potential to develop online simulations of cylindrical motion down an inclined plane, since it is currently not available.

In conclusion, studying this topic is of paramount importance due to the huge number of important real-life applications it carries along with it such as the motion of cartwheels, moving heavy objects on a ramp, and more. This literature review provides an in-depth analysis of the different factors that affect the motion of a cylinder down an inclined plane in the presence and absence of slippage, and I hope that it provides a strong basis for future, more in-depth studies.

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